Weibull Articles
From Dr. Robert B. Abernethy's Library

Dr. Robert B. Abernethy author of The New Weibull Handbook, (visit this hyperlink to download a PDF copy of Chapter 1 from his book) has a collection of Weibull articles in his library. Some of the articles in Dr. Bob's library were written directly by Ernst Hjalmar Waloddi Weibull. His library collections of papers are published below as downloadable PDF files. Many of the Weibull articles are copies from other bad copies, thus legibility is a little difficult in preserving these old, important, historical, documents pertaining to Weibull analysis.

How do you pronounce Waloddi Weibull's name? If you're-

Swedish-	Wa-Loddi Vay-Bull
Norwegian-	Wa-Loddi Vay-Bull
German-	Vah-Lodi Vy-Bull
French-	Wah-Lodi Way-Bull
Polish-	Vah-Lodi Vy-Bull
American-	Wah-Low-D Y-Bull

No one says Wee-Bull!

In the words of a popular song from years ago with the vegetable tomato spelled phonetically: "…you say tow-may-toes and I say ta-mat-toes….".

Dr. Bob has graciously allowed his Weibull documents to be scanned and published on this website in PDF format. The PDF format allows wide available to students and Weibull analysis practitioners around the world. You can download a PDF reader at no cost from the Adobe website or **upgrade** to the no-cost **Adobe Acrobat Reader** version **10.0** to correct the error messages.

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Weibull Articles Posted For PDF Downloads

The articles are listed in chronological order. PDF download files are hyperlinked.

1. **Waloddi Weibull** author, Professor, Royal Technical University, Stockholm, **1939**
   "**A Statistical Theory Of The Strength Of Materials**", Ingeniörsvetenskapsakademiens Handlingar Nr 151, 1939, Generalstabens Litografiska Anstalts Förlag, Stockholm.

   This document, in English, was provided by Isamu Yoshimoto, 1-26-23-301 Kamiyoga, Setagaya-ku, Tokyo, 158-0098, JAPAN to Dr. Abernethy. This document shows the mean rank

Y-axis plotting position used by Dr. Weibull. Later, Dr. Weibull adopted **Benard's median rank** plotting position.
Download this 2.8Meg PDF file which includes 45 pages.

2. **Waloddi Weibull** author, **1939**
"**The Phenomenon of Rupture in Solids**", Ingeniörs Vetenskaps Akademien-Handlingar, Nr 153, 1939, Generalstabens Litografiska Anstalts Förlag, Stockholm.

Document Search/Request:
If you have a copy of this document in English (or worst case in Swedish), please send a copy for posting and downloading to Paul Barringer.

3. **Waloddi Weibull** author, **1948**
"**En Statistisk Teori För Utmattmingshallfastheten**",
Report TM 344 written for AB Bofors (in Swedish) dated 2.9.**1948** [*If you could translate this to English, please send a copy to the address noted in Section 2 for posting*] This 23 page report is broken into three sections for easier download.
The first section includes pages 1-8
779KB PDF file (8 pages)
The second section includes pages 9-17
891KB PDF file (9 pages)
The third section includes 18-23
362KB PDF file (6 pages)

4. **Waloddi Weibull** author, **1949**
"**A Statistical Representation Of Fatigue Failures In Solids**",
Transactions Of The Royal Institute Of Technology, Stockholm, Sweden, Number 27,
This report is broken into four sections for easier download.
The first section includes the Introduction and Statistical Aspects
554KB PDF file (12 pages)
The second section includes Complete Fatigue Diagram and S-N Curves
633KB PDF file (17 pages)
The third section includes P-S Curves Size Effect, Computation of the Parameters, and Numerical Examples
790KB PDF file (15 pages)
The fourth section includes Appendix by Bengt W. Weibull, Punched-Card Methods
790KB PDF file (9 pages)

5. **Waloddi Weibull** author, **1950**
"Explosion of Spherical Charges in Air: Travel Time, Velocity of Front, and Duration of Shock Waves"
Abstract-
Travel times of shock waves emitted from bare spherical charges of high explosives have been measured. The experimental arrangements are described. Observed values may be reproduced with great precision by the formulae given in Figures 4 and 5.
The scaling laws have been verified over the whole range.
The relations between the front velocity and the ambient density of the surrounding atmosphere have been determined.
The front velocities of shocks from PETN and from different mixtures of TNT and A1 have

been determined. (Figure 8).

The resulting data do not verify the theories of Rüdenberg and Sedov.

Report Number: X-127
Referenced as AD-0704695
BALLISTIC RESEARCH LABS ABERDEEN PROVING GROUND MD
TI (6) EXPLOSION OF SPHERICAL CHARGES IN AIR: TRAVEL TIME, VELOCITY OF FRONT, AND DURATION OF SHOCK WAVES (Detonation av Klotformiga Laddningar iLuft; Gangtid, Fronthastighet och Vaglangd hos den Ustsanda Stotvagen)
Prof. Waloddi Weibull
Feb 1950, 27 Pages
Unclassified report
Trans. from Tidskrift fuer Kustartilleriet (Sweden) n1 1947, by Lars Niclas Enequist.
Download this 1.3 meg file as AD-0704695.

6. **Waloddi Weibull** author, **1951**
"A Statistical Distribution Function of Wide Applicability", **Journal Of Applied Mechanics** ASME Paper.
4 Meg PDF file (7 pages) **←This is THE hallmark Weibull paper and includes discussion in this one file!**
See Weibull's Example 1. ←Yield strength
See Weibull's Example 2 . ←Fly ash size distribution
- Also see Goren Weibull's comments about his father's 1951 paper from the Garfield Library classics of 1981 (thanks to Fritz Scholz for this reference).
- Also see the Birnbaum tribute by Sam Saunders , page xxi, top paragraph, of why the Weibull distribution may have gained much wide use (thanks to Fritz Scholz for this reference).

7. Benard, A and Bos-Levenbach, E. C. authors, **1953**
"*Het uitzetten van waarnemingen op waarschijnlijkdeids-papier" (The Plotting of Observations on Probability Paper)*, **Statististica Neerlandica**, Volume 7, pages 163-173, 1953. ***Translated from Dutch into English by Ronald Schop in 2001*** (In Weibull's later papers, Dr. Weibull acknowledged the superiority of using Benard's median rank equation as providing a better plotting position for ranked data on Weibull probability paper).
This paper is available for download as a 191 KB PDF file (8 pages)

8. **Waloddi Weibull** author, **May 1955**
"**Basic Aspects Of Fatigue**"
A paper presented at the Stockholm Colloquium On Fatigue, 6 pages.
9. **Waloddi Weibull** author, **September 1959**
"**Statistical Evaluation Of Data From Fatigue And Creep-Rupture Tests Part 1. Fundamental Concepts And General Methods**",
Wright Air Development Center Report 59-400, Report # AD229450—September **1959**, 1
This report is broken into four sections for easier download.
The first section includes Introduction, Classification of Test Series, and Statistical Methods and Tools.
1.438MB PDF file (25 pages)
The second section includes Estimation Of Distribution Parameters.
1.088MB PDF file (22 pages)
The third section includes Fitting Curves To Observations and Bibliography.
368KB PDF file (8 pages)

The fourth section includes Tables and Figures.
1.165MB PDF file (25 pages)

10. **Waloddi Weibull** author, **1960**
"**Size Effects On Fatigue Crack Initiation and Propagation In Aluminum Sheet Specimens Subjected To Stresses Of Nearly Constant Amplitude**", Report 86 to Flygtekniska Försöksanstalten The Aeronautical Research Institute Of Sweden.
Summary:
Fatigue tests on axially-loaded 24S-T and 75S-T aluminum sheet specimens of various sizes were conducted with a constant nominal stress amplitude, established by a stepwise reduction of the load in proportion to the remaining cross-sectional area.
Earlier observations were confirmed, showing that, in this type of test, the crack propagates with a constant, stable rate of growth after a---generally short---transition period has been passed. This transition period was found to increase with the number of cycles necessary to initiate a visible crack.
A law, deduced by means of dimensional considerations and stating that the stable rate of growth should be proportional to the size of geometrically similar specimens, was well substantiated, which implies that the relative rate of growth is, for a given stress amplitude and mean stress, a material constant.
For geometrically similar specimens, the period of crack initiation was observed to depend on the size of the notch and to increase with decreasing size, which is in accordance with the statistical theory of strength. Stockholm, June, 1960.
2.4Meg PDF file (30 pages)

11. **Waloddi Weibull** author, **1961**
Fatigue Testing And Analysis Of Results
published for and on behalf of Advisory Group For Aeronautical Research and Development North Atlantic Treaty Organization..
This old book is broken into chapters for easier downloading as PDF files.
The first section contains a title page with a hand written note from Waloddi Weibull "To Professor A. M. Freudenthal with best wishes from the author." , Table of Contents, Forward by Theodore von Kármán, Chapter 1 on Symbols and Nomenclature, Chapter 2 on Fatigue Testing Methods
1.833MB PDF file (18 pages)
The second section contains Chapter 3 on Fatigue Testing Machines And Equipments
3.075MB PDF file (21 pages)
The third section contains Chapter 4 on Instruments And Measuring Devices
1.063MB PDF file (8 pages)
The fourth section contains Chapter 5 on Test Pieces: Design, Preparation, Measurement And Protection
799KB PDF file (6 pages)
The fifth section contains Chapter 6 on Factors Affecting Test Results
2.878MB PDF file (20 pages)
The sixth section contains Chapter 7 on Planning Of Test Programmes and Chapter 8 on Presentation Of Results [*Weibull probability plots appear in this section*]
3.179MB PDF file (27 pages)
The seventh section contains Chapter 9 on Analysis Of Results [Dr. Weibull shows how to transform two-dimension S-S_e into a one-dimensional function, and he mentions Benard's median rank, discusses graphical methods for plots, etc.]

3.642MB PDF file (33 pages)
The eight section contains the Bibliography
3.758MB PDF file (28 pages)

12. **Waloddi Weibull** author, **August 1962**

The Effect Of Size And Stress History On Fatigue Crack Initiation And Propagation

Abstract-

The first part of investigation deals with the effect of size and preloading on the duration N_i of the crack initiation period. Geometrically similar sheet specimens of two different aluminum alloys were subjected to various load cycles. It was found that each size had its individual S-N_i curve, considerable differing from those of other sizes. A reduction to one and the same size by means of the Neuber stress concentration factor K_N was only partially successful. A static preload was found to increase the value of N_i from 11 kc to 205 kc.

The second part of the investigation deals with the propagation period. Equations relating crack length to number of cycles are derived for two alternatives: constant stress cycles and constant load cycle applied to a sheet specimen. The formulas are verified by tests including various combinations of material, size, and stress amplitude.

In the first alternative, it was found that the rate of crack growth is, independently of the crack length, constant after a transition period has been passed. The duration of this period is dependent on the duration of the preceding initiation period. For small values of N_i it does not even exist.

In the second alternative a convenient method for interpreting the results was obtained by plotting crack length vs. number of cycles. It was found that the propagation period starts with a transition period, followed by one, two, or even three stable propagation periods, the number depending on the magnitude of the applied load. As examination of the broken specimens showed that these periods correspond to different fatigue mechanisms.

It is concluded that total fatigue life cannot be predicted without considering separately the parts of which it is composed. Consequently, it cannot be expected tat the shape of the conventional S-N curve that relates total life to applied load can be corrected to relevant testing conditions.

Forward-

This report was prepared by Prof. Dr. Waloddi Weibull, Bockamöllan, Sweden under USAF Contract No. AF 61(052)-522. The contract was initiated under Project No. 7351, "Metallic Materials", Task No. 735106, "Behavior of Metals." The contract was administered by the European Office, Office of Aerospace Research. The work was monitored by the Directorate of Materials and Processes, Aeronautical Systems Division, under the direction of Mr. W. J. Trapp.

This report covers work conducted during the period September 1960 to September 1961.

Report Number:
ASD-TDR-62-785
Originating Activity:
Bockamöllan, Sweden
Referenced as:
AD-287962 / xag
Thanks to Sam Chan of The Boeing Company for finding this document.
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13. **Waloddi Weibull** author, **February 1963**

Outline Of An Algebra Of Stochastic Quantities

Abstract-

The aim of the work reported has been to develop methods of solving random equations, that is, equations involving variates (random variables). The main difficulty of this task arises from the fact that no variate, if not degenerate, is invertible, or, algebraically expressed, even if the set V of variates is a commutative monoid under both addition and multiplication, it does not constitute a field.

For this purpose a set S of elements, called stochastic quantities (for brevity, stochastics), of which V is a subset, has been constructed with the property that it constitutes a field. This implies that there exists for every element of it an inverse element relative to both the additive and multiplicative laws of composition, and thus it will be possible to compute with the stochastics just as easily as is done with the rational numbers with respect to the four fundamental operations +, -, •, :.

Considering a variate as a finite or infinite set of ordered pairs, denoted by f(x) [x], where the first projection f(x) is a real-valued, non-negative function, defined for a continuous set of values of x or for an at most denumerable set of points x_i and interpreted as a mass density or as discrete parts of a unit mass, respectively; and the second projection [x] is anyone of the values that the variate can take, the notation of a stochastic is $f(x) \bullet j_z^n[x]$, where f(x) is a real valued, positive or negative, function and the symbol $j_z^n[x]$ is defined, for n = 1, by $j_z[x] = (1/dz)[x] - (1/dz)[x+dz]$. Thus j_z may be interpreted as a duplex mass, composed of two infinitely large masses (1/dz) and –(1/dz) located at an infinitesimal distance dz from each other. For n = 2 we have $j_z^2[x] = (1/dz^2)[x] - (2/dz^2)[x+dz] + (1/dz^2)[x+2 \bullet dz]$ and j_z^2 may be interpreted as a triplex mass, composed of three infinitely large masses at an infinitesimal distance dz from each other, and so on for arbitrary values of n. Since, by definition, $j_z^0[x] = 1[x]$, the general expression includes the variates as a special case obtained by setting n = 0.

From the definition above it follows, if f(x) is a continuous function, that $f(x) \bullet j_z^n[x]$ is equal to the ordinary derivative $d^n(fx))/dxn$. Thus, j_z^n can be regarded not only as a multiplex mass but also as an operator. In the same way, j_x^{-n}, defined as the inverse of j_x^n, can be interpreted both as a mass distribution and as a repeated integration, further, the operator $d^n/dx^n = j^n$ may be defined, as is demonstrated, also for the general case that n^x is an arbitrary real number.

Since some problems leading to random equations have been presented, general properties of variates and multiplex stochastics are indicated. Based on the know laws of composition of variates, corresponding laws and some general theorems valid for multiplex stochastics have been deduced. Owing to the dual nature of the symbol j^n, simplified methods for composition and inversion of variates can be developed as is demonstrated. Finally, classification and some solutions of random equations and criteria for the existence of real roots are indicated.

Forward-

This report was prepared by Prof. Dr. Waloddi Weibull, La Rosiza, Lausanne, Switzerland under USAF Contract No. AF 61(052)-522. The contract was initiated under Project No. 7351, "Metallic Materials", Task No. 735106, "Behavior of Metals". The contract was administered by the European Office, Office of Aerospace Research. The work was monitored by the Directorate of Materials and Processes, Aeronautical Systems Division, under the direction of Mr. W. J. Trapp.

The report covers work conducted during the period February 1960 to August 1962. [Formulas presented in this ASD-TRD-63-63, also referenced as AD-298991, were used in the March 1969 report AF61(052)-522 as worked out examples, also referenced as AD-816012.]

Report Number:
ASD-TDR-63-63
Originating Activity:
La Rosiaz
Lausanne, Switzerland
Referenced as:
AD-0298991
Thanks to Sam Chan of The Boeing Company for finding this document.
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14. **Waloddi Weibull** coauthor—**December 1963**
"First Seminar On Fatigue And Fatigue Design", by A. M. Freudenthal, W. Weibull, and A. O. Payne, Sponsoring Agencies: Office of Naval Research, Air Force Materials Laboratory, Advanced Research Projects Agency, Contract No. NONR 266(91), Project NO. NR 064-470, Technical Report No. 2, Columbia University Department Of Civil Engineering And Engineering Mechanics, December 1963
The report is broken into six sections for ease of download:
1. **Fatigue Mechanisms and Fatigue Damage Accumulation** by A. M. Freudenthal, Cover page through page 23, 1.3 Meg PDF file. Provides an introduction and the state of fatigue in 1963.
2. **Fatigue Crack Propagation In Sheet Specimens** by Waloddi Weibull, page 24 through page 52, 1 Meg PDF file.
Discusses a failure model for fatigue crack propagation which is independent of the crack length.
3. **Fatigue Design And Reliability** by A. M. Freundenthal, page 53 through page 67, 0.5 Meg PDF file.
Discusses load-strength interference and structural reliability under conditions of fatigue with a probabilistic approach to safety analysis.
4. **Analysis Of Fatigue Test Results** by Waloddi Weibull, page 68 through page 89, 0.7 Meg PDF file.
Discusses probit tests for life and stress levels and the three parameter Weibull equation to model fatigue life or fatigue strength.
5. **Fatigue Of Structures** by A. O. Payne, pages 90 through page 137, 2 Meg PDF file.
Discusses fatigue failures for aircraft, welds and welded construction fatigue, fatigue of welded pressure vessels, riveted aluminum fatigue, notched material fatigue, effects on preloads on fatigue life [Mustang wings refer to the P-51 airplane], safe life structures modeled by the log normal distribution, submarine hulls fatigue life, tanker trucks fatigue life,
6. **Appendix**, pages 138 through page 172, 1.1 Meg PDF file.
Contains the graphs and tables associated with the Fatigue of Structures paper. Appendix I discusses Methods of Improving the Fatigue Resistance of Welds. Appendix II discusses Design of Program Load Test For Aircraft Wing, Tables, and Figures.

Report Number:
TR-2
Referenced as:
AD-619075 / xag – also see AD-611414

Thanks to Sam Chan of The Boeing Company for finding this document.

15. **Waloddi Weibull** author—**March 1967**
"Estimation Of Distribution Parameters By A Combination Of The Best Linear Order Statistic Method And Maximum Likelihood".
Abstract-

This report consists of three parts, the first one dealing with the unbiased, minimum-variance estimation of location scale parameters, assuming the shape parameter to be known, the second one presenting formulae for computing the likelihood of a given sample, the third one specifying the estimation procedure. The first part develops general formulae for computing the coefficients of linear estimators, composed of all or part of the elements of a random sample. These formulae are specialized for the cases of exponential distributions and also for estimations, using two of the order statistics only. Formulae for expected values, variances and covariances of standardized Weibull order statistics are deduced and applied to a system of equations, which determines the linear coefficients. For the solution of such systems, a program has been written and applied to a IBM 7090 computers, which delivers the results extremely fast, thus eliminating the need of extensive tables. Tables of expected values and covariance matrices are presented for sample sizes n = 5, 10, 15, 20 and α = 0.1, 0.3, 0.5, 0.7, 0.9, 1.0, useful when no computer is available. The second part presents formulas for computing the likelihood of a given sample for the most general situation, that is, for arbitrarily censored, truncated or grouped samples, and, for the special case of life testing, when the sample may be composed of one subset of items, which have failed after observed time units, a second subset of items, which have accumulated observed time units, without failure, and a third subset of items, which have failed during one or more inspection periods, without knowing their exact life times. The third part defines the procedure of combining the preceding formulas for best estimation, when none of the parameters is known.

[Formulas presented in this AFML-TR-67-105, also referenced as AD-816206, were used for in report AFML-TR-69-143, also referenced as AD-689407.]

USAF report AFML-TR-67-105 / AF 61(052)-522 —**1967**
This report is broken into two parts for easier download.
The first is a narrative of the BLUES (best linear unbiased estimate)
581KB PDF file (16 pages)
The second contains tables
597KB PDF file (15 pages)
AD-816206 / XAG

16. **Waloddi Weibull** author, **June 1967**
"The Order Statistics $y_i = \log(z_i^m)$, Their Properties And Use For Parameter Estimation (z = standardized Weibull variate)"
USAF AFML-TR=67-161; indexed as report AD-818104
Abstract-

Pertinent formulas for the confidence limits, expected values, variance and covariances of the order statistics $y_i = \log(z_i^m)$ have been developed and used for application of the generalized least-squares method, resulting in unbiased, minimum-variance estimates of the distribution parameters. Approximation formulae, based on simplified covariance matrices, have been proposed and examined, Extensive tables of the required statistics, computed by use of an IBM 7090 computer, are presented.

The first part contains Table Of Contents and Symbols 184KB PDF file (6 pages)

The second part contains the body of the report in Sections 1-3. 952KB PDF (26 pages)
The third part contains Tables 1: Values of Percentage Points, and Table 2: Differences between Benard and Exact Percentage Points 644KB PDF file (8 pages)
The fourth part contains Table 3: Values of Percentiles $y_{i,p}$; N=1(1)25 533KB PDF file (6 pages)
The fifth part contains Table 4: Expected Values E(yi), Percentage Points P-bar-i and Variances 1,122KB PDF file (17 pages)
The sixth part contains Table 5: Values of Correction Term, Table 6: Accuracy of Proposed Formulas for Plotting Positions, and Table 7: Coefficients ai, bi of Linear, Unbiased, Minimum Variance Estimators: N = 2(1)5;10;20 145KB PDF file (3 pages)

Report Number:
AFML-TR-67-161
Originating Activity:
La Roziaz
Lausanne
Referenced as:
AD-818104
Thanks to Jim Breneman of Pratt & Whitney for finding this document.

17. **Waloddi Weibull** author, **June 1967**
"Estimation Of Parameters From Large Samples Arbitrarily Censored Or Truncated"

Abstract-
Approximation formulas for the expected values, variances and covariances of the order statistics y_i, which provide a very good approximation for sample sizes equal to or larger than N=20, are developed. Their applications to graphical analysis and parameter estimations by use of desk computing machines and digital computers are demonstrated. Tables which simplify the computing procedure are presented.

Report Number-
AFML-TR-67-197

Referenced as-
AD-0657318
Waloddi Weibull LAUSANNE (SWITZERLAND)
Download this 1.1 Meg file as AD-0657318.

18. **Waloddi Weibull** author, **December 1967**
"Moments About Smallest Sample Value"

Abstract-
A new type of moments has been achieved by substituting in the central moments the smallest value of the sample for its mean. The new moments have the same advantage as the central moments of being independent of the location parameter but for certain value of the shape parameter they have less variance and thus are preferable for estimating purposes. The asymptotic properties of four estimators, three of them composed of the new moments and one of them of central moments have been examined. It could be concluded that for the shape parameter $\alpha \geq 0.5$ the estimator, which was composed of the first and second order moments of the new type, was by far the most efficient one. Small-sample properties of the new-moments estimators

have been appraised by use of extensive Monte-Carlo studies and it could be stated that the same conclusion applies also to small and moderated sample sizes.

Report Number-
AFML-TR-67-375 Referenced as AD-0664049
Waloddi Weibull LAUSANNE (SWITZERLAND)
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19. Allis/Seimens, **1968**
"**Instructions For How To Perform A Weibull Analysis By Hand**"—**1968**.
This report is broken into three parts for easier download.
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Unfortunately, every other page is missing from the original file!!

20. **Waloddi Weibull** author, **1969**
A List Of USAF reports
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21. **Waloddi Weibull** author, **1969**
A list of books and papers
264KB PDF file (7 pages)

22. **Waloddi Weibull** author, **March 1969**
"**Composition And Decomposition Of Bounded Variates With Special Reference To The Gamma And The Weibull Distributions"**.
USAF report AF61(052)-522; indexed as report AD-816012
Abstract-
The algebra published in Technical Report No. ASD-TDR-63-63 has been further developed, and its use has been illustrated by some worked examples. After some modifications of the notations, the differentiation and integration of stochastics, including the variates as a special case, have been more thoroughly examined, in particular with respect to the concept of broken derivatives and integrals. A generalized distribution function has been set up. By proper specification of its two shape parameters, it can be brought to reproduce the density functions of the Exponential, Gamma, Pearson Type III, Chi-square, Rayleigh, Weibull, and some more distributions of practical importance. This general function has been expanded in a power series which is transformed in a series, called the integral series. Based on these formulae, rules of summation and multiplication of independent variates are presented and applied to some distributions. Inverse addenda for various variates have been developed and used for decomposition of sums of Gamma and Weibull variates.

Forward-
This report was prepared by Prof. Dr. Wallodi Weibull, Lausanne, Switzerland, under USAF Contract No. AF 61(052)-522. The contract was initiated under Project No. 7351,

"Metallic Materials", Task No. 735106, "Behavior of Metals". The contract was administered by the European Office, Office of Aerospace Research. The work was monitored by the Air Force Materials Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp.

This report covers work conducted during the period February 1964 to February 1965. The manuscript was released by the author in November 1966 for publication as an AFML Technical Report.

[Worked out formulas presented in this March 1967 report AF61(052)-522, also referenced as AD-816012, were derived from the February 1963 report ASD-TDR-63-63, also referenced as AD-298991.]

The first part contains the Abstract, Table of Contents, and the Introduction
227KB PDF file (5 pages)

The second part contains Notations For Discontinuous Functions, Notations For Variates And Other Stochastics, Differentiation and Integration Of Stochastics
582KB PDF file (12 pages)

The third part contains A Generalized distribution Function, Expansions In Power Series and Integral Series, Composition Of Independent Variates, Decomposition Of Sums Of Independent Variates, and Tables
531KB PDF file (11 pages)

Thanks to Jim Breneman of Pratt & Whitney for finding this document.

23. **Waloddi Weibull** author, **April 1969**
"A Criterion for The Acceptability Of Assumed Distributions"

Abstract-

The usual way to solve the fundamental problem of deciding whether an assumed distribution function is acceptable or not, consists – if at all done – in estimating the parameters and checking the attained goodness of fit by some accepted criterion, in most cases the Chi-square test. In this way, the decision depends not only on the assumed function, so it may happen that an acceptable function may be rejected on the basis of results from poor estimating or fitting procedures.

The purpose of this research was to find a criterion which eliminates such fatalities and depends entirely on the assumed function alone. Such a criterion, based on the "number-of-runs" has been proposed. The properties of this statistic and its usefulness as a measure of departure from the true distribution have been demonstrated. The concept "maximum number of runs" (MAXNOR) of a given sample and methods for its ascertaining have been introduced. Its use as a criterion for deciding whether the assumed function is acceptable or not has been studied by applying it to data from tests on strength of brittle materials, fatigue life of aluminum alloys, etc.

Forward-

This report was prepared by Prof. Dr. Waloddi Weibull, La Rosiaz, Lausanne, Switzerland, under USAF Contract No. AF 61(052)-943. The contract was initiated under Project No. 7351, "Metallic Materials", Task No. 7351, "Metallic Materials", Task No. 735106, "Behavior of Metals". The contract was administered by the European Office, Office of Aerospace Research. The work was monitored by the Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp.

This report covers work conducted during the period February 1968 to December 1968.

The manuscript of this report was released by the author January 1969 for publication as a technical report.

Report Number:
AFML-TR-69-124
Originating Activity:
La Rosiaz
Lausanne, Switzerland
Referenced as:
AD-689406 / xag
Thanks to Sam Chan of The Boeing Company for finding this document.
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24. **Waloddi Weibull** author, **April 1969**
”**The Efficiencies Of Unbiased, Linear Estimators For Scale and Location Parameters Composed Of One, Two, Or Three Order Statistics**”
Abstract-

The result of this investigation is, from a practical point-of-view, important in so far as it proves that the procedure of estimating scale and location parameters can be simplified by suppressing a large part of the sample without much loss in efficiency provided that the proper ordered observations are used. These order numbers are presented for estimators composed of one, two, or three observations. It is remarkable that neglecting, for instance 39 out of 40 observations reduces the efficiency by 35% only. The actual reduction in efficiency, being in many cases quite small when using two or three observations, can be read from the tables.

[Formulas presented in AFML-TR-67-105, also referenced as AD-816206, were used for this report.]

Forward-

This report was prepared by Prof. Dr. Waloddi Weibull, Lausanne, Switzerland, under USAF Contract No. AF 61(052)-943. The contract was initiated under Project No. 7351, “Metallic Materials”, Task No. 735106, “Behavior of Metals”. The contract was administered by the European Office, Office of Aerospace Research. The work was monitored by the Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp.

This report covers work conducted during the period February 1967 to February 1968. The manuscript of this report was released by the author March 1968 for publication.

Report Number:
AFML-TR-69-134
Originating Activity:
La Rosiaz
Lausanne, Switzerland
Referenced as:
AD-689407 / xag
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25. **Waloddi Weibull** author, April 1969
”Moment Estimators For Weibull Parameters And Their Asymptotic Efficiencies”

USAF report AFML-TR-69-135; indexed as AD-690162
Abstract-

The classical method of moments for estimating distribution parameters which consists in equating as many of the populations as the number of unknows to the corresponding sample moments has been much appreciated, because it is quite easy to use and does not need any ordering of the observation. However, in some cases its efficiency is very poor, so it has to be used with some precaution. In order to elucidate this statement, formulas for the asymptotic efficiency of the most used estimators have been derived for the alternatives of one, two and three unknown parameters. Numerical values corresponding to several values of α and for the cases of one or two unknown parameters have been computed and are presented.

Forward-

This report was prepared by Prof. Dr. Waloddi Weibull, Lausanne, Switzerland, under USAF Contract No. AF 61(052)-522. The contract was initiated under Project No. 7351, "Metallic Materials," Task No. 735106, "Behavior of Metals". The contract was administered by the European Office, Office of Aerospace Research. The work was monitored by the Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the Direction of Mr. W. J. Trapp.

This report covers work conducted during the period 1964. The revised manuscript was released by the author August 1968,
This report is 535KB PDF file (15 pages)
Thanks to Jim Breneman of Pratt & Whitney for finding this document

26. **Waloddi Weibull** author, April 1969
"A General Method For Estimating Distribution Parameters"
Abstract-

The method presented is applicable to complete, censored, or truncated samples and to grouped data drawn from any population having a continuous distribution function, simple or composed, involving an arbitrary number of unknown parameters. The estimates are consistent and asymptotically efficient (in some cases for any sample size) and easily determined by use of a versatile computer program. The efficiency can be stated for any individual case, even when only a part of the sample is used for the estimation. Two criteria of goodness-of-fit, which complete each other, makes it possible to decide whether the fit attained is acceptable or not.

Two applications may be mentioned: the evaluation of data from bending and torsional tests on brittle materials, a problem up-to-now not quite satisfactorily solved due to the complicated distribution functions arising; and the analysis of bimodal fatigue-life distributions.

Forward-

This report was prepared by Prof. Dr. Waloddi Weibull, Lausanne, Switzerland, under USAF Contract No. AF 61(052)-943. The contract was initiated under Project No. 7351, "Metallic Materials," Task No. 735106, "Behavior of Metals". The contract was administered by the European Office, Office of Aerospace Research. The work was monitored by the Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the Direction of Mr. W. J. Trapp.

This report covers work conducted during the period April 1967 to April 1968. The revised manuscript was released by the author June 1968 for publication.

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AFML-TR-69-136

Originating Activity:
La Rosiaz
Lausanne, Switzerland
Referenced as:
AD-689405 / xag
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27. **Waloddi Weibull** author, **February 1971**
Outline Of A Theory Of Powerful Selection Of Distribution Function
Abstract-

The conventional method of analyzing a given set of test data consist in assuming a distribution function and estimating its parameters. The only way of deciding whether the function is acceptable or not and which of two assumed functions is the better one is by means of a test of goodness-of-fit. For small and moderate sample sizes this test makes a very unreliable basis for a decision, and the confidence that can be put in the choice is practically unknown.

In order to eliminate these deficiencies a new method, called the method of powerful selection, is proposed. By use of a test statistic, called the selector, it is possible, without preceding parameter estimators, to state the acceptability of a function on the basis of a pre-assigned level-of-significance and the decision power, that is, the chance of making a true decision between two functions. The tools of this method are presented and their applications illustrated by numerical analyses of some fatigue-test series.

It will not too seldom occur that none of several assumed functions will be accepted. In this situation the selectors can be used for diagnosing the rejected functions with regard to causes such as outlying observations, composed populations, and contaminated data.

Forward-

This report was prepared by Prof. Dr. Waloddi Weibull, La Rosiaz, Lausanne, Switzerland under USAF Contract No. F61052-69-C-0029. This contract was initiated under Project No. 7371, "Metallic Materials", Task No. 735106, "Behavior of Metals". The contract was administered by the European Office, Office of Aerospace Research. The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson AFB, Ohio, under the direction of Mr. W. J. Trapp.

This report covers work conducted from 1 February 1969 to 28 February 1971. The manuscript of this report was released by the author February 1971 for publication.

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15 Ch. Fontanettaz
1012 Lausanne, Switzerland
Referenced as:
AD-725037 / xag
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28. **Waloddi Weibull** author, **June 1972**
The Concept Of Pseudo-Standardized Variables and Its Use As Elements Of Shape Operators

Abstract-

The concept of pseudo-standardized variable is explained and the fundamental properties of this variable are indicated. Its most important property of being scale and location invariant makes it useful as elements of shape operators, and its space being equal to the closed interval (0,1) has practical advantages.

Four types of shape operators are defined and examined. Twenty-five tables which simplify their practical applications have been prepared and are presented. Two examples concerning data of rotating beam fatigue performance illustrate the different numerical procedures.

Forward-

The research work reported herein was conducted by Prof. Dr. Waloddi Weibull, Chemin Fontanettaz 15, 1012 Lasusanne, Switzerland under USAF Contract No. F44620-72-C-0028. This contract, which was initiated under Project No. 7351, "Metallic Materials", Task 735106, "Behavior of Metals", was administered by the European Office, Office of Aerospace Research. The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp, AFML/LL.

This report covers work conducted during the period 1 February 1971 to May 1972. The manuscript was submitted by the author for publication in June 1972.

Report Number:
AFML-TR-73-98
Later identified as **AD-764361**
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29. Announcement of Waloddi Weibull receiving **1972** ASME Medal for distinguished engineering achievement at the Society's Winter Annual Meeting November 26-30, 1972. The announcement also includes a brief biography.
93KB PDF file. (2 pages)

30. **Waloddi Weibull** author, **May 1973**
The Rank-Score Test-An Improvement Of The Rank Sum Test

Abstract-

The hypothesis that two random samples are from identically distributed but unknown populations can be tested by use of the rank-sum as the test statistic. It is proposed to substitute for it the rank-score, which, as pointed out. Supplies more information about the populations of the samples under examination.

This new statistic is defined and its main properties are indicated. Tables for its application to pseudo-standardized samples have been established by use of two computer programs, thereby enabling tests of the hypothesis that the samples are from populations with identical distribution function, including its shape parameter, if any, but possible different scale and location parameters.

The practical use of the new test has been demonstrated by numerical examples concerning samples of fatigue test data from a large collection prepared at The Boeing Company.

Forward-

The research work reported herein was conducted by Prof. Dr. Waloddi Weibull, Chemin Fontanettaz 15, 1012 Lausanne, Switzerland under USAF Contract No. F44620-73-C-0066. This

contract was administered by the European Office of Aerospace Research. The contract, which was initiated under Project No. 7351, "Metallic Materials:, Task 735106, "Behavior of Metals", as administered by the European Office of Aerospace Research. The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp, AFML/LL.

This report covers work conducted during the period 15 March 1973 to 15 April 1973. The manuscript was submitted by the author for publication in May 1973.

Report Number:
AFML-TR-73-203
Originating Activity:
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Referenced as: **AD-769451** / xag
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31. **Waloddi Weibull** author, **May 1973**
A New Test Operator, VJ, Based On Class Frequencies

Abstract-
The test statistic X^2 of the Chi-square test may, if applied to a proper sample, be used for selecting distribution functions. When examining its use for this purpose its decision power was found to be very small due to a kind of pooling, an inherent property of its definition. In order to eliminate this pooling, a new test statistic, denoted by VJ, was introduced. It is defined by the number v_i of sample elements which fall within each of r properly defined classes into which the space of the variable x has been divided. In fact X^2 may be regarded as a statistic obtained from VJ by a pooling procedure. For this reason VJ was expected to have a much larger decision power than X^2 as was verified by the example that the decision power for a specified case being 6.6% for X^2 was raised to 69.1% for VJ.

The properties of VJ have been thoroughly examined. In particular the class limits yielding the largest decision power have been determined with the result that, in some cases, the decision power was found to be somewhat larger than anyone so far attained.

The statistic VJ can also be used for stating whether a hypothetical distribution is acceptable or not and also for selecting the most probable one within a set of such distributions. Necessary tables for the practical use have been prepared.

Forward-

The research work reported herein was conducted by Prof. Dr. Waloddi Weibull, Chemin Fontanettaz 15, 1012 Lausanne, Switzerland under USAF Contract No. F44620-72-C-0028. This contract was administered by the European Office of Aerospace Research. The contract, which was initiated under Project No. 7451, "Metallic Materials:, Task 735106, "Behavior of Metals", as administered by the European Office of Aerospace Research. The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp, AFML/LL.

This report covers work conducted during the period 1 February 1971 to 30 April 1972. The manuscript was submitted by the author for publication in May 1972. [This date may be incorrect as the Document Control Data – R&D transmittal document shows May 1973]

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32. **Waloddi Weibull** author, **May 1973**
The EKS-Square Test Of Goodness Of Fit—An Improvement Of The Chi-Square Test
Abstract-

When applying the classical Chi-square test of goodness of fit, it is always assumed that the test statistic is χ^2-distributed. Since this is true only for very large samples, some restrictions on the class frequencies have to be introduced. It is generally accepted that none of the expected frequencies should be less than ten, which makes this test useless for small and moderate samples.

In order to eliminate these – from a practical viewpoint server – restrictions, it is proposed to use the exact sampling distribution instead of the limiting χ^2-distribution. When doing so, the test will be called the Eks-square test.

Programs have been written for computing these distributions and the improvements attained have been stated.

The possibilities of using the modified test statistic as a location, scale, and shape operator have been examined and illustrated by numerical examples. Several tables have been prepared.

Forward-

The research work reported herein was conducted by Prof. Dr. Waloddi Weibull, Chemin Fontanettaz 15, 1012 Lausanne, Switzerland under USAF Contract No. F44620-72-C-0028. This contract was administered by the European Office of Aerospace Research. The contract, which was initiated under Project No. 7351, "Metallic Materials:, Task 735106, "Behavior of Metals", as administered by the European Office of Aerospace Research. The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp, AFML/LL.

This report covers work conducted during the period 1 February 1971 to 25 February 1972. The manuscript was submitted by the author for publication in March 1972. [This date may be incorrect as the Document Control Data – R&D transmittal document shows May 1973]

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Originating Activity:
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Referenced as: **AD-762546** / xag
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33. **Waloddi Weibull** author, **August 1973**
The Concept Of Score Of A Random Sample

Abstract-

To any give random sample there may be assigned a number called its score and denoted by $SC(r,N_{os})$, where r = the number of classes into which the space of the random variable has been divided and N_{os} = the number of order statistics actually used. It is easily determined from the sample elements and offers some definite advantages as a test statistic for selecting the most probable population from which the given sample has been drawn. Its decision power tends with increasing r to the largest power attainable for the given sample size. By means of some versatile computer programs the sampling distributions for several combinations of r and N_{os} have been determined. Tables have been prepared from which the probabilities of twelve different hypothetical populations can be immediately read and their acceptability stated.

Foreword-

The research work reported herein was conducted by Prof. Dr. Waloddi Weibull, Chemin Fontanettaz 15, 1012 Lausanne, Switzerland under USAF Contract No. F44620-72-C-0028. This contract, which was initiated under Project No. 7351, "Metallic Materials", Task No. 735106, "Behavior of Metals", was administered by the European Office, Office of Aerospace Research. The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp, AFML/LL.

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1012 Lausanne, Switzerland
Referenced as:
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34. **Waloddi Weibull** author, **1973**

Scientific Reports & Lectures prepared for the U.S. Air Force on contract-**1973** (with Waloddi Weibull's signature).

341KB PDF file (5 pages)

35. Dumonceaux & Antle's paper, **1973**

"**Discrimination Between the Log-Normal and the Weibull Distribution**"
This paper shows 20 samples are needed to distinguish between Weibull distributions and log-normal distributions—**1973**.

294K PDF file (4 pages)

36. **Waloddi Weibull** works and biography--**1975**

A translation into English by Stig Elg for the Swedish Academy of Engineering Sciences.

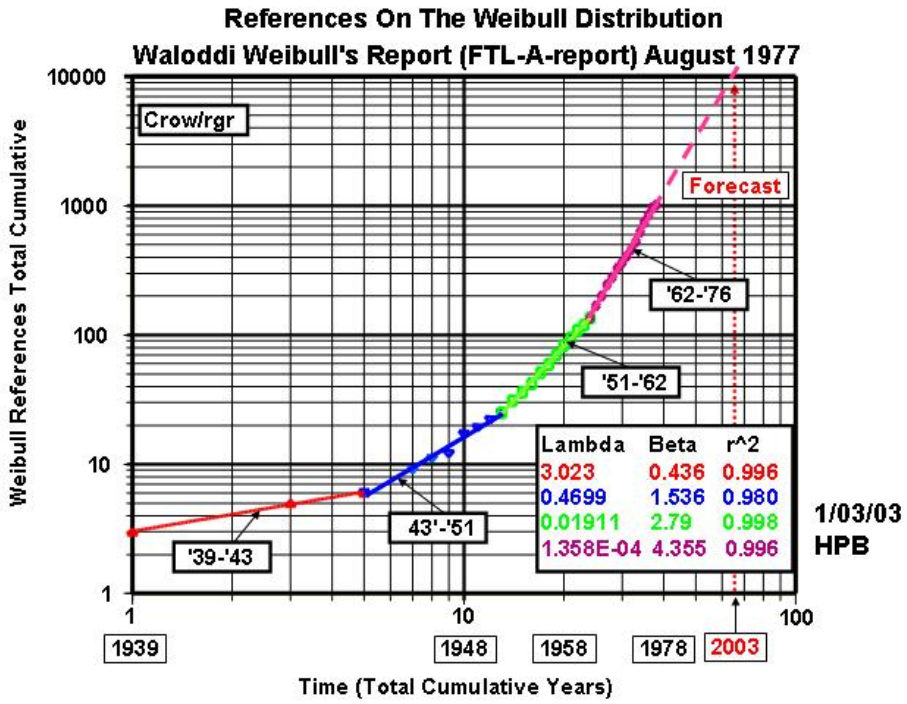
397KB PDF file (5 pages)

37. **Waloddi Weibull** Summary of papers on the Weibull distribution--**1977**

"**References On The Weibull Distribution**", Försvarets Teletekniska Laboratorium [Stockholm],

FTL A-report, August 1977 [contains 1019 references to the Weibull distribution]. Thanks to Dr. Glenn Bowie, retired scientist from the Structures and Materials Laboratory at Lockheed-California, Burbank, CA and his company CorTech Training, Red Wing, MN who loaned his report for conversion into the PDF files listed in this section. You will find excerpts from Dr. Bowie's website at the bottom of this page concerning private conversations he had with members of the Weibull family.

0.3 MB PDF File (pages cover-5) introduction
0.8 MB PDF File (pages 6-20) reference 1-115
1.1 MB PDF File (pages 21-40) reference 116-286
1.2 MB PDF File (pages 31-60) reference 287-453
1.1 MB PDF File (pages 61-80) reference 454-617
1.1 MB PDF File (pages 81-100) reference 618-785
1.1 MB PDF File (pages 101-120) reference 786-951
1.0 MB PDF File (pages 121-141) reference 952-1019 + Table of classified references + cumulative references versus cumulative time. Page 141 contains a graph of cumulative references to cumulative time. Putting this data into a Crow/AMSAA format, we can forecast that in 2003 we should expect the list of references citing Weibull's work will reach 10,000 documents.



38. **Waloddi Weibull** author, **September 1977**

The Probability Of Failure Of A System Subjected To The Joint Effect Of Cyclic Loading And Randomly Distributed Discrete Load Peaks

Abstract-

If a specimen is subjected to some kind of cyclic loading with a maximum load level S, which is larger than the fatigue limit of the specimen, then the strength of the specimen will gradually

decrease until its residual strength R reaches the value S, when static failure occurs.

This deterioration process may, as indicated, be described graphically by and R-S-N diagram or analytically by a set of parameter functions of S and N.

If now a discrete load peak of known level L is imposed upon the specimen and N fatigue cycles, then the probability of failure may be directly read from the diagram or computed by use of the parameter functions.

A generalization of the R=S=N=diagram is proposed in order to make it applicable to the case, when the discrete load peaks are replaced by sequences of different cyclic loadings.

By use of this diagram it has been proved that Miner's measure of cumulative fatigue damage $M = \Sigma n_i/N_i$ depends on the order in which the different sequences are applied, a defect which has been repeatedly verified by experiment.

Keywords-
Structural failure probability
Cumulative fatigue damage
Fatigue failure distribution
Structural reliability

Report Number-
AFML-TR-77-169
Originating Activity
Waloddi Weibull LAUSANNE (SWITZERLAND)
Referenced as-
AD-A055243
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39. **Waloddi Weibull** author, **September 1977**
"A New Test Of Normality And Of Exponentiality Called The Q-Test"

Abstract-
A new test statistic, denoted by Q and equal to the quotient of two unbiased estimates of the standard deviation of a normal distribution is proposed for testing the hypothesis that a given sample is drawn from a normal population or, alternatively, from and exponential population. The sampling distributions of Q have been computed and used for setting the limits of rejection regions corresponding to 1%, 2%, and 5% levels of significance and also for stating the decision power of Q used as a shape estimator.

Key words-
Distribution functions
Statistical sampling
Fatigue life distribution

Report Number-
AFML-TR-77-168
Originating Activity
Waloddi Weibull LAUSANNE (SWITZERLAND)
Referenced as-
AD-A055242
Download this 0.6 Meg file as AD-A055242

40. **Waloddi Weibull** author, **April 1978**
A Test Of Homogeneity Based On Maximum Likelihood Estimates

Abstract-
A sample is said to be homogeneous, if all its elements are drawn from the same population, otherwise heterogeneous. It is self-evident that there is no sense in estimating the parameters of an assumed distribution from heterogeneous sample. Considering the fact that samples of fatigue test data are frequently composed of elements drawn from tow or even three populations, it is an important rule, frequently violated, to start statistical analysis of a given sample by stating whether it is homogeneous or not. A simple test based on the alternative maximum likelihood estimates of the complete sample and of the sample more or less truncated is presented.

Keywords-
structural failure probability
cumulative fatigue damage
fatigue failure distribution
structural reliability

Report Number:
AFML-TR-78-27
Originating Activity:
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FRANCE
Referenced as:
AD-A058102
Thanks to Jerry Sterling of NASA for finding this document.
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41. **Waloddi Weibull** author, **September 1977**
The Concept Of Maximum Reliability Selection Of Unknown Distribution Parameters

Abstract-
The classical method of estimating an unknown parameter of a given distribution function by means of a unique function of the observations has been replaced by a procedure which consists in deciding between several possible values of the parameter by use of a test statistic, called the selector. Its merit is appraised by use of a new concept, the reliability of the selector, which is equal to the probability of selecting the true value of the parameter. It has been proved that the maximum likelihood estimation method possesses maximum reliability.
Pertinent formulas have been developed and applied to the Weibull distribution.

Keywords-
Distribution parameter selection
Structural reliability
Distribution functions

Report number-

AFML-TR-77-170

Referenced as-
AD-A055630
Waloddi Weibull LAUSANNE (SWITZERLAND)
Download this 0.9 Meg file as AD-A055630.

42. **Waloddi Weibull** author, **April 1978**
Statistical Analysis Of One Hundred And Twelve Groups Of Fatigue Performance Data: Testing the Homogeneity Of the Samples

Abstract-
Three different tests for the hypothesis that the sample is homogeneous, denoted by MLE, OMLE and TI, are described, and their applications to 112 completed samples taken from an extensive list of fatigue performance data collected at the Boeing Company. The hypothesis of homogeneity was rejected for 60 of 112 examined samples by MLE, 45 of 107 samples by OMLE and 17 of 96 samples by TI. The number of samples for which the hypothesis was rejected at least by one of the tests was 64 of 112. The rejections are mainly due to the high-time outliers, but in some cased to low-time outliers, which indicate two-component distribution.

Keywords-
Distribution functions
Random sampling techniques
Statistical population sampling
Two-component distributions
Sample homogeneity testing

Report Number:
AFML-TR-78-28
Originating Activity:
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Referenced as:
AD-A058098
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43. Mischke, C. R., **A Distribution-Independent Plotting Rule for Ordered Failures**, **1979**. PDF file size is 976K.
Various plotting rules have been developed for associating a cumulative density, F_i, or a reliability, R_i [R_i=1-F_i] with an ordered life measure, x_i. These suffer from various biases when rules formulated for one distribution family are used with another. The plotting rationale presented:
1) is distribution independent,
2) frees the plotter from tabular information while retaining high precision,
3) easily accommodates to censored data, and

4) suggests the proper regression model.
10 pages.

44. This Weeks Citation Classic, September 10, 1981.
This document refers to Waloddi Weibull's publication in the ASME Journal of Applied Mechanics, **18**:293-7, 1951 with comments from from Weibull's son Goren Weibull.

45. Weibull Symposium: Stockholm, Sweden, June 19-21, **1984** To the Memory of Waloddi Weibull, Excerpts from "Probabilistic Methods in the Mechanics of solids and Structures", editors: S. Eggwertz and N.C. Lind,
1.95MB PDF file (24 pages)

46. Mr. Chi-Chao Lui, PhD **1997** dissertation from University of Nottingham:
A Comparison Between The Weibull And Lognormal Models Used To Analyse Reliability Data.
Abstract, Contents, List of Tables, List of Figures, Acknowledgements, Glossary, and Notations And Abbreviations. (use this to decide if your want the longer document)
1,251 KB PDF file (22 pages)
Complete Dissertation as one file
15,800 KB PDF file (293 pages)
Appendices-
Appendix A – Derivation of the Weibull related functions-6 pages
Appendix B – Derivation of the Lognormal related functions-8 pages
Appendix C – Correlation coefficient for the Weibull distribution-47 pages
Appendix D – Correlation coefficient for the Lognormal distribution-47 pages
Appendix E – Comparison of results for data from the Weibull distribution-22 pages
Appendix F – Comparison of results for data from the Lognormal distribution-23 pages

Dr. Abernethy interprets and summarizes Mr. Lui's three most important conclusions regarding life data as:
1. "Median rank regression is recommended as the overall best method for life data analysis for data sets with and without all kinds of suspensions."
2. "The two best methods of goodness of fit are the likelihood ratio test and the p-value of the coefficient of determination (both methods are available in SuperSMITH Weibull software)."
3. "For 20 or less failures, always use the two-parameter Weibull distribution even if you know the underlying failure mechanism demonstrates a different distribution---the reason for selecting the two-parameter Weibull is both a more stable predictor and a more conservative predictor
(Dr. Abernethy believes the log normal distribution for small data sets is so optimistic in the lower tail, he sees less reason to use it because it is too optimistic—meaning too few failures are predicted. This comment is derived from personal communications with Dr. Abernethy and he viewed it as a surprise that he and Chi-Chao Liu saw the problem with a similar viewpoint.).

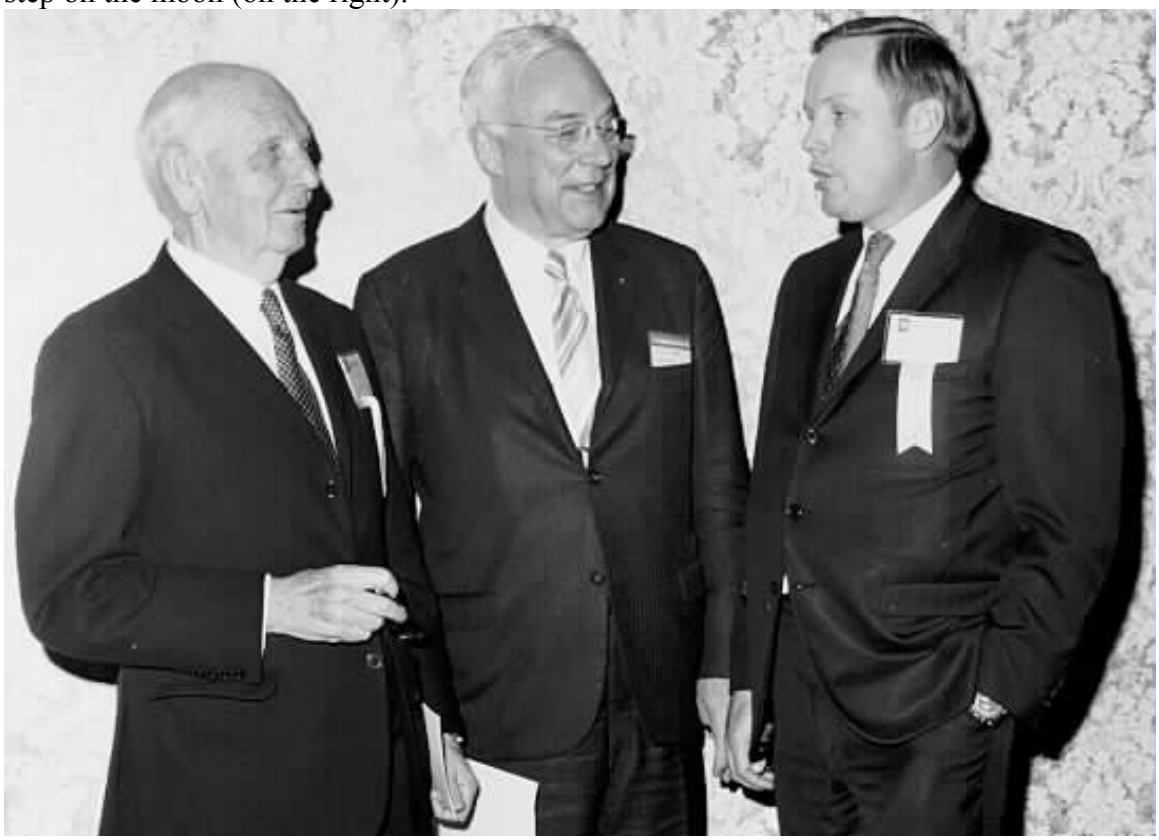
******

Excerpts (extracted by permission from Dr. Bowie) from Dr. Bowie's personal communication with the Weibull family as reported at http://glennbowie.tripod.com:

- Mrs. Inga Britta (Ibbi) Weibull on August 20, 1996 and on September 18, 1996 he received a letter from Dr. Weibull's wife Ibbi and his son Göran Weibull. They reported Dr. Weibull's full name was Ernst Hjalmar Waloddi Weibull which was shortened to the nickname of Doddi in the family and by use of close friends.
- Dr. Weibull had five children. The oldest was Hans Waloddi Weibull and the youngest was Göran Waloddi Weibull.
- The last known mailing addresses of the Weibull family members are:

Ibbi Weibull	Göran W. Weibull
Björnstorps torg	Bäckamöllan
S-240 13 Genarp	S-227 55 Brösarp
SWEDEN	SWEDEN

- In 1972, Dr. Weibull (on the left) received an ASME Medal and his photo was captured with Dr. Richard Folsom (in the middle), along with former Astronaut Neil Armstrong, the first man to step on the moon (on the right).



Return to the list of references.

****

Dr. Abernethy's book, The New Weibull Handbook, and the Weibull techniques are supplemented by software written by Wes Fulton Fulton Findings. The Fulton Findings software is SuperSMITH Weibull

for making probability plots and SuperSMITH Visual for making Crow-AMSAA growth plots. Prices for the software are modest.

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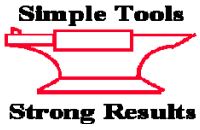

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