A COMPARISON BETWEEN THE WEIBULL AND LOGNORMAL MODELS USED TO ANALYSE RELIABILITY DATA

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ABSTRACT

Reliability and failure data, both from life testing and from in-service records, are often modelled by the Weibull or Lognormal distributions so as to be able to interpolate and/or extrapolate results. This research aims to compare the Weibull and Lognormal models for complete and randomly censored samples using a Monte Carlo simulation procedure. The results of the comparison should give engineers some guidance in selecting the distribution which is more appropriate, from the results of the mathematical simulation, to their particular application.

This thesis is divided into 8 chapters. After a general introduction and a statement of the objectives of the research, there is a review of related published research. A practical engineering application is used as an example. Both reliability models, Weibull and Lognormal, are then illustrated. Median rank regression (MRR) and maximum likelihood estimation (MLE) data-fitting methods are described and a p-value model based on a graphical goodness-of-fit is advanced. There follows a derivation of the relationships between the parameters describing the two distributions. Monte Carlo simulation is chosen and explained as a methodology of relating the results of the two models to each other. The simulation results are then compared and analysed. The thesis concludes with critiques and suggestions for further work. The appendices containing detailed mathematical derivations, results and data are on a diskette which is inside a pocket within the back cover of this thesis.

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GLOSSARY

Terms

Accelerated life testing – Accelerated life testing of items involves tests which can shorten the lives of items and obtain the results quickly. Accelerated life testing is often done under conditions such as high levels of temperature, pressure, voltage, load, cycling rate, vibration etc., or some combination of these [75]. Accelerated life testing is also carried out at the same conditions as an item will experience in use, but the testing is carried out faster so that it is always in advance of the real life in service of an item, (e.g. for an aircraft component).

B-life – Another term used for expressing percentile life. This is the life by which a certain proportion (B%) of the population can be expected to have failed.

Burn-in – 'This is the name given to the processes to stimulate early failure in defective electronic components and assemblies by accelerating the stresses that will cause defective items to fail, without damaging good items.' [77]

Censored data – Life data which have not failed by the end of test or removed from the test without failing. Generally, censored data can be categorised into Type I (time censored) and Type II (failure censored) censored data. Or alternatively censored data can be classified into the left censored, right censored and randomly censored data. These terms are explained in more detailed in Chapter 1.

Confidence interval – Any parameter varies. We can establish limits for that variation by choosing a level of confidence. For example a 90% confidence interval could be defined by the interval between two confidence limits, the 5% limit and the 95% limit. The 5% confidence limit is the value below which we expect 5% of the variation to lie. The 95% confidence limit the value below which we expect 95% of the variation to lie. So the interval which will contain 90% of the variation is that interval between the 5% and 95% limits and so is called the 90% confidence interval. 90%, 95% and 99% confidence levels are used very often in industry. For hypothesis testing, levels of the significance are defined in a similar way to confidence intervals.

Critical correlation coefficient – The correlation coefficient (CC) is a statistical measure of the strength of a relationship between two variables. The value of CC is between -1 to +1. The CC can also be used as a gauge of goodness-of-fit of the Weibull and Lognormal probability plots. For comparison purpose, if setting 90% confidence one-sided level, then the lower 90% of the distribution of CC, is called the critical correlation coefficient (CCC) [4].

Estimate – 'A numerical value obtained from an estimator by substituting a numerical value for the sample statistic is called an estimate.' [47]

Estimator – 'A mathematical expression which estimates a population parameter as a function of a sample statistic is called an estimator.' [47]

Hazard function – The hazard function h(t) is the conditional probability of an item failing in the interval t to $(t + dt)$ given that it has not failed by time t [77]. This is also known as the instantaneous failure rate. Taking the bathtub curve for example, the early failure period has a decreasing hazard function as time goes by. The useful life period has a constant hazard function. The wear-out period has an increasing hazard function.

Infant mortality period – A failure of items (such as components, equipment and systems) which occur in the early part of the life phase. Such failures usually have a decreasing hazard function with time and generally are caused by initial production, assembly, test, installation or commissioning errors [37]. This is also known as 'early failure period'.

Maximum likelihood estimation – A technique which introduced by Fisher in early 1920's [34]. This technique can be used to determine good estimates of the parameters of a probability from observed data [67].

Mean time to failure (MTTF) (for non-repairable items) – Mean life or average time to failure. The *MTTF* is defined by BS 4778 as 'For a stated period in the life of an item, the ratio of the cumulative time for a sample to the total number of failures in the sample during the period under stated conditions.' The other term, ***mean time between failures (MTBF)***, is also used very often. The *MTBF* is defined by BS 4778 as 'For a stated period in the life of an item, the mean value of the length of time between consecutive failures computed as the ratio of the cumulative observed time to the number of failures under stated conditions.' The MTBF is only applicable for the repairable items under the condition of a constant hazard function [77].

Median rank regression – A method which can be used to best fit the data plotted on the probability paper to determine the parameters of the distributions. The best fit line which has been shown is the least squares line. The term 'median rank' is used because it is a popular method of estimating the Y-axis plotting coordinates.

Monte Carlo simulation – 'A mathematical model of a system with random elements, usually computer-adapted, whose outcome depends on the application of randomly generated numbers.' [4]

Parameter – A value which can be used to describe the population (e.g. the characteristic life of the Weibull population or the median life of the Lognormal population)[4].

Randomly censored data – also known as progressively censored, hyper-censored, multiply censored and arbitrarily censored data. Such data usually come from the field, because units go into service at different times and have different running times when the data are recorded. Such data may be time-censored or failure-censored [75]. These terms are explained in more detailed in Chapter 1.

Reliability – 'The probability that an item will perform a required function without failure under stated conditions for a stated period of time.'[77]

Residual – It is defined as the observation values minus the fitted values [27].

Suspended items – Items which have dropped out or been withdrawn from life test either because they have failed in a different way or because they have survived to the end of the test. Some experts use the term 'censored data' as a synonym for suspended data.

Useful life period – A period or phase between early failure and wear-out periods. Such failures have a constant hazard function with time, that is to say, during this period failures are independent of time. Failures within this period often come from human errors, maintenance errors and nature such as lightning strikes [4][37].

Wear-out period – A period or phase when failures of items (components, equipment or systems) occur after the end of the useful life. Such failures have an increasing hazard function with time and generally are caused by factors such as wear, degradation, corrosion, erosion and fatigue [37].

NOTATION AND ABBREVIATIONS

β	Shape parameter of the Weibull distribution (> 0) and also the slope of the Weibull probability plot.
η	Scale parameter of the Weibull distribution (> 0) and also the characteristic life.
ρ	$1/\sigma$, alternative shape parameter of the Lognormal distribution (> 0) and also the slope of the Lognormal probability plot.
θ	e^{μ}, scale parameter of the Lognormal distribution (> 0) and also the median life.
λ	Location parameter of the type I EV distribution.
κ	Scale parameter of the type I EV distribution.
μ	Location parameter of the Normal distribution.
σ	Scale parameter of the Normal distribution if specified.
1-α	Confidence level (α = significance level).
$\Gamma(\cdot)$	Gamma function.
γ_i	$\Gamma\left(1+\frac{i}{\beta}\right)$.
ω	$e^{\left(\frac{1}{\rho^2}\right)}$.
$\Phi(\cdot)$	CDF of the standard Normal distribution.
$\phi(\cdot)$	PDF of the standard Normal distribution.
2-p	2-parameter.
3-p	3-parameter.
c	Constant.
CC	Correlation coefficient.
CCC	Critical correlation coefficient.
CDF	Cumulative distribution function.
CHF	Cumulative hazard function.
CK	Coefficient of kurtosis.
CS	Coefficient of skewness.
CV	Coefficient of variation.
e or exp	The base of natural logarithms (= 2.718281828…).
E(·)	Expected value operator.

EV	Extreme value.
F(·)	Cumulative distribution function (CDF).
f(·)	Probability density function (PDF).
g(·)	Arbitrary function.
H(·)	Cumulative hazard function (CHF).
h(·)	Hazard function (or instantaneous failure rate).
HF	Hazard function.
k	Number of suspensions.
L	Subscript $_L$ indicates the Lognormal distribution. e.g. $f_L(\cdot)$ is the Lognormal PDF.
MAD	Median absolute deviation.
Mean	Mean life of underlying distribution.
Median	Median life of underlying distribution.
MLE	Maximum likelihood estimator.
MLR	Maximum likelihood ratio.
Mode	Mode life of underlying distribution.
MPI	Most powerful invariant.
MRR	Median rank regression.
MTTF	Mean-time-to-failure (= mean life).
muAL	Anti-log of μ (i.e. e^{μ}).
N	Subscript $_N$ indicates the Normal distribution. e.g. $f_N(\cdot)$ is the normal PDF.
n	Sample size.
PDF	Probability density function.
r	Number of failures.
R(·)	Reliability function (RF).
RF	Reliability function.
RMSE	Root median square error.
S	Suspension.
SD	Standard deviation.
sdF	Standard deviation factor (i.e. e^{σ}).
SE	Standard error.
S_{MAD}	Standardised median absolute deviation (= MAD/0.6745).
SSI	Scale-shape invariant.
T	Random variable.
TS	Test statistics.
t	Time-to-failure (or miles, cycles-to-failure in different instances).
t_0	Location parameter.
t_p	Percentile of a distribution.
VAR	Variance.
W	Subscript $_W$ indicates the Weibull distribution. e.g. $f_W(\cdot)$ is the Weibull PDF.

z_p Percentile of the standard Normal distribution.

Note: That on page 12, I have used the convention:

$$n = 3(1)100.$$

This means that the values of n start at 3 and step forward by 1 until reaching 100. The same convention will be used in other places for other variables.