Appendix

Computation of the parameters E, k and m by means of punched-card machines

by

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THE PUNCHED-CARD METHOD. The basic thing in this method is a card containing 12 rows and 80 columns. (see figure 26) Ten of the rows represent the digits 0 to 9 and the two rows left are used for special purposes. If we now punch a hole in one of these »net points» on the card, this punch will give the card a special meaning. As an example we shall number a file of cards from 1 to 99. We need a two-digit space which means that we need two of the 80 columns in the card, e. g. columns 30 and 31. Hence we punch a hole in the "1"s row column 31 of the first card. In the second card we punch a hole in the "2"s row same column, etc.; in the tenth card we punch a hole in the row 1 column 30 and a hole in the row 0 column 31. In the ninety-ninth card there is. consequently a punch in the row 9 both in column 30 and 31. (in figure 26: number 34 in col. 30 to 31). This shows that a single card might contain an 80-digit number.

The two special rows 11 and 12 mentioned above are used to either codify a card or to indicate a negative amount (see figure 1: column 45 to 49).

The punches are made on a card-punching machine.

When the cards are punched, they must be sorted, which is done with a sorting-machine. This machine sorts the cards column after column, one column at a time. So if the cards are numbered with a four-digit number they must be sorted four times.

For the following steps of the calculation, there are different kinds of machines. There is e. g. a machine that multiplies two fields in a card and punches the result in a third field. There is another machine that adds a special field in a file of cards, and prints the sum.

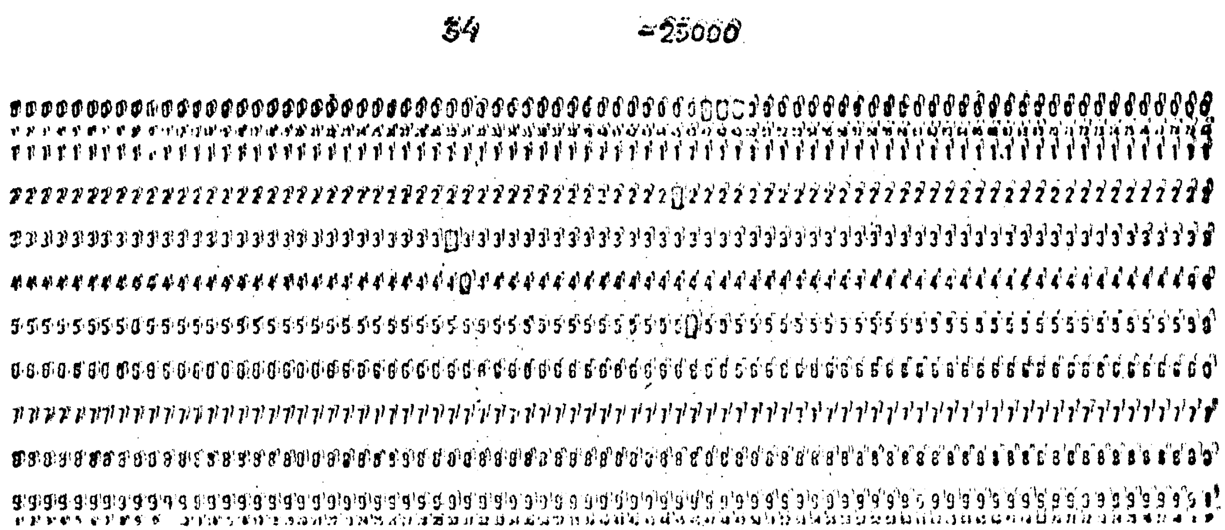

Fig. 20. A punched card.

These are the most important machines in our case, but there are other types for special purposes.

Using the punched-card machines in mathematical computation, one must remember that they are fit for only three of the rules of arithmetic. If we wish to apply them to higher problems, a combination of manual and mechanised operations will give very good results, as will be shown further on. Another solution is to set up table cards for the functions we need. Any operation in a work scheme could be mechanised, but one has to compare the time required for a manual solution and for setting up a machine. Which method is the better must be decided in each case.

THE PARAMETERS E, k AND m. In our case there are given samples from fatigue failure tests, represented by the measured values of N, as is shown above. From above we also have the equations

$$\left.\begin{aligned} y &= \log N \\ x &= \log (S - E) \\ a &= \log k \\ \text{and} \qquad & \\ y &= a - m\,x \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

According to these equations we have to compute the given values of N with arbitrary values of the parameter E_r to get a maximum of the correlation coefficient r in (39).

As is seen in the equations, the factors are the results of products, squares, and sums of x and y, and so the first step of the work is to compute the x's and y's. Manually, this is a very simple job and if the number of values is 400, 300, or less, it is no use to let the punched-cards come in. If the number is greater, however, we could still use table-cards for the logarithmic function as mentioned above. In this connection we must decide how to codify the values of x and y, and in our example the following method is used:

The values of x and y are set up in columns and rows. Thus arranged, each value will be named after its column and row,

y_{0101}	y_{0201}	y_{0301}	y_{0401}
y_{0102}	y_{0202}	y_{0302}	y_{0402}
y_{0103}	y_{0203}	y_{0303}	y_{0403}
y_{0104}	y_{0204}	y_{0304}	y_{0404}

the two first digits in the index telling the column, and the following two telling the row. As will be shown further on, the column numbers are doubled for the y's and on the cards the indices contain three groups of digits:

$$y_{010101} \qquad y_{020201} \qquad y_{030301} \qquad y_{040401}$$
$$y_{010102} \qquad y_{020202} \qquad y_{030302} \qquad y_{040402}$$

The values of x and y are now ready to be put in a punching scheme.

note	η	ν	μ	card type	E	x
columns to be punched	1—2	3—4	5—6	7	8—12	13—17
	01	01	01	5	xxxx	xxxxx
	01	01	02	5	xxxx	xxxxx
	01	01	03	5	xxxx	xxxxx
				⋮		
— — —	— — —	— — —	— — —	— — —	— — —	— — —
	ν	ν	μ		X	y
	01	01	01	6	xxxx	xxxxx
	01	01	02	6	xxxx	xxxxx
	01	01	03	6	xxxx	xxxxx
				⋮		

where each line represents a card to be punched. Setting it up like this, every value is represented by a card. The cards are used to compute the values of r to find the E's wanted. When we have obtained the right E's, they are used to determine corresponding values of k and m. Whether to do this manually or not depends on the number of k and m. If this is great, about 100 or more, it is better to do it on the punched cards, but in our case with 20 values it is faster to compute them manually.

THE WORK SCHEME. The first thing to do in the punched-card computation is, as mentioned above, to set up the actual values on a punching scheme. The next thing to do is to plan how to place

…e fields on the cards for the following operations. A very useful …y is to make a space chart. We will find in our example that …ere are four kinds of cards. Two of them are the x and y cards …d the others, the so-called summary cards. We get the summary …cards when we sum together on the tabulator the values in each …roup of the x-cards. We get the summary y-cards in the same …y. On the space chart we also find some unused fields, e. g. …n the x-cards columns 24 to 29. The idea is to place the fields so …s to simplify the operations as much as possible, and to let both … and y-cards be prepared at the same time.

As the operations are very easy there is no need to go into all the details, because a »punched-card man» will very soon find out how to solve it. A simplified description of the work scheme is as follows:

0. Setting up the values on the punching scheme.
1. Punching the x and y cards.
2. Computing the squares x^2 and y^2.
3. Doubling the x cards (see below)
4. Computing the products $x \cdot y$
5. Summary-punching the values Σx, Σy, Σx^2, Σy^2, and Σxy per group η (r) and n^0 (r)
6. Computing $(\Sigma x)^2$, $(\Sigma y)^2$ and $\Sigma x \, \Sigma y$ on the summary cards.
7. Listing (on the tabulator) the values x, Σx, Σx^2, $(\Sigma x)^2$, y, Σy, Σy^2, $(\Sigma y)^2$, Σxy, $\Sigma x \, \Sigma y$.
8. Computing $\dfrac{\Sigma x \, \Sigma y}{i} - \Sigma xy$
9. » $\Sigma x^2 - \dfrac{(\Sigma x)^2}{i}$ and $\Sigma y^2 - \dfrac{(\Sigma y)^2}{i}$
10. *Manually* computing $f(x) = \sqrt{\Sigma x^2 - \dfrac{(\Sigma x)^2}{i}}$ and $f(y) = \sqrt{\Sigma y^2 - \dfrac{(\Sigma y)^2}{i}}$
11. Punching the results in the summary cards.
12. Computing $(f(x) \cdot f(y)) = F$
13. » $\left[\dfrac{\Sigma x \, \Sigma y}{i} - \Sigma xy\right] : F = r$
14. Listing on the tabulator the values F, x, y and r.
15. Sorting out the highest value of r in each group.

N. B. Every y_i column shall be computed with all the x_i — columns. To do it quickly the x-cards are doubled as many times as the value r (= 20 in this case)

At this moment we have found the actual values of E, and if we then want to compute k and m manually, we could use the list from operation 7 where we will find all the factors needed.

If we want to compute k and m on the machines, however, the following additional scheme will be used:

16. Reproducing E, Σx, Σy, Σxy, Σx^2, $(\Sigma x)^2$, Σxy, to new cards.
17. Computing $[i \Sigma x^2 - (\Sigma x)^2]$
18. *Manually* finding $1/(i \Sigma x^2 - (\Sigma x)^2) = f(z)$
19. Punching $f(x)$
20. Computing $(\Sigma y \Sigma x^2)$ and $(\Sigma x \Sigma xy)$
21. » $[(\Sigma y \Sigma x^2) - (\Sigma x \Sigma xy)]$ and $[i \Sigma xy - \Sigma y \Sigma x]$
22. » $f(x)\, x\, (\Sigma y \Sigma x^2) - (\Sigma x \Sigma xy) = \log k$ and $f(z)\, x\, (i \Sigma xy - \Sigma y \Sigma x) = m$
23. List E, $\log k$, and m.

In our example x and y were used with 4 decimals, $f(x)$, $f(y)$ and r with 5 decimals.

The cards used in the computation are shown below. Note that the "x's" written to the right on the cards are identifying punches. (x 50, x 60 etc.)

x-cards x 50

i	r	n	E	x	x^2	xy
1—2	3—4	5—6	8—12	13—17	18—23	30—35

y-cards x 60

i	r	n	N	y^2	y
1—2	3—4	5—6	8—12	18—23	24—29

Summary x-cards x 70

i	r	—	E	Σx	Σx^2	Σxy	$(\Sigma x)^2$
1—2	3—4	5—6	8—12	15—20	21—26	33—38	39—45

$\Sigma\ \Sigma y$	$\Sigma x^2\ \frac{(\Sigma x)^2}{i}$	$f(x)$	$\frac{\Sigma x \Sigma y}{i} - \Sigma xy$	F	r
46—52	53—58	59—63	64—69	70—74	75—80

Summary y-cards x 70, 75

c	r	--	N	Σy^2	Σy	$(\Sigma x)^2$
1—2	3—4	5—6	8—12	21—26	27—32	39—45

$\Sigma y^2 - \frac{(\Sigma y)^2}{i}$	$f(y)$
53—58	64—68

CHECKING METHODS. The checking methods are of two kinds.

A. Checking of every operation, which is done in a common punched-card way.

B. Checking of the computing method. This is usually done by comparing two or three manually-computed values.

If the different operations are checked and the manually-computed values are exactly as the punched-card values, one can be satisfied.

CONCLUDING REMARKS. The time required for the computation of twenty y_ζ, ten y_η, ten x_η and ten x_η from operation 0 to operation 15, including the manual operation 10, was about 15 hours. With several sampling groups and continuous working, however, the time required will be about six hours.

The punched-card machines of to-day, indeed, are very useful in several kinds of calculations, especially when it is manually a very tedious task and a great number of values are involved. Thus it must be said once again that every problem, technical or scientific, can be solved on the punched card. Whether or not a problem is to be solved on the punched card or manually, is of course a matter of time, and which way is the better must be found out from case to case. However, the machines of to-morrow will be sure to simplify the work and even the operations. The descriptions of new machines in some technical papers show that the new machines are able e. g. to notice if an amount is positive or negative. It is shown too, that the machines can perform two, three, or four continuous operations in one.

Furthermore, at all real, not idealized materials there is a dispersion of the properties of such a magnitude that statistical points of view have to be introduced.

The final question calling for an answer is accordingly: How great is the probability that a specimen at a given load will endure a certain number of load cycles? We have, thus, three factors, the relation between which must be known:

1) The probability of failure P, 2) The load S, defined by the range of stress applied during a cycle, and 3) The life N, given as the number of load cycles.