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REFERENCES ON THE WEIBULL DISTRIBUTION

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Summary

A little more than one thousand publications (books and papers) which illuminate the various aspects of the Weibull distribution have been collected and classified with regard to the theoretical properties of the distribution, its use for the statistical analysis of test data, and its practical applications. A graphical representation of the number of publications as a function of time is presented.

Sammanfattning

Något mer än ettusen publikationer (böcker och tidskriftsartiklar), som belyser de olika aspekterna av Weibullfördelningen har sammanställts och klassificerats med hänsyn till fördelningens teoretiska egenskaper, dess användning för statistisk analys av försöksdata samt dess praktiska tillämpningar. En grafisk framställning av antalet publikationer som en funktion av tiden presenteras.

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Contents

1. Introduction

In order to explain the - at that time - well known but unexplained facts that the relative strength of a specimen decreases with increasing dimensions and that its bending strength is larger than its tensile strength, the author proposed in 1939 a statistical theory of strength of materials (Ref.916).

This theory was based on the assumption that the strength is a stochastic quantity, which has to be specified by a distribution function including one or more parameters.

It was also supposed that this function is a characteristic of the material and that the above-mentioned size effects should be reflected by changes in the values of the parameters of the given function.

At this time, the distribution function proposed by Gauss was dominating and distinguishly called the normal distribution. By some statisticians it was even believed to be the only possible one (In fact, a paper by the author (Ref.923) was with this motivation refused by the editor of a British scientific journal).

However, by means of the following elementary example it was proved that the normal distribution is unacceptable for the present purpose.

Let T be the ultimate tensile strength of a bar or a wire of length $L=1$ and $F(t)=\text{Prob}(T \leq t)$ its cumulative distribution function. Then, the probability of failure at a load equal to t will be $F(t)$ and the probability of non-failure equal to $1-F(t)$. If now the length of the bar is doubled, then it is evident that the probability of non-failure $1-F_2(t)$ is equal to the probability that none of the two halves of the bar fails, that is

$$1 - F_2(t) = \left[1-F(t)\right]^2$$

and, in general, for any arbitrary length L

$$1 - F_L(t) = \left[1-F(t)\right]^L \qquad (1)$$

or

$$F_L(t) = 1 - \left[1-F(t)\right]^L \qquad (2)$$

Let us now assume that $F(t)$ is the normal distribution function, then equ (2) proves that $F_L(t)$ is not a normal distribution function, that is, if the strength of a bar is normally distributed for a certain length, then the normality will be definitely excluded for any other length. Consequently the normal distribution was found unacceptable for the present purpose, and another distribution function had to be found.

This problem was solved in the following way. From equ (1) it follows that

$$\log \left[1-F_2(t)\right] = L \cdot \log \left[1-F(t)\right] \qquad (3)$$

Thus it will be required that $\log \left[1-F(t)\right]$ is a function of t, that is,

$$\log \left[1-F(t)\right] = -g(t) \qquad (4)$$

The most simple two-parametric function is given by

$$g(t) = (t-a)/b \qquad (5)$$

and the most simple three-parametric function by

$$g(t) = \left[(t-a)/b\right]^c \qquad (6)$$

Accepting equ (6) we arrive at the distribution function

$$F(t) = 1 - e^{-\left[(t-a)/b\right]^c} \qquad (7)$$

which was proposed in 1939 (Ref.917,equ (37)

At that time the author was unaware of the fact that this function had been derived by Fisher & Tippet (1928) in a quite different way and as such called the third asymptotic distribution of the smallest value.

From the preceding it is clear that the function (7) was initially proposed for the purpose of describing stochastic properties of materials. It was, however, demonstrated in a paper by the author (Ref.923) that this function had a much wider applicability. A strong evidence of this fact is given in the Table of classified references.

2. Modern means of collecting references

In order to master the overwhelming flow of scientific papers it has been necessary to introduce the concept of search word (also called key words). If such words are added to a paper, it can be searched by a computer. This type of searching has been carried through by Hans W.Weibull using the Space Documentation Service REDON in Frascati (Roma), Italy.

The name Weibull was used as a search word, to be found in the title of the article, in the abstract, or as a descriptor. It was not used as a search word of the author.

This procedure does not claim that all relevant references have been found. It is evident that quite a lot of them may have escaped detection.

3. Classification of the references

The references are divided into three main groups

T. Theoretical properties of the distribution

M. Methods for statistical analysis of test data

A. Practical applications

Each of these groups is further classified as indicated in the Table of classified references.