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THE ORDER STATISTICS $y_i = \log(z_i^m)$, THEIR PROPERTIES AND USE FOR PARAMETER ESTIMATION

(z=standardized Weibull variate)

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FOREWORD

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ABSTRACT

Pertinent formulae for the confidence limits, expected values, variances and covariances of the order statistics $y_i = \log(z_i{}^m)$ have been developed and used for application of the generalized least-squares method, resulting in unbiased, minimum-variance estimates of the distribution parameters. Approximation formulae, based on simplified covariance matrices, have been proposed and examined. Extensive tables of the required statistics, computed by use of an IBM 7090 computer, are presented.

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TABLES

SYMBOLS

i	order number
N	sample size
x	Weibull distributed variate
$\alpha = 1/m, \beta, \mu$	distribution parameters
y	$\log z^m$
$z = (x-\mu)/\beta$	standardized Weibull variate
$x_{i,N}$	i^{th} order statistic in a sample of size N. The subscript N will be omitted when no confusion arises
$x_{1,1}$	a single value drawn at random from a Weibull population
$x_{i,1}$	the i^{th} of such randomly drawn values not rearranged according to magnitude and thus independent of other values
$x_{i,p}$	the percentile of the i^{th} order statistic corresponding to the percentage 100 p
$\check{x}_i = x_{i,.50}$	the median of the order statistic x_i
$E(x_i)$	the expected value of x_i
$E(x_i^2)$	the second moment of x_i
$E(x_i, x_j)$	the product moment of x_i and x_j
$V(x_i)$	the variance of x_i, also denoted by σ_{ii}
$Cov(x_i, x_j)$	the covariance of x_i and x_j, also denoted by σ_{ij}
$C_i^N = N!/i!(N-1)!$	the binomial coefficient
C_1, C_2	constants
$\bar{P}_i$	the percentage point of Ex_i
$\check{P}_i$	the percentage point of $\check{x}_i$