Small Sample Size Datasets: Help or Hindrance

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Abstract

Some common sense issues will be presented concerning the size and effectiveness of small datasets for Weibull analysis. A few examples will illustrate the issues for aiding common sense evaluation of the data. Some guide lines will be listed from a management perspective.

What Do Engineers Know And How Do They Know It?

Two great quotations set the stage for the worries about small datasets. Theodore von Kármán's [Director of the Guggenheim Aeronautics Laboratory at the California Institute of Technology] quotation rides on my computer monitor from some unknown source:

"*The Scientist studies what is, the engineer creates what has never been*."

Oliver Heaviside [the famous British electrical engineer and inventor of operational calculus could not give mathematically rigorous proofs for all his formulas which he approached from a heuristic basis] also has a neat quotation (Kármán and Biot 1940) in response to criticism for using formal mathematical manipulations without understanding how they worked:

"Shall I refuse my dinner because I do not fully understand the process of digestion?"

Dr. John Lienhard, University of Houston Engineering Professor and author of the nationally syndicated daily radio show **"Engines of Our Ingenuity"** refers to Heaviside [who coined the word impedance] as the "wounded genius" (http://www.uh.edu/engines/epi425.htm). Heaviside was partly deaf and compensated for his disability with shyness and sarcasm as you see in the quotation above. Lienhard says about Heaviside:

> ***"He loathed all that business of deducing one fact from another. He meant to invent knowledge—not to compute it."***

Dick Reiman, historian, comments that Heaviside's adversaries claimed he was a "first rate eccentric" [he was] (http://ieee.cincinnati.fuse.net/reiman/04_1990.html) and they attacked telephony's genius with a quote that today we know was wrong:

"His methods were said to be "imperfect" and "of no consequences".

So, when the scientific community is aggravated with uncertain ideas and data, they attack, and "wars" begin. Statisticians enjoy "wars", and with small data sets, many will dismiss the frugal information as imperfect and of no consequences and they will dismiss methods which can not pass the test of time. Unfortunately, we engineers do not have the luxury of waiting on voluminous information--we are always pressed by time and budgets as noted by Vincenti. To paraphrase Heaviside: ***Should I refuse my dinner because I do not understand all the aspects of my small data set and my personal clock will run out on my life before I know the perfect answer which is free from error***. We engineers are stuck in the time/cost warp that requires us to take action rather than contemplating our navel. The Accreditation Board for Engineering and Technology defines engineering:

> ***"Engineering is the profession in which a knowledge of the mathematical and natural sciences, gained by study, experience, and practice, is applied with judgment to develop ways to utilize, economically, the materials and forces of nature for the benefit of mankind."***

Some engineering definitions will also include phrases addressing the "art and science" of engineering as important [the issue of art is enclosed in the judgment word of the ABET definition]. Others (Vincenti 1993) include organizing the design, construction, and operation of the artifice [An artful or crafty expedient.]. Vincenti also includes the engineering requirement for:

1) economy,
2) freedom from error, and
3) adoption of standards as important elements of the engineering definition.

Vincenti also argues that the engineering profession works with imperfect definitions of scope and poorly defined data from which they must extract important information from charts, graphs, and tables using methodologies that do not always have neat and tidy closed solutions. Additionally engineering information must fit the immediate timeline need for getting answers to pressing problems within the tightly compressed time frames with a design that produces a product at a price the customer is willing to pay. We engineers need to an idea of when to accept the risk and when to reject the risk of small data sets—good engineering judgment helps with this decision.

Now we've come full circle to the "unknowns" addressed by von Kármán and Heaviside. We engineers must do the best we can from what we have available. Perfection will not fit the timeline or cost budget but it will increase our anxieties. You will never have enough data for a risk free decision—if handling risk is unpalatable, then perhaps you should consider a different line of work than engineering.

Engineers agonize about the uncertainties, wide confidence limits, and all sorts of errors on the basis of: "How can I possible answer, with precision, all the questions from their managers". As working engineers move into management positions their tolerance for uncertainty and lack of perfect answers increases on one hand but on the other hand they worry about the money that might result from the errors.

This means that engineers must have methods to bracket the results and convert details into time and money so they can sell their managers to take action. Remember, even your old fishing buddy when you were both engineers now acts and reacts differently when he moved up the ladder to management.

Managers will not accept "root canals" from an over abundance of techie details—they want to know the issue, how it will be resolved, how much money is at stake (along with some ± bracketing), and what time will the problem go away. Uncertainty will always exist (more with small datasets and less with big datasets). Keep the risk$'s manageable. Know the cost brackets. Solve the problem and move to the next one quickly in keeping with the Pareto distribution.

Ayyub (Ayyub 2001) presents two graphics summarizing the process of knowledge and ignorance. Ayyub says engineering and science depend on development and use of predictive models that require knowledge, information, and subjective opinions of experts (even though the experts have their opinions and pet theories). On one side you have knowledge which is described by Ayyub in Figure 1, and on the other side you have ignorance as shown in Figure 2.

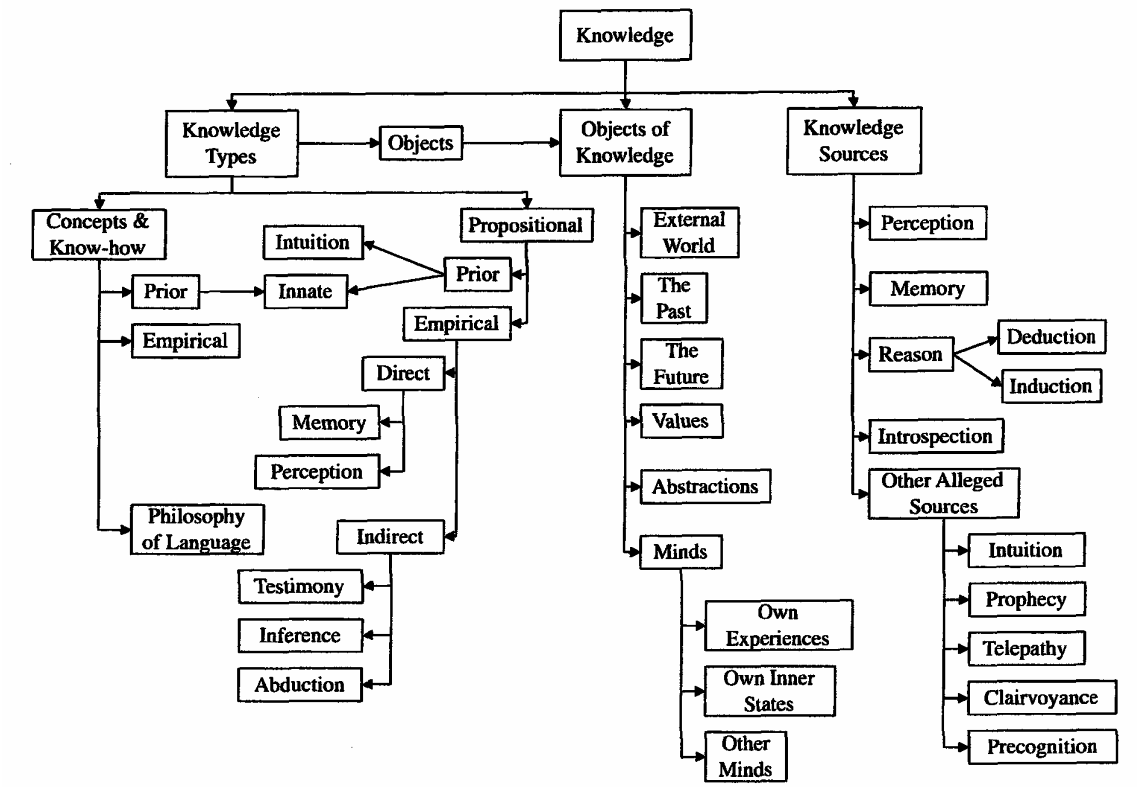

Figure 1: Knowledge

What I know, you think of as my ignorance and so another war begins. Ignorance is shown in Figure 2.

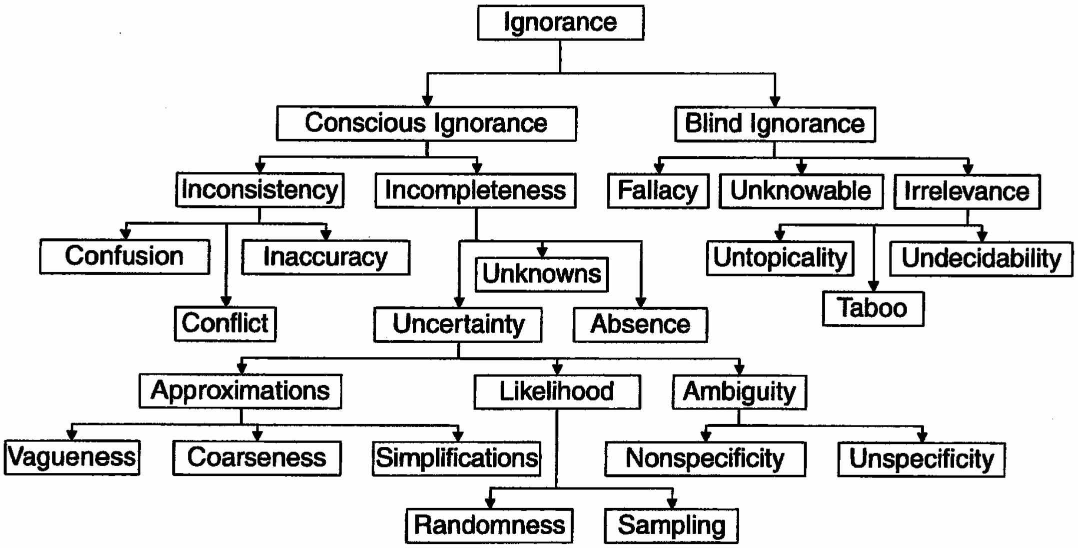

Figure 2: Ignorance

Reducing Errors in Small Datasets

The greatest un-measurable change in uncertainty occurs when you move from opinions, and no data, to the first piece of data.

Uncertainty becomes measurable when you move from one piece of data to two pieces of data. Even though you can calculate the uncertainty, do you know enough to do the right thing?

Since you will never ever have enough data for a risk free decision, and you cannot live long enough to get all the clean, orderly, data you would like to have—what do you do? Obviously you're perpetually caught in a situation of tradeoffs between "good" and "bad" data which is bounded by insufficient time and insufficient money. Avoid being caught like a deer at night spellbound by the headlights of an automobile and at the last moment makes a suicidal dash into peril. Use judgment and experience to consider how wrong or how right your information may be.

Consider the following dataset for a heat exchanger.

1) Last year we completed a turnaround and discovered (by test) a heat exchanger with three leaking tubes. Before turnaround, we did not suspect tube leaks as a problem. Now we're worried—Is the heat exchanger at end of life? Should we hold the course or buy a new tube bundle for installation at the next turnaround?

2) The data shows one tube plugged after two years of service—the tube was plugged because of damage during the 2 year turnaround.. One tube was plugged with a demonstrated leak after 5 years of service which was found by test during the year 3 turnaround.

3) Three tubes were plugged (age 8 years) during the three year turnaround and the leaks were discovered by test.

4) We're now at year 9 and have 432 tubes still working and we do not know of any failures but we know the next turnaround will occur two years into the future.

5) If more than 10% of the tubes are lost, we have reached end of life for the tube bundle because it represents a functional failure for adequate heat transfer.

6) Should we continue? Retube now with a new $125,000 bundle?

7) The data set is -2, 5, 8*3, -9*432. **←Do I have enough data to make the decision?**

Using WinSMITH Weibull's inspection option to handle the course data with the resulting stack of data we see the results in Figure 3 which suggests end of life for the tube bundle after 18 years of service. The results of an Abernethy risk says that we should expect to see 6 tubes fail during the next 24 months and the number could be as high as 10 tubes or as low as 3 tubes based on a 90% confidence. The decision based on this analysis is to take the risk and continue running—if we couldn't observe three leaks from the previous test, do you think we can detect 6 leaks predicted for the next inspection interval. Therefore take the risk and continue operating.

We know that small data sets produce greater errors in beta than eta. Do you think the $\beta = 3.5$ in Figure 3 is reasonable? Consider Nordman and Meeker's (Nordman 2002) reporting of Nelson's work with heat exchangers in nuclear power stations where $\boldsymbol{\beta = 3.3}$—thus adding experience says it's in the same range noted in Figure 3 and we feel better that our results are not silly.

Experience by Beamer (Beamer 1997) from the ASME Weibull Workshop reports $2.3 < \beta < 10$ for heat exchangers in a refinery. Analysis, by the author, of a heat exchanger from a refinery in February 2002 showed $\beta \approx 11$ for 6 cases and $\beta \approx 0.8$ for one case on the same heat exchanger during it's 35 year life.

Using the Weibayes feature of WinSMITH Weibull (Fulton 2002) to impose additional information on the data (this assumes β = 3.548 is incorrect and our selection of β = 11 is a better choice) we get the plot shown in Figure 4

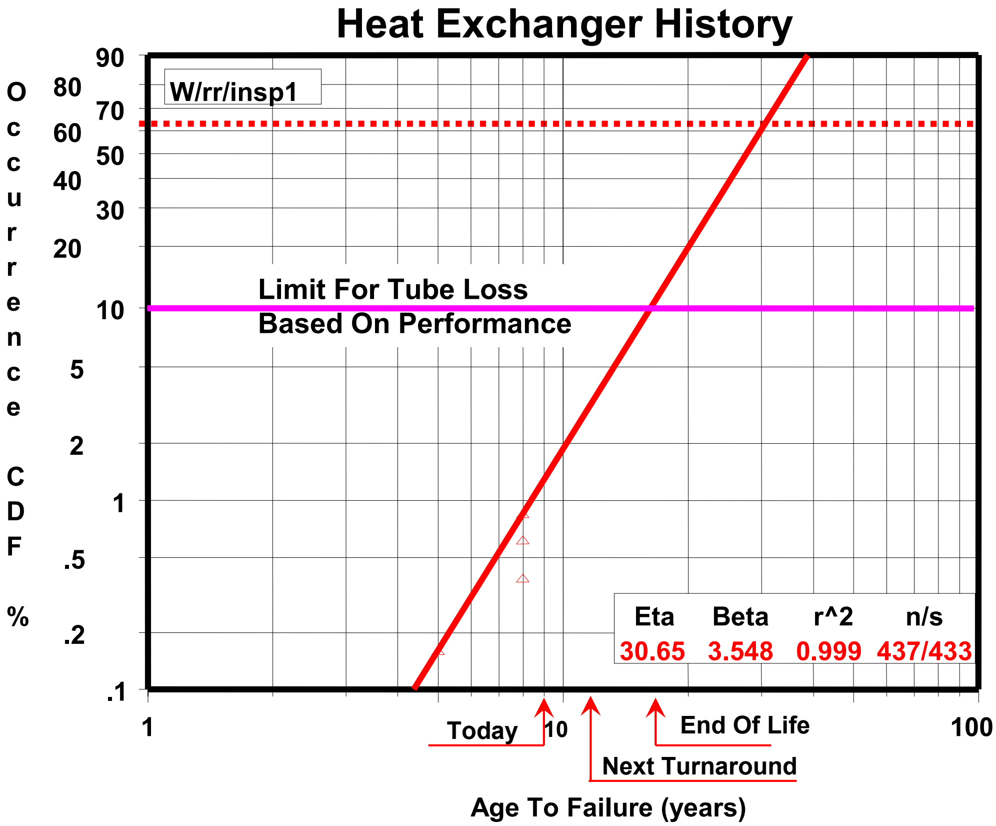

Figure 3: Weibull Life Of Heat Exchanger

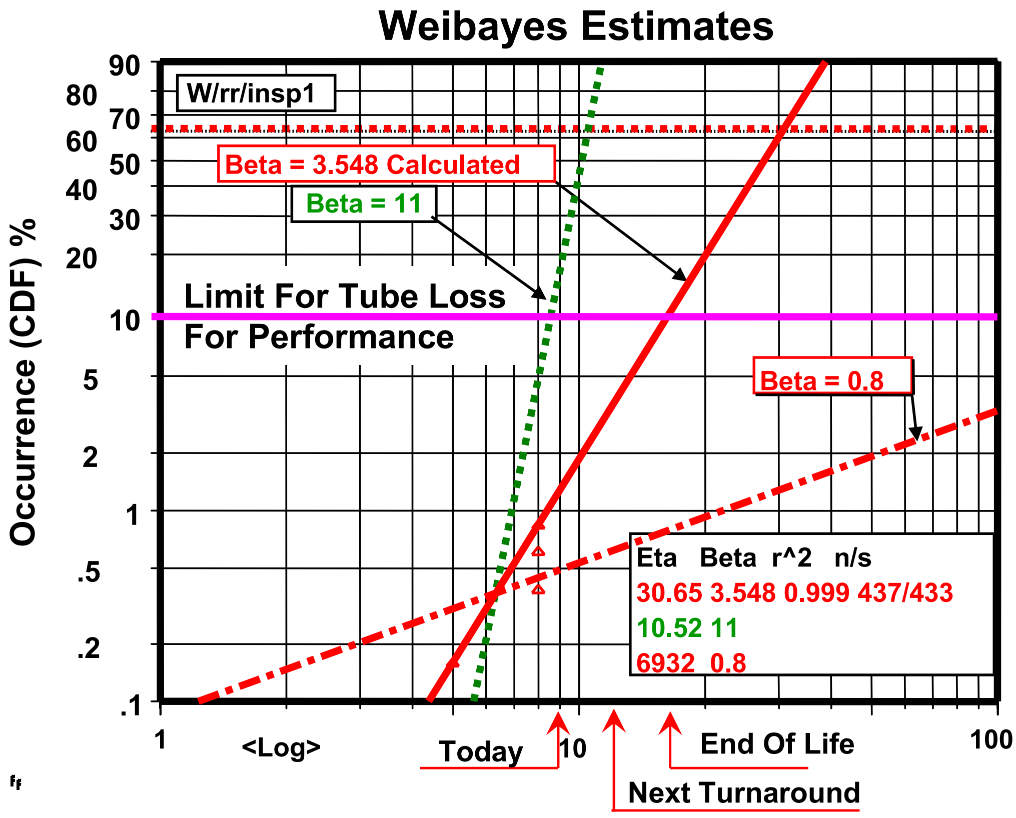

Figure 4: Weibull What If On β

In Summary

So which value of β is correct? One way to answer the question is to prepare a hypothesis and test it. Based on Figure 4, the large β would tell you the heat exchanger is suffering from big performance problems—it's not, so reject the hypothesis for huge betas. Based on Figure 4, the small β would have fewer failures than the beta calculated—therefore keep running.

Don't get bogged down in arguing what is the "right" beta because you'll never know. To demonstrate the lack of the "right" beta, run a small simulation in Excel with a know beta and know eta and draw samples randomly and watch what happens to the calculated values for each random group of 10 data points. Remember, in real life you'll rarely see the large number of data points you'll find in an Excel simulation. Follow the principles of The New Weibull Handbook (Abernethy 2000). Use WinSMITH Weibull software to ease the calculation load (Fulton 2002)

Life is too short to argue endlessly about numbers, tempus fugits, costs accumulate. Solve the problem the best you can and move on. Use small datasets the best you can—perfect answers rarely exist.

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Biography

Paul Barringer is a reliability, manufacturing, and engineering consultant with more than thirty-five years of engineering and manufacturing experience in design, production, quality, maintenance, and reliability of technical products. Experienced in both the technical and bottom-line aspects of operating a business with management experience in manufacturing and engineering for an ISO 9001 facility. Industrial experience includes the oil and gas services business for high pressure and deep holes, super alloy manufacturing, and isotope separation using ultra high speed rotating devices.

He is author of training courses: **Reliability Engineering Principles** for calculating the life of equipment and predicting the failure free interval**, Process Reliability** for finding the reliability of processes and quantifying production losses, and **Life Cycle Cost** for finding the most cost effective alternative from many equipment scenarios using reliability concepts.

Barringer is a Registered Professional Engineer, Texas. Inventor named in six U.S.A. Patents and numerous foreign patents. His education includes a MS and BS in Mechanical Engineering from North Carolina State University.

For other issues on process reliability refer to **Problems Of The Month** at **http://www.barringer1.com.**

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