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A Distribution-Independent Plotting Rule for Ordered Failures

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Various plotting rules have been developed for associating a cumulative density, F_i, or a reliability, R_i, with an ordered life measure, x_i. These suffer from various biases when rules formulated for one distribution family are used with another. The plotting rationale presented: 1) is distribution independent, 2) frees the plotter from tabular information while retaining high precision, 3) easily accommodates to censored data, and 4) suggests the proper regression model.

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ABSTRACT

Various plotting rules have been developed for associating a cumulative density, F_i, or a reliability, R_i, with an ordered life measure, x_i. These suffer from various biases when rules formulated for one distribution family are used with another. The plotting rationale presented 1) is distribution independent, 2) frees the plotter from tabular information while retaining high precision, 3) easily accommodates to censored data, and 4) suggests the proper regression model.

NOMENCLATURE

a	Marginal difference between 1 and first median rank reliability, $1 - R_i$, or last median rank reliability and zero, $R_n - 0$, for completely failed sample of census n.
b	Internal difference between adjacent median rank reliability assignments, $R_i - R_{i+1}$
B	Complete beta function
B_s	Incomplete beta function
f(x)	Probability density function of random variate, x
F(x)	Cumulative density function of random variate, x
$F_r(x)$	Cumulative density function of rth order statistic
$g_r(x)$	Probability density function of rth order statistic
i	ith ordered failure
I	normalized incomplete beta function, B_s/B
n	sample census
p	argument
q	argument
r	rth ordered failure
R(x)	Reliability function of random variate, x
R_1	first median rank reliability assignment
ΔR	internal difference between reliability assignments with complete sample failure
$\Delta R'$	internal difference between reliability assignments after suspended item
$_\gamma R_r$	γ-fraction reliability assignment corresponding to rth order statistic
u	Dummy variable of integration
x	Random variate
x_i	ith ordered failure life measure
x_r	rth ordered failure life measure
$\tilde{x}_r$	median value of rth failure life measure

γ fraction

Γ Gamma function

$\square$ suspended item

RATIONALE FOR DISTRIBUTION INDEPENDENCY

The incentive for developing plotting rules originates, in part, with the undefined nature of the cumulative density (or reliability) function at the life measure of a sample item failure and with the multiplicity of advice in the literature. Bliss [1][1] and Ipsen and Jerne [2] recommended a CDF assignment of

$$F(x_i) = \frac{i - \frac{1}{2}}{n}$$

where i is the ordered failure number (ranked first to last), and n is the original sample census. Weibull [3] proposed and Gumbel [4] discussed an assignment of

$$F(x_i) = \frac{i}{n + 1}$$

For Gaussian CDF,

$$F(x_i) = \frac{i - \frac{3}{8}}{n + \frac{1}{4}}$$

has been suggested. Blom [5] indicated

$$F(x_i) = \frac{1 - \alpha}{n - \alpha - \beta + 1}$$

Benard and Bosi-Levenbach [6] recommended $\alpha = \beta = 0.3$ in the above assignment, and Johnson [7] suggested the median as the plotting position.

To answer the question as to the magnitude of F_i (or R_i), which should be associated with a failure life measure (x_i), one can proceed as follows. Consider a sample of census, n, tested to failure, as depicted in Fig. 1. The probability that r-1 sample observations are less than x is $[F(x)]^{r-1}$. The probability that one sample observation is x (is x_r) is f(x) dx. The probability that n-r sample observations are greater than x is $[R(x)]^{n-r}$. The probability of an instance of r-1 sample observations with life measure less than x and one sample observation with life measure x and n-r sample observations with life measure greater than x is

$$[F(x)]^{r-1} \; [R(x)]^{n-r} \; f(x) \; dx$$

[1]Numbers in brackets denote References at end of paper.

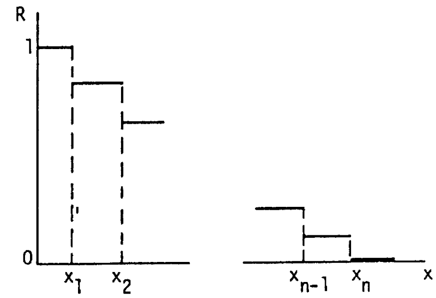

Fig. 1. The reliability of a sample of n items as a function of life measure, x.

The number of ways that n trials can consist of r-1 outcomes less than x, n-r outcomes greater than x, and one outcome that is x, is

$$\frac{n!}{(r-1)! \; 1!(n-r)!}$$

Consequently, the probability that the rth order statistic x_r lies between x_r and $(x_r + dx_r)$ is

$$g_r(x) \; dx = \frac{n!}{(r-1)!(n-r)!} \; [F(x)]^{r-1} \; [R(x)]^{n-r} \; f(x) \; dx$$

If the right side of the above equation is expressed in terms of R(x), we have

$$g_r(x) \; dx = - \frac{n!}{(r-1)!(n-r)!} \; [1 - R(x)]^{r-1} \; [R(x)]^{n-r} \; dR(x)$$

The failure fraction of the rth order statistic $F_r(x)$ is given by

$$\gamma = F_r(x) = \int_{-\infty}^{\gamma x_r} g_r(x) \; dx = - \frac{n!}{(r-1)!(n-r)!} \int_{1}^{\gamma R_r} R^{n-r}(1-R)^{r-1} \; dR \qquad (1)$$

The median rth order statistic is associated with the reliability, ${}_{0.5}R_r$. From Eq. (1),

$$\gamma = \frac{1}{2} = F_r(\tilde{x}_r) = \int_{-\infty}^{\tilde{x}_r} g_r(x) \; dx = - \frac{n!}{(r-1)!(n-r)!} \int_{1}^{0.5^{R_r}} R^{n-r}(1-R)^{r-1} \; dR$$

or

$$\frac{1}{2} = - \frac{n!}{(n-r)!(r-1)!} \int_{1}^{0.5^{R_r}} R^{n-r}(1-R)^{r-1} \; dR \qquad (2)$$

The value of ${}_{0.5}R_r$ is independent of the distribution from which the sample was drawn, as indicated by inspection of the right side of Eq. (2). Thus, the assignment of ${}_{0.5}R_r$ to the ordered failure x_r is equally likely to associate a reliability that is too high or too low, regardless of the distribution of x. The complete beta function is defined as [8]

$$B(p,q) = \int_0^1 u^{p-1}(1-u)^{q-1}\,du\,. \qquad (3)$$

In our nomenclature, p-1 = n-r, q-1 = r-1, and consequently, p = n-r+1 and q = r. The beta function for integer values of p and q may be expressed in factorial terms:

$$B(p,q) = \frac{\Gamma(p)\ \Gamma(q)}{\Gamma(p+q)} = \frac{(p-1)!(q-1)!}{(p+q-1)!} = \frac{(n-r!)(r-1)!}{n!} \qquad (4)$$

Consequently,

$$1 - \gamma = \frac{B_{\gamma R_r}(n-r+1,r)}{B(n-r+1,r)} = I_{\gamma R_r}(n-r+1,r) \qquad (5)$$

where I is the ratio of the incomplete beta function to the complete beta function (as tabulated in Ref. [9], pages 251+). Tables of median rank CDF's have been constructed by abstracting incomplete beta function tables. This can be done for median rank reliabilities, as shown in Table 1.

Table 1. Median rank reliability assignments.

Sample Census n	R_1	R_2	R_3	R_4	R_5	. . .
1	0.50000					
2	0.70711	0.29289				
3	0.79370	0.50000	0.20630			
4	0.84090	0.61427	0.38573	0.15910		
5	0.87055	0.68619	0.50000	0.31381	0.12945	
.						
.						

FREEING THE PLOTTER FROM TABULAR INFORMATION

Of interest are the properties of the median rank reliability table, such as depicted in Table 1. Let us define $(1 - R_1)$ as the marginal difference and construct the table of difference shown in Table 2. Examination of Table 2 reveals that the internal reliability differences for a given sample size are substantially equal, i.e., precise to a plotting accuracy of three significant figures, or slightly less. This observation [10] leads to the concept depicted in Fig. 2.

All reliability assignments are equally spaced internally, linear measure b apart, and centered in the interval with marginal differences of linear measure a. From the geometry of Fig. 2, for any sample of census, n, the unit of width of the reliability interval consists of two segments of length a, and n-1 segments of length b, or

$$2a + (n-1)b = 1$$

Table 2. Interassignment spacing.[a]

Sample Census n	Marginal Difference					
	$1-R_1$	R_1-R_2	R_2-R_3	R_3-R_4	R_4-R_5	. . .
2	0.29289	0.41422	<u>0.29289</u>			
3	0.20630	0.29370	0.29370	<u>0.20630</u>		
4	0.15910	0.22663	0.22854	0.22663	<u>0.15910</u>	
5	0.12945	0.18436	0.18619	0.18619	0.18436	
6	0.10910	0.15535	0.15696	0.15718	0.15096	
7	0.09428	0.13421	0.13563	0.13588	0.13588	
8	0.08300	0.11013	0.11939	0.11964	0.11968	
9	0.07413	0.10549	0.10662	0.10684	0.10692	
10	0.06697	0.09529	0.09631	0.09653	0.09659	
11	0.06107	0.08689	0.08783	0.08801	0.08809	
12	0.05613	0.07985	0.08071	0.08089	0.08095	
:						

[a] Underlined tabular entries are right-hand marginal differences, (R_n-0).

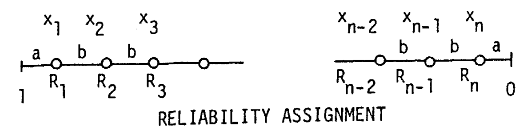

Fig. 2. The distribution-independent reliability assignment displays an interassignment spacing of b and a marginal spacing of a.

Division by a and solution for b/a yields

$$\frac{b}{a} = \frac{\frac{1}{a} - 2}{n-1} \qquad (6)$$

The value of a for the right side of Eq. (6) is taken as the marginal difference in Table 2, and Table 3 is formed by evaluation of Eq. (6). The striking observation from Table 3 is that the ratio, a/b, is substantially constant. Through sample size 15, this ratio is constant to two significant figures and in this range, it is either 0.69 or 0.70. For simplicity, we will assign the constant as 0.7 and pose the following plotting rule: Space the reliability assignments equally, centered in the interval with marginal differences equal to 7/10's of the internal differences. Thus, from the geometry of Fig. 2,

$$2a + (n-1)\,b = 1$$

$$2(0.7)\,b + (n-1)\,b = 1$$

from which

$$b = \frac{1}{n+0.4} = \Delta R \qquad (7)$$

The first reliability assignment is

$$R_1 = 1 - a = 1 - 0.7\Delta R = 1 - \frac{0.7}{n+0.4} = \frac{n-0.3}{n+0.4} \qquad (8)$$

Table 3. Spacing ratios.

Sample Census n	b/a	a/b
2	1.41425	0.70709
3	1.42365	0.70242
4	1.42845	0.70006
5	1.43125	0.69869
6	1.43318	0.69775
7	1.43445	0.69713
8	1.43546	0.69664
9	1.43623	0.69627
10	1.43690	0.69594
11	1.43747	0.69567
12	1.43780	0.69551
13	1.43837	0.69523
14	1.43876	0.69504
15	1.43882	0.69501
⋮		
20	1.44000	0.69445
30	1.44079	0.69406
60	1.44122	0.69385
∞	$(\ln 2)^{-1}$	$\ln 2$

The ith reliability assignment can be written as [10]

$$R_i = \frac{n - i + 0.7}{n + 0.4} \qquad (9)$$

A comparison can be made between the median rank reliability assignments, abstracted from the beta function table and as given by the plotting rule for a sample of census n = 11 (depicted in Table 4).

Table 4. Comparison of median rank and plotting rule plotting position for sample census, n = 11.

i	Median Rank, R_i, from Beta Function Table	Plotting Rule
1	0.93893	0.939
2	0.85204	0.851
3	0.76421	0.763
4	0.67620	0.675
5	0.58811	0.588
6	0.50000	0.500
7	0.41189	0.412
8	0.32380	0.325
9	0.23579	0.237
10	0.14796	0.149
11	0.06107	0.061

ACCOMMODATING TO ORDINARY AND CENSORED DATA

Example 1

Consider the case of four ordered failures at life measures x_1, x_2, x_3, and x_4. From Eq. (7), the reliability decrement is

$$\Delta R = \frac{1}{n + 0.4} = \frac{1}{4.4} = 0.227$$

and it follows that

$$R_1 = 1 - 0.7\Delta R = 1 - 0.7(0.227) = 0.841$$
$$R_2 = R_1 - \Delta R = 0.841 - 0.227 = 0.614$$
$$R_3 = R_2 - \Delta R = 0.614 - 0.227 = 0.386$$
$$R_4 = R_3 - \Delta R = 0.386 - 0.227 = 0.159$$

Figure 3 depicts the associated geometry. We check that 0.227(0.7) agrees with the right hand marginal difference of 0.159.

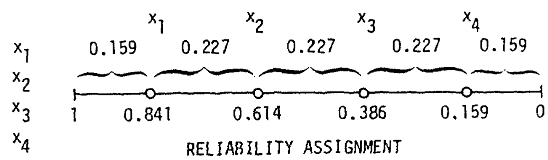

Fig. 3. The median rank reliability assignment for the sample (x_1, x_2, x_3, x_4), all failing.

Example 2

Consider a suspended item after the third failure, x_1, x_2, x_3, □ . From Eq. (7), the reliability decrement is

$$\Delta R = \frac{1}{n + 0.4} = \frac{1}{4.4} = 0.227$$

and it follows that

$$R_1 = 1 - 0.7\Delta R = 1 - 0.7(0.227) = 0.841$$
$$R_2 = R_1 - \Delta R = 0.841 - 0.227 = 0.614$$
$$R_3 = R_2 - \Delta R = 0.614 - 0.227 = 0.386$$

The fourth failure does not occur, and the suspended item cannot be any of the ordered failures since it survived them all. Consequently, the reliability spacing is the same as with four failures with the fourth failure omitted. Figure 4 depicts the associated geometry.

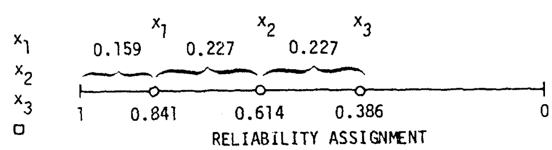

Fig. 4. The median rank reliability assignment for a sample of four, with three failures, (x_1, x_2, x_3, □).

Example 3

Consider a suspended item after the second failure, x_1, x_2, □, x_3. It follows that the reliability decrement is

$$\Delta R = \frac{1}{n + 0.4} = \frac{1}{4.4} = 0.227$$

and the first two reliability assignments are

$$R_1 = 1 - 0.7\Delta R = 1 - 0.7(0.227) = 0.841$$

$$R_2 = R_1 - \Delta R = 0.841 - 0.227 = 0.614$$

In the remaining interval of 0.614, the third failure must be placed so that the distance from the last failure and the marginal distance on the right are in the ratio 1:0.7. Consequently, the new interfailure reliability difference $\Delta R'$ is

$$\Delta R' = \frac{0.614}{1.7} = 0.361$$

and the third reliability assignment is

$$R_3 = R_2 - \Delta R' = 0.614 - 0.361 = 0.253$$

Figure 5 depicts the associated geometry, and we check that 0.7(0.361) = 0.253.

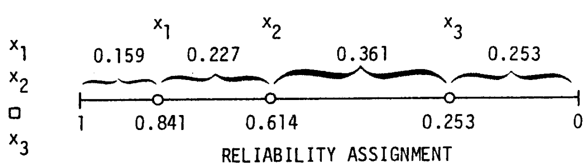

Fig. 5. The median rank reliability assignment for a sample of four, with three failures, (x_1, x_2, □ , x_3).

Example 4

Consider a suspended item after the first failure, x_1, □ , x_2, x_3. It follows that

$$\Delta R = \frac{1}{n + 0.4} = \frac{1}{4.4} = 0.227$$

and that the first reliability assignment is

$$R_1 = 1 - 0.7\,\Delta R = 1 - 0.7(0.227) = 0.841$$

The remaining distance of 0.841 must be divided to accept two equally spaced failures with a marginal difference on the right of 0.7 of the interfailure spacing. Thus,

$$\Delta R' = 0.841/2.7 = 0.311$$

and it follows that the remaining reliability assignments are

$$R_2 = R_1 - \Delta R' = 0.841 - 0.311 = 0.530$$

$$R_3 = R_2 - \Delta R' = 0.530 - 0.311 = 0.219$$

Figure 6 depicts the associated geometry, and we check that 0.7(0.311) = 0.219.

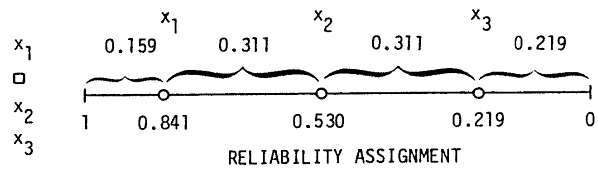

Fig. 6. The median rank reliability assignment for a sample of four, with three failures, (x_1, □ , x_2, x_3).

A generalization of the preceding experience is that to find $\Delta R'$, take the last reliability assignment prior to the suspension, which is the distance to the right end of the reliability assignment interval, and divide this remaining distance into $j + 0.7$ parts, where j is the number of active specimens remaining after the suspension.

$$\Delta R' = \frac{R_i}{j + 0.7} = \frac{n - i + 0.7}{(n + 0.4)(j + 0.7)}$$

$$= \frac{0.7 + n - i}{0.7 + j}\,\Delta R$$

or, in words,

$$\Delta R' = \frac{0.7 + n - \begin{pmatrix}\text{number of failures}\\ \text{preceding the suspension}\end{pmatrix}}{0.7 + \begin{pmatrix}\text{number of active specimens}\\ \text{after suspension}\end{pmatrix}}\,\Delta R \qquad (10)$$

In the case of x_1, x_2, x_3, □ , which was the circumstance of Example 2,

$$\Delta R' = \frac{0.7 + 4 - 3}{0.7 + 0}\,\Delta R = \frac{1.7}{0.7}\,(0.227) = 0.551$$

The right hand marginal difference is $0.7\,\Delta R' = 0.7(0.551) = 0.386$, which may be confirmed by inspecting Fig. 4. In the case of x_1, x_2, □ , x_3, Eq. (10) gives

$$\Delta R' = \frac{0.7 + 4 - 2}{0.7 + 1}\,\Delta R = \frac{2.7}{1.7}\,(0.227) = 0.361$$

confirming the value of $\Delta R'$ in Example 3. In the case x_1, □ , x_2, x_3, Eq. (10) gives

$$\Delta R' = \frac{0.7 + 4 - 1}{0.7 + 2}\,\Delta R = \frac{3.7}{2.7}\,(0.227) = 0.311$$

confirming the value of $\Delta R'$ in Example 4.

Example 5

As an example of using Eq. (10) as an aid in making reliability assignments, consider the case of x_1, □ , □ , x_2. We determine the initial reliability decrement to be

$$\Delta R = \frac{1}{n + 0.4} = \frac{1}{4.4} = 0.227$$

and establish the first plotting position as

$$R_1 = 1 - 0.7\,\Delta R = 1 - 0.7\,(0.227) = 0.841$$

The new reliability decrement, due to the suspended items, is established using Eq. (10)

$$\Delta R' = \frac{0.7 + 4 - 1}{0.7 + 1}\,\Delta R = \frac{3.7}{1.7}\,(0.227) = 0.494$$

and the second plotting position is assigned as

$$R_2 = R_1 - \Delta R' = 0.841 - 0.494 = 0.347$$

The reliability assignment geometry is depicted in Figure 7.

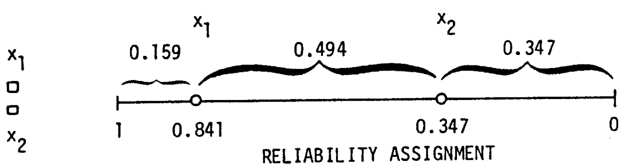

Fig. 7. The median rank reliability assignment for a sample of four, with two failures (x_1, □, □, x_2).

Example 6

Equation (10) is valuable for the case where the first item is suspended, i.e., □ , x_1, x_2, x_3. The modified reliability decrement is

$$\Delta R' = \frac{0.7 + 4 - 0}{0.7 + 3}\,\Delta R = \frac{4.7}{3.7}\,\frac{1}{4.4} = 0.289$$

Since it is not immediately obvious where to plot R_1, we work from the other end of the interval using $\Delta R'$ as a reliability increment.

$$R_3 = 0.7\,\Delta R' = 0.7(0.289) = 0.202$$

$$R_2 = R_3 + \Delta R' = 0.202 + 0.289 = 0.491$$

$$R_1 = R_2 + \Delta R' = 0.491 + 0.289 = 0.780$$

The reliability assignment geometry is depicted in Figure 8.

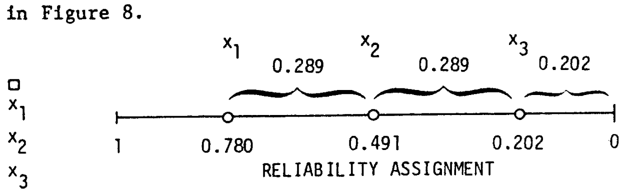

Fig. 8. The median rank reliability assignment for a sample of four, with an initial suspension, (□, x_1, x_2, x_3).

Example 7

For more complex cases, Eq. (10) may be more involved than the direct approach. Consider x_1, □ , x_2, □, x_3. The initial reliability decrement is

$$\Delta R = \frac{1}{n + 0.4} = \frac{1}{5.4} = 0.185$$

The first reliability assignment is

$$R_1 = 1 - 0.7\,\Delta R = 1 - 0.7(0.185) = 0.870$$

The modified reliability decrement, due to the first suspended item, is

$$\Delta R' = \frac{R_1}{0.7 + j} = \frac{0.870}{0.7 + 3} = 0.235$$

and the second reliability assignment is

$$R_2 = R_1 - \Delta R' = 0.870 - 0.235 = 0.635$$

The second modified reliability decrement due to the second suspended item is

$$\Delta R'' = \frac{R_2}{0.7 + j} = \frac{0.635}{0.7 + 1} = 0.373$$

and the third reliability assignment is

$$R_3 = R_2 - \Delta R'' = 0.635 - 0.373 = 0.262$$

The geometry of the reliability assignment is depicted in Figure 9.

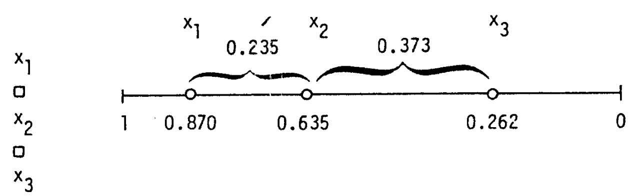

Fig. 9. The median rank reliability assignment for a sample of five with two suspensions, (x_1, □ , x_2, □ , x_3).

COINCIDENT FAILURES

Plotting rules sometimes treat coincident failures (identical life measures) such that a single point results. If such occurrences are viewed as one early failure and one later failure, the plotting places two reliability assignments on the same life measure. If confidence bounds are established by the usual regression procedures, the double plot results in the proper influence of the scatter. Since the usual plotting procedure uses ordinary linear regression on CDF (or reliability) rectification paper (such as Weibull or Gaussian paper), coincident failure life measures, treated as if they occur sequentially and infinitesmally spaced, is recommended.

CHOOSING THE PROPER REGRESSION MODEL

If two samples of census n are prepared for rectification plotting, it is observed that the reliability assignments are the same in both instances (barring suspensions). Consequently, the variable to be considered error-free is the reliability assignment, the variable subject to random error is the ordered life measure, and the regression scheme is mandated.

The usual initial step is to plot ordered life measure data on graph paper designed to rectify CDF or reliability functions. Examples are Weibull, normal, or lognormal papers. After assigning reliabilities according to the scheme recommended herein, a best straight line is sought, the equation of which is a useful predictor of reliability as a function of life measure. The best straight line may be established by eye, viewing the plotted points through a transparent straightedge, or by regression procedures.

Confidence bounds on the line, either by point-by-point or line-as-a-whole method as appropriate to user's intent, should be established and plotted. The 5% and 95% median ranks available in tables accompanying median rank tables [11] are not confidence bounds on the line.

SUMMARY

From the preceeding discussion, the following conclusions can be made:

- The median rank CDF (or reliability) assignment has a rational basis for any continuous distribution.
- For plotting or regression purposes, the median rank CDF (or reliability) assignments are equally spaced internally and centered in the range 0, 1 with margins of 0.7 of the internal spacing.
- Suspended items and coincident failures are easily accommodated.
- The least-squares regression (x on y, or y on x) is not arbitrary, the variable free of error being the median rank CDF (or reliability) assignment.
- When data are transformed by plotting on reliability paper which rectifies the CDF (or reliability) -life measure locus, established statistical methods for linear regression are available for placing confidence bounds on the line-as-a-whole as well as estimates of future points.

Reference [12] is recommended reading to anyone confronted with the statistical presentation and interpretation of data.

ACKNOWLEDGMENT

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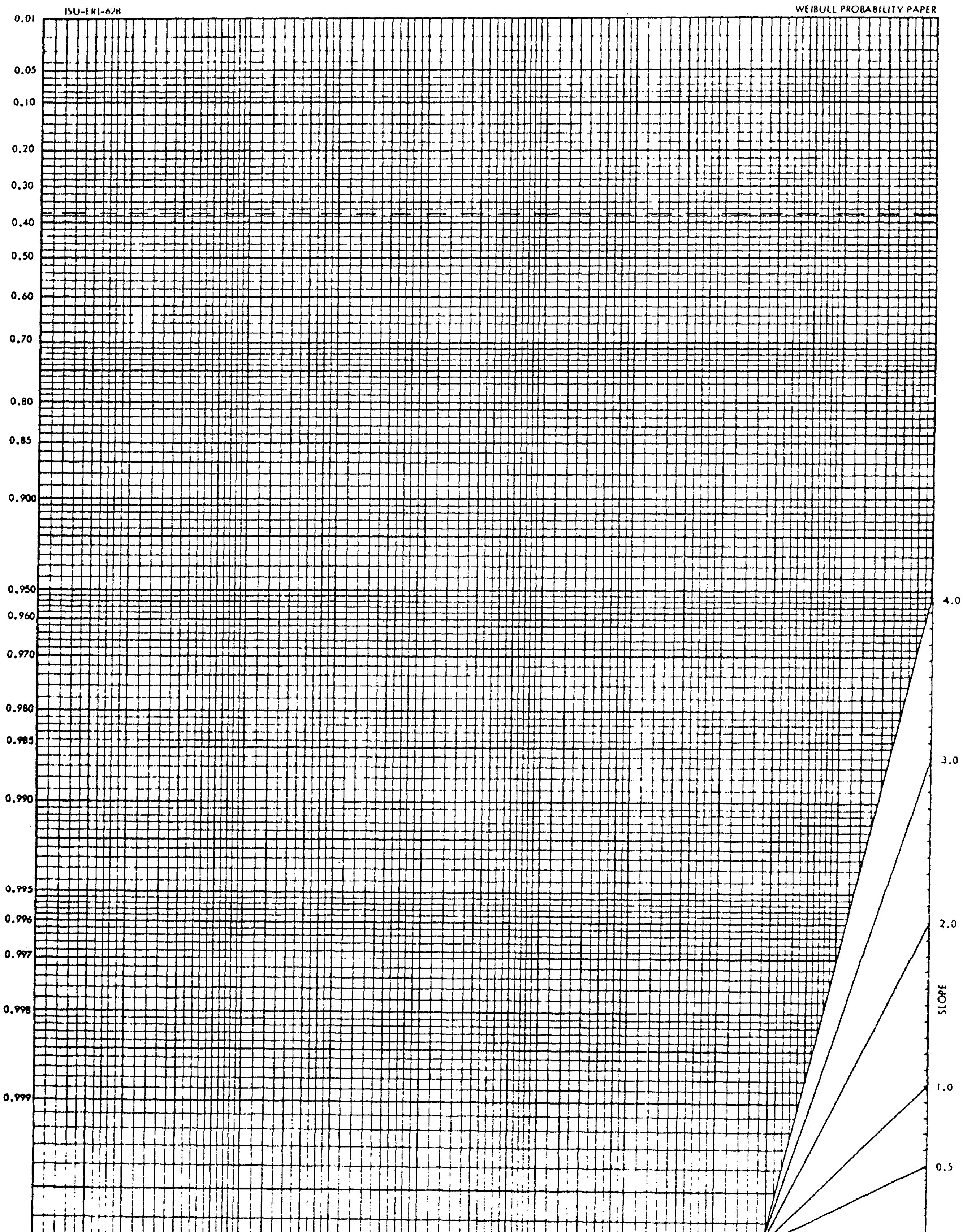
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WEIBULL PROBABILITY PAPER
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SLOPE
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1 2 3 4 5 6 7 8 9 10 2 3 4 5 6 7 8 9 10 2 3 4 5 6 7 8 9 10