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Basic aspects of fatigue

By

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With 5 figures

It is convenient to think of a specimen under repeated stresses as undergoing a progressive, accumulating damage. This concept has been applied to the total fatigue process with more or less success, in some cases leading to controversial opinions.

The purpose of this paper is to demonstrate that some of the difficulties may be eliminated, if due consideration is taken to the fact that the fatigue process consists of two stages of quite different nature: the crack *initiation* and the crack *propagation*, and if the concept of cumulative damage is applied, not to the total complex process, but to a single point of the specimen, during the first stage that point where the crack is suspected to start, and during the second stage an arbitrary point at the prospective path of the crack.

If D denotes the damage, by definition equal to 100% when fracture occurs, and N any number of stress cycles, the damage factor k may be defined by

$$D = k \cdot N \qquad (1)$$

The damage factor is a function as well of the stress as of the strength of the material (and possibly also of N which for the present will be neglected).

The real stress field within the specimen is composed of the smooth nominal stress, calculated according to the theory of elasticity and denoted by σ, and local stress concentrations, caused by scratches, inclusions or similar stress raisers, whereas the real strength may be that of the idealized material, weakened by dislocations and imperfections of microscopical and macroscopical nature.

In this way, we may visualize a k-field within the specimen with statistically distributed local maxima.

If this field does not change under the influence of the pulsating load (which in fact it sometimes does), it is obvious that the first crack starts at the point with the highest k-value. In most cases this point is located at the surface of the specimen.

After some time, other cracks may appear. It is not absolutely excluded that the first crack stops to grow and that someone of the later ones will develop into the final rupture, because of the possibility that the first crack may change the initial k-field.

From a phenomenological viewpoint, we may describe the k-field in the vincinity of a local stress raiser at x_0 by the expression

$$k = f_0[\sigma, (x - x_0)] \tag{2}$$

where σ is the nominal stress and x a coordinate along the prospective path of the crack (limiting us for the present to the one-dimensional case). This expression is illustrated by fig. 1. The value of k falls off rapidly from its maximum value at x_0 to zero at $(x_0 + c)$, that point where the real stress is equal to the endurance limit. (It is tacitly understood that there is a one-to-one relationship between the real and the nominal stresses).

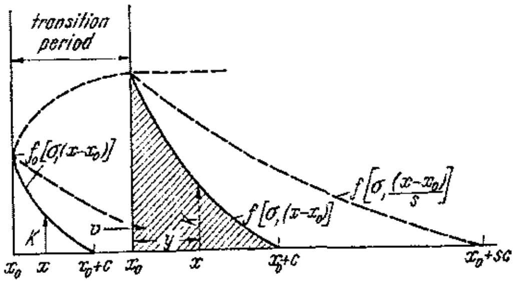

Fig. 1. Geometric distribution of the damage factor k.

The first stage is characterized by the condition that x_0 is a fixed point, for instance the bottom of an initial surface scratch. Accordingly, the damage D in an arbitrary point x increases in proportion to the value of k under the influence of a constant stress amplitude. After a certain number of stress cycles, denoted by N_0, the damage reaches the value 100% at x_0 and the crack starts moving. This is the end of the first stage.

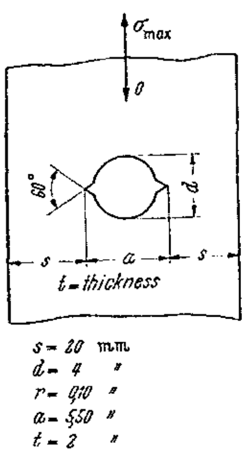

Fig. 2. Test specimen.

By (1) and (2), we have

$$N_0 = 1/f_0(\sigma, 0) \tag{3}$$

As a consequence of the preceding considerations, two general conclusions may be drawn. First, since $f_0(\sigma, 0)$ depends on the surface conditions (scratches, imperfections and the like) the quantity N_0 is a random variable with large scatter. Second, the probability of encountering heavy stress raisers increases with the surface area. In other words, a considerable size effect is to be expected on N_0.

Both conclusions have been verified experimentally by means of plate specimens as shown in fig. 2. In most of the tests, the central

hole was provided with two sharp V-notches in order to facilitate the simultaneous start of the cracks (left and right). The stress was pulsated between a maximum value σ_{max} and zero. The quantity N_{10} i.e. the number of cycles at which the crack has developed to 10% of the width, has been used as a measure of the duration of the first stage, thus including the transition period (see below). As a measure of the propagation time, the difference $(N_B - N_{10})$ has been chosen, where N_B denotes the number of stress cycles at failure. The coefficients of variation V computed show (see table 1) that the scatter of the crack initiation is extremely high, particularly if the extra notches are absent.

Table 1. *Scatter of* N_{10}, N_B, and $(N_B - N_{10})$

Material	σ_{max} kg/mm²	Coefficient of variation of N_{10} %	of N_B %	of $(N_B - N_{10})$ %	Extra notches
24 S-T	15.6	37.1	34.0	2.2	No
24 S-T	12.0	31.2	27.6	3.8	No
24 S-T	8.0	8.2	5.3	3.4	Yes
75 S-T	7.0	19.8	17.0	10.6	Yes
75 S-T[1]	7.0	24.4	20.5	8.5	Yes
Cr-Mo Steel					
$\sigma_B = 90$ kg/mm²	19.0	51.1	36.7	14.5	Yes
$\sigma_B = 70$ kg/mm²	19.0	2.6	4.1	9.0	Yes

[1] Coated with kerosene

The size effect on geometrically similar specimens is demonstrated in table 2. It may be concluded that the diameter of the hole is responsible for the large size effect. The difference between the values of the 50 · 10 and 10 · 10 mm specimens, 76 and 111 respectively, is considered to be due to the different stress concentration. The values of K_t

Table 2. *Size effect on* N_{10}
Material: 24 S-T, Alclad; $\sigma_{max} = 80$ kg/mm²

s mm	d mm	r mm	N_{10} kc	K_t	Plate marked
10	2	0.05	506	2.64	1
20	4	0.10	99	2.64	1
30	6	0.15	82	2.64	1
50	10	0.25	76	2.64	2
10	10	0.25	111	2.27	2
50	50	1.25	54	2.64	2

Tabled values of N_{10} are averages of 3 tests each.

give the theoretical stress concentration of holes without extra notches which are assumed to raise the stress concentration proportionally because of the geometrical similarity.

The second stage is characterized by the condition that x_0, *now* denoting the tip of the crack, is a moving point. The rate of crack propagation v is defined by

$$v = \mathrm{d}x_0/\mathrm{d}N \tag{4}$$

At the beginning of this stage, the material just in front of the crack has suffered a certain amount of damage which is very nearly 100%. The curve of f_0 (but not f) thus represents the distribution of the damage by using a scale which makes the ordinate at x_0 equal to 100%. It is easily understood that a less sloped curve, as indicated by the dotted lines in fig. 1, corresponds to a larger rate of propagation.

During the crack propagation, the k-curve is moving with x_0, in the most simple case without changing its shape. This seems to happen, when the tip of the crack is well away from the initiating scratch. Thus k is changing during a "*transition period*" from f_0 successively to a stable form f. It seems as generally $f_0(\sigma, 0) < f(\sigma, 0)$, but sometimes the reverse may happen, depending on the stress raising effect of the initial crack.

A fixed point x is apparently subjected to a pulsating stress of increasing amplitude. When the tip of the crack is moving from x_0 to $x_0 + \mathrm{d}x_0$, the material at x suffers an increased damage $\mathrm{d}D$ which by (1) is

$$\mathrm{d}D = k \cdot \mathrm{d}N \tag{5}$$

Considering (2) and (4), the total damage accumulated at this point, after it has been passed by the whole k-curve, is

$$D = \int_{x-c}^{x} \frac{f[\sigma, (x - x_0)]\,\mathrm{d}x_0}{v} \tag{6}$$

with the necessary condition that

$$D = 1 \text{ for } x = x_0 \tag{7}$$

Substituting

$$y = x - x_0 \tag{8}$$

we have

$$\int_0^c \frac{f(\sigma, y)\,\mathrm{d}y}{v} = 1 \tag{9}$$

In general, f and σ and accordingly v are functions of x_0. If f and σ do not change with x_0, it follows that v = constant and then from (9)

$$v = \int_0^c f(\sigma, y)\,\mathrm{d}y = A \tag{10}$$

where A denotes the shaded area in fig. 1.

If we now enlarge all the dimensions s-fold, keeping σ unchanged i.e. increasing the load s-fold, it follows from the law of similarity that the area will be $s \cdot A$ and from (10) that the rate of propagation will

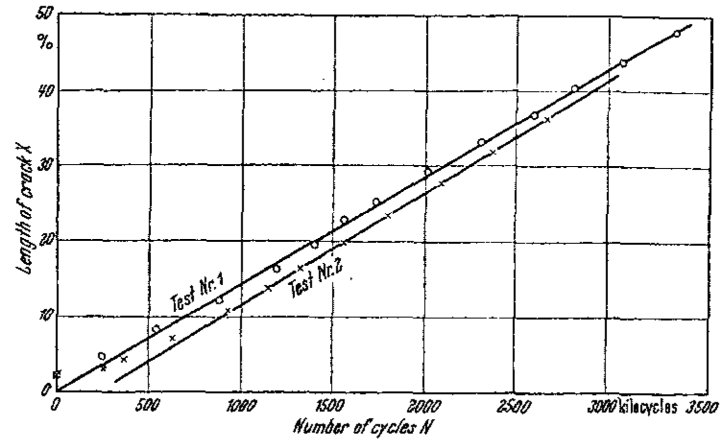

Fig. 3. Crack propagation at constant stress amplitude. Material 24 S-T. σ_{max} = 4,0 kg/mm².

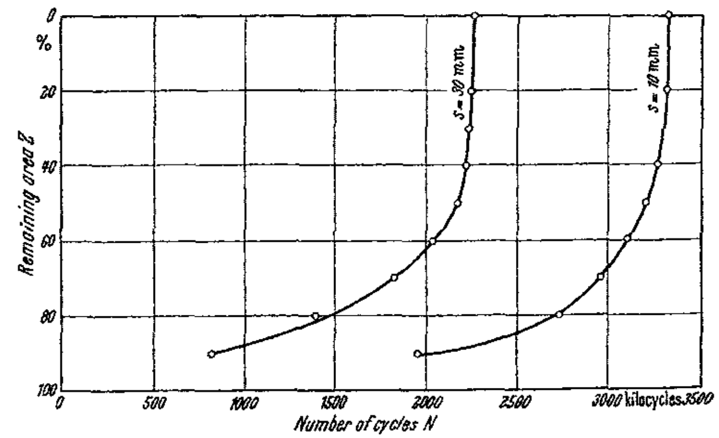

Fig. 4. Crack length vs. number of stress cycles at constant load amplitude.

also be s times enlarged. This means that the propagation time is independent of the width of the specimen.

It is possible to carry out this deduction also in that case when $\sigma = f_1(x_0)$, as for instance when the load amplitude is constant and accordingly the stress amplitude increases reciprocally with the remaining cross-sectional area.

The statement that the rate of propagation after the transition period is independent of the length of the fatigue crack has been verified in the following way: The stress was pulsated between σ_{max} and zero. By decreasing the load at intervals ($\Delta x \sim 1$ mm), given in table 3, the value of σ_{max} was kept between 3.90 and 4.10 kg/mm². The mean velocity $\Delta x/\Delta N$ over an interval does not show any systematic trend. This result is still more conspicuous in fig. 3 from which it is readily found that the rate of propagation is constant as soon as the transition period has been passed. The theoretical conclusion that the propagation *time* is independent of the width of the specimen is verified by fig. 4, where the (z vs $\cdot N$) curves for the widths 10 mm and 30 mm are plotted (z = remaining area in percentage of initial). These curves are parallel but offset 110 kc (size effect as to hole diameter). There was no complete

Table 3. *Growth of fatigue cracks at nearly constant stress amplitude*
Material: 24 S-T; $\sigma_{max} = 3.90 - 4.10$ kg/mm²
Dimensions: $s = 30$; $d = 6$; $r = 0.15$; $t = 2$ mm

Test Nr. 1				Test Nr. 2				
N kc	Length of crack mm	Length of crack %	Rate of propagation 10^{-3} mm/kc	N kc	Length of crack mm	Length of crack %	Rate of propagation 10^{-3} mm/kc	
0	0.63[1]	2.1	—	0	0.66[2]	2.2	—	Transition period
250	1.43	4.8	3.20	259	1.03	3.4	1.43	
536	2.53	8.4	3.85	375	1.30	4.3	2.33	
866	3.70	12.3	3.55	625	2.13	7.1	3.32	
1184	4.90	16.3	3.77	915	3.20	10.7	3.69	
1387	5.83	19.4	4.58	1130	4.13	13.8	4.33	
1555	6.88	22.9	6.25	1313	4.93	16.4	4.37	
1725	7.58	25.3	4.12	1553	5.90	19.7	4.04	
2010	8.80	29.3	4.28	1798	7.05	23.5	4.69	
2300	9.93	33.1	3.90	2068	8.33	27.8	4.74	
2583	11.00	36.7	3.78	2361	9.58	31.9	4.27	
2793	12.10	40.3	5.12	2656	10.88	36.3	4.41	
3078	13.10	43.7	3.57	—	—	—	—	
3366	14.30	47.7	4.17	—	—	—	—	
			M. v. 4.24 ± 0.77				M. v. 4.41 ± 0.24	

[1] The crack length $x = 0.63$ was obtained by applying
$\sigma_{max} = 4.5$ kg/mm² during $\Delta N = 850$ kc (crack visible)
8.0 „ „ 30 „
6.0 „ „ 220 „
4.0 „ „ 480 „
5.0 „ „ 100 „

[2] The crack length $x = 0.66$ was obtained by applying
$\sigma_{max} = 8.0$ kg/mm² during $\Delta N =$ 30 kc (crack visible)
6.0 „ „ 117 „
5.0 „ „ 121 „

similarity, the diameter of the hole in the first series being 4 mm instead of 2 mm, but the diameter does apparently not play an important role as soon as the fatigue crack is fully developed. Table 4 gives the observed data of N, z, and x.

Table 4. *Crack propagation at different widths of specimen*

	Series 80142x		Series 80362x		
z %	x mm	N_1 kc	x mm	N_2 kc	$N_1 - N_2$ kc
90	1	196	3	82	114
80	2	274	6	139	135
70	3	297	9	182	115
60	4	311	12	204	107
50	5	320	15	217	103
40	6	327	18	222	105
30	7	330	21	223	107
20	8	332	24	224	108
	$N_B = 332$		$N_B = 225$		107
					M. v. = 111.2

Values of N_1 and N_2 are averages of 6 values, viz., left- and right-hand cracks from 3 test specimens.

The second stage ends with tensile rupture, when the mean stress over the remaining area reaches a value which is very little influenced by the crack length as demonstrated in table 5. The specimens have been run in a fatigue machine. When the crack had developed to a predetermined value of z, the test was stopped and the ultimate tensile strength measured. The heading "100" means that the specimens were drawn without preceding fatigue. The tensile strength over the remaining area is remarkably independent of the various factors which means, among other things, that machined V-notches (bottom radius 0.05 and 0.1 mm) are just as severe stress raisers as the fatigue crack.

Tables 5. *Tensile strength in kg/mm² of remaining cross-sectional area*

Series	Remaining cross-sectional area in percentage of initial						
	100	90	80	70	60	50	42
120362x	38.3	38.1	38.1	37.6	37.8	37.4	—
80362x	37.5	—	36.0	—	—	35.3	—
120262x	39.3	—	38.7	—	38.2	—	38.2
120363x	36.4	—	36.2	—	36.5	—	36.7
120342x	39.2	—	38.1	—	37.8	—	39.2
M. v.	38.2	38.1	37.4	37.6	37.6	36.4	38.0

The preceding considerations have some bearing on the experimental determination of the P-S-N-relationship. The central part of the P-S-N-field may be investigated with reasonable means, but the boundaries $P = 0$ and $N = \infty$ have to be determined by extrapolation. This is an unsafe procedure even with large samples under the actual condition that the distribution function of fatigue life and the S-N equations are not exactly known (and probably never will be). An illustration is given in fig. 5 which shows 103 test values plotted on a normal probability paper.

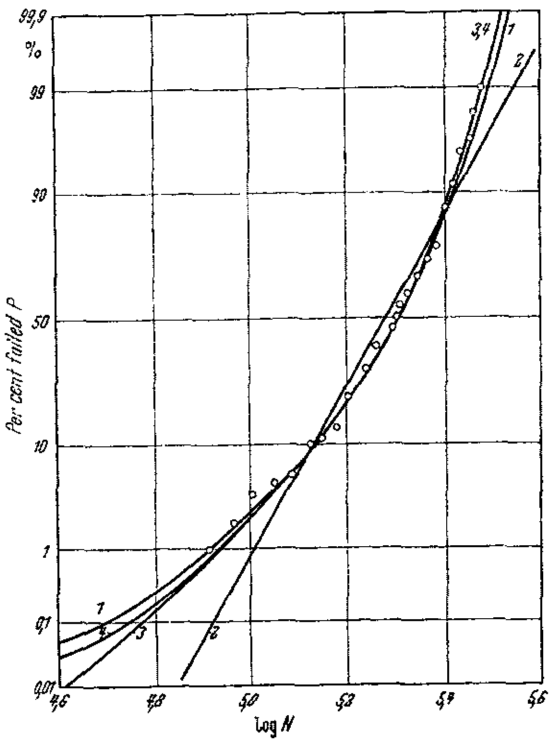

Fig. 5. Distribution of fatigue life of 24 S-T flat specimen, notched by two holes. Stress pulsating between 14 kg/mm² and zero.
1. $P = \Phi(N)$; 2. $P = \Phi(\log N)$;
3. $P = F(N)$; 4. $P = F(\log N)$.

Four different distributions have been examined and corresponding curves drawn. The normal distribution function is denoted by Φ, whereas $F(x)$ denotes the function

$$F(x) = 1 - e^{-\left(\frac{x - x_u}{x_0}\right)^{1/\alpha}} \qquad (12)$$

The only obvious result is that the distribution is not log-normal (Curve 2). An experimental decision as to the three remaining functions is quite impossible without increasing the number of tests to several thousands, in spite of the fact that the functions give very different values of the lower bound N_u and $(\log N)_u$, respectively, of the distribution, viz.,

Curve 1: $N_u = -\infty$
2: $(\log N)_u = -\infty$
3: $N_u = 24.9$ kc.
4: $(\log N)_u = 2.108$

The statistical treatment of fatigue data should be very much simplified and the results safer, if it could be proved that the S-N-curve for $P = 0$ may be identified with the crack propagation time, as ascertained in a large series with flat, drilled specimens of 24 S-T, and the lower bound of the endurance limit with that stress at which a crack does not propagate.

Preliminary tests have however shown that the latter hypothesis is complicated by the fact that the non-propagating stress does not take a fixed value but depends highly on the preceding fatigue history.

The present theoretical considerations have been induced by results from extensive experimental investigations, made possible by liberal grants from the Saab Aircraft Company and carried out in close co-operation with Mr. Fred Turner, Head of the Stress Department of that company, and Mr. Gunnar Wållgren, Head of the Structures Department of the Aeronautical Research Institute of Sweden.

Discussion

R. B. Heywood: Tests on crack propagation have been made at the Royal Aircraft Establishment on BTD 546 clad aluminium alloy sheet panels of 20 g thickness by loading in repeated tension (zero to maximum) at about 30 r/min., and giving failure in 70 to 100,000 cycles. The specimens were 72 in. by 36 in., and contained a central transverse slot 0.5 in. long by 1/16 in. wide, with semi-circular ends.

The results were in agreement with Weibull's finding that considerable scatter in the initiation of cracks was obtained, but that little scatter in the propagation was found. Thus the crack from one end of the slot might be 0.2 in. long before the crack from the other end had started, but with progression of the two cracks across the panel, this difference in length became no greater.

However the results were not in accordance with Weibull's finding that cracks propagate themselves uniformly with cycles, for the cracks grew at a progressively faster rate as their length increased. This result was obtained even when the load was reduced during the test, so that the nominal stress across the remaining section was constant. It is suggested that the difference in behaviour of these specimens with those of Weibull's may have been due to the fact that the theoretical elastic stress at the extremity of a crack in an infinite sheet is proportional to the square root of the length — conditions which were approximated in the large sheets — but that the maximum stress may actually fall as the crack approaches the edges of the sheet — conditions which are more nearly approached in Weibull's experiments.

In other tests in which the maximum load was kept constant but was of a different value in different tests, the final catastrophic failure of the slotted panels was approximately in accordance with a relationship that has been suggested by Wells, namely that $\sigma^2 L$ is a constant, where σ is the nominal stress on the full area of the panel and L is the overall length of cracks from one crack extremity to the other.

J. Schijve: The difference between the results of Weibull and of Heywood may probably be explained by a speed effect. In Heywood's tests the stress was higher and the frequency was considerably lower (30 r/min instead of 2,000 r/min) than in the tests of Weibull. So in each load cycle of Heywood's tests a higher peak stress was maintained for a larger time at the end of the crack. It then may be expected that a kind of creep will come into the fatigue process of the crack propagation. The speed effect at high fatigue stresses has been noted by various authors.

E. GASSNER had made the same observations as SCHIJVE on aluminum alloy 24S-T.

B. LUNDBERG: WEIBULL's statement that most of the scatter is attributed to the pre-crack period, whereas the time or number of load cycles for the propagation of the crack has little scatter, is a very interesting one. The observation seems also rather natural if one assumes that a certain "unit damage" — the greater the higher the stress or strain range is — is caused by each load cycle. This unit damage should not be confused with a crack. In the pre-crack period such a unit damage would occur at outer surfaces or at interior surfaces of cavities at weak places subjected to high stresses or strains. It seems natural to assume that these unit damages, which might have the nature of unbonding of atoms according to SHANLEY's theories, might occur at quite a large number of differently located weak spots. Sooner or later a number of unit damages occur at one and the same spot, and this implies the formation of a sub-microscopic crack. This occurrence is, quite naturally, characterized by a large scatter as the surfaces in a "submicroscopic sense" normally are widely different from each other even for nominally identical specimens, and it is thus more or less a question of probability when a number of unit damages begin to occur at one and the same spot.

After the crack has been formed, all, or at least the majority, of the subsequent unit damages will probably occur at the root of the crack, as this then has become the weakest spot and is subjected to high stresses. It seems natural that the continued propagation of the crack is of the nature of a straight-forward mechanical procedure (or rupture) which follows a certain law and would not be expected to have a large scatter between nominally identical specimens.

P. E. WIENE: As a practical confirmation of the slow propagation of cracks I can mention that some ten years ago we had at Burmeister and Wain some cases of broken piston rods (how this was cured, see ref. [*1*] in my paper).

We therefore controlled all piston rods by magnetic tests once a year, i. e. about every 40 million stress-cycle, and nearly always found starting cracks, but we never had a broken rod; this indicates that the sharpest cracks are not always the most dangerous.

W. WEIBULL: In most cases, the rate of propagation has been observed to increase during the "transition period" which ends, when the crack has reached a length depending on the applied stress-amplitude. Between the values 10% and 50% (which is the highest value studied), the rate of propagation is remarkably constant. If the results of HEYWOOD refer to crack lengths smaller than 10% only, there seems to be no discrepancy between his and the author's observations.