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**THE ORDER STATISTICS  $y_i = \log(z_i^m)$ , THEIR PROPERTIES  
AND USE FOR PARAMETER ESTIMATION**

**( $x$ =standardized Weibull variate)**

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## FOREWORD

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## ABSTRACT

Pertinent formulae for the confidence limits, expected values, variances and covariances of the order statistics  $y_i = \log(z_i^m)$  have been developed and used for application of the generalized least-squares method, resulting in unbiased, minimum-variance estimates of the distribution parameters. Approximation formulae, based on simplified covariance matrices, have been proposed and examined. Extensive tables of the required statistics, computed by use of an IBM 7090 computer, are presented.

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## SYMBOLS

$i$	order number
$N$	sample size
$x$	Weibull distributed variate
$\alpha = 1/m, \beta, \mu$	distribution parameters
$y$	$\log z^m$
$z = (x-\mu)/\beta$	standardized Weibull variate
$x_{i,N}$	$i^{\text{th}}$ order statistic in a sample of size $N$ . The subscript $N$ will be omitted when no confusion arises
$x_{1,1}$	a single value drawn at random from a Weibull population
$x_{i,1}$	the $i^{\text{th}}$ of such randomly drawn values not rearranged according to magnitude and thus independent of other values
$x_{i,p}$	the percentile of the $i^{\text{th}}$ order statistic corresponding to the percentage $100 p$
$\tilde{x}_i = x_{i,.50}$	the median of the order statistic $x_i$
$E(x_i)$	the expected value of $x_i$
$E(x_i^2)$	the second moment of $x_i$
$E(x_i, x_j)$	the product moment of $x_i$ and $x_j$
$V(x_i)$	the variance of $x_i$ , also denoted by $\sigma_{ii}$
$\text{Cov}(x_i, x_j)$	the covariance of $x_i$ and $x_j$ , also denoted by $\sigma_{ij}$
$C_i^N = N!/i!(N-i)!$	the binomial coefficient
$C_1, C_2$	constants
$\bar{p}_i$	the percentage point of $E x_i$
$\tilde{p}_i$	the percentage point of $\tilde{x}_i$