

# Corrosion Problems Quantified With Gumbel Lower Distribution

**Abstract:** Several case studies show how to separate general corrosion from accelerated corrosion and how to predict end of useful life of products.

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## Gumbel Upper or Gumbel Lower?

- The Gumbel upper distribution is used when you have **BIG** numbers. It's best know for flood data (you only record the deepest [**largest**] stream gage reading for a single year).
- The Gumbel lower distribution is used when you have **LITTLE** numbers. It's used where you've only recorded the thinnest [**smallest**] wall in a single corrosion area.

The Gumbel Smallest Extreme Value is considered a model for a system having  $n$  elements in a series and where the failure distributions of components are reasonably uniform and similar (See British Standard BS 5760).

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The Weibull distribution straight line equation  
 $\ln\left(\ln\left(\frac{1}{1-F(t)}\right)\right) = \beta \cdot \ln(t) - \beta \cdot \ln(\eta)$

## What's The Math Difference?

Both are also known as the double exponential →

The **Gumbel largest extreme value** CDF is:

$$F(t) = e^{-e^{-\frac{(t-\xi)}{\delta}}}$$

Rearranging the equations to read

$$F(t) = e^{-e^{-\frac{(t-\xi)}{\delta}}} = \frac{1}{e^{e^{-\frac{(t-\xi)}{\delta}}}} \quad \text{Or} \quad \frac{1}{F(t)} = e^{e^{-\frac{(t-\xi)}{\delta}}}$$

Taking the log of both sides you get:

$$\ln\left(\frac{1}{F(t)}\right) = e^{-\frac{(t-\xi)}{\delta}}$$

Again, taking the log of both sides you get:

$$\ln\left(\ln\left(\frac{1}{F(t)}\right)\right) = \frac{-(t-\xi)}{\delta} = \frac{-t}{\delta} + \frac{\xi}{\delta}$$

$Y = mX + b$

$$t = \xi - \delta \cdot \ln\left(\ln\left(\frac{1}{F(t)}\right)\right)$$

For Monte Carlo modeling:

$$t = \xi - \delta \cdot \ln(-\ln(a_{\text{random}_n}))$$

The **Gumbel smallest extreme value** CDF is:

$$F(t) = 1 - e^{-e^{\frac{t-\xi}{\delta}}}$$

Rearranging the equations to read

$$1 - F(t) = e^{-e^{\frac{t-\xi}{\delta}}} = \frac{1}{e^{e^{\frac{t-\xi}{\delta}}}} \quad \text{Or} \quad \frac{1}{1 - F(t)} = e^{e^{\frac{t-\xi}{\delta}}}$$

Taking the log of both sides you get:

$$\ln\left(\frac{1}{1 - F(t)}\right) = e^{\frac{t-\xi}{\delta}}$$

Again, taking the log of both sides you get:

$$\ln\left(\ln\left(\frac{1}{1 - F(t)}\right)\right) = \frac{t-\xi}{\delta} = \frac{t}{\delta} - \frac{\xi}{\delta}$$

$Y = mX + b$

$$t = \xi + \delta \cdot \ln\left(\ln\left(\frac{1}{1 - F(t)}\right)\right)$$

For Monte Carlo modeling:

$$t = \xi + \delta \cdot \ln(-\ln(1 - a_{\text{random}_n}))$$

Observations:

- Same Y-axis
- Weibull has log X-axis
- Gumbel has uniform X-axis

**$\xi$  is a scale factor**  
 **$\delta$  is a shape factor**  
**Small  $\delta$  → steep lines for G- & G+ distributions**

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## Problem 1: Heat Exchanger Thin Tubes?

- We have a shell & tube heat exchanger
- Process fluids are inside the tubes and the tubes are losing wall thickness with use
- Outside the tubes are cooling water
- Periodic inspections have recorded the **minimum** wall thickness in each tube selected randomly. We have only one wall thickness for each tube inspected.

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## What's The Issue? How To Resolve?

- Heat exchanger is 17 years old—460 tubes
- At turnaround, eddy current wall thickness inspection occurred—**We're worried!**
- Did an IRIS inspection on 10% of tubes—**Now we're more worried**—what does the data say?
- Retube **NOW** at 17 years with T/A delays?  
Retube next turnaround in 3 years at 20 years?  
Retube at 2<sup>nd</sup> turnaround in 6 years at 23 years)?

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Time Issues

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## What Are Cost Consequences?

- Failure **\$** is dependent on outside temperatures:
  - Summer failure = \$750,000 lost margins & retube
  - Fall failure = \$500,000 lost margins & retube
  - Winter failure = \$100,000 lost margins & retube
  - Spring failure = \$250,000 lost margins & retube
- Another key issue is environmental impact along with the cost issues if failure occurs

Murphy says: Big Money Issues Will Prevail

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## Why Did They Inspect?

- Rule of thumb for this facility-
  - **Inspect tubes** if wall thickness has been reduced by 1/3, i.e. from 0.083” to 0.055”
  - **Consider retubing** heat exchangers when tube wall thickness has been reduced to ½ of original wall thickness, i.e. when wall thickness has been reduced from 0.083” to 0.0415”
- This exchanger has environmental concerns

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## Eddy Current vs IRIS Inspection

- Eddy current inspection is the usual quick and inexpensive inspection of each tube—minimum wall is reported for each tube
- IRIS inspection is a **more detailed and more expensive inspection** with a rotating head ultrasonic tool—minimum wall is reported for each tube and **tube ID's must be very clean for an accurate IRIS inspection.**

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# What Did IRIS Inspection Find?

- The minimum wall thickness report shows:

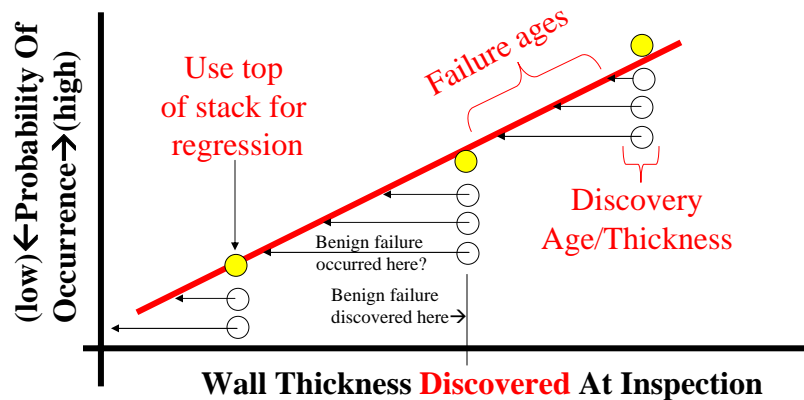
Rule of thumb triggers inspection at 0.050" →	Wall*qty		
	0.050*1	0.063*9	Wall thickness measured in inches
	0.055*1	0.064*9	
	0.056*2	0.065*4	
	0.058*2	0.066*5	
	0.059*1	0.067*2	
	0.061*6	0.069*4	

- Minimum allowed wall thickness is 0.036" for structural integrity.**

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# Stacks Of Data—Use Sherwin’s Inspection Option

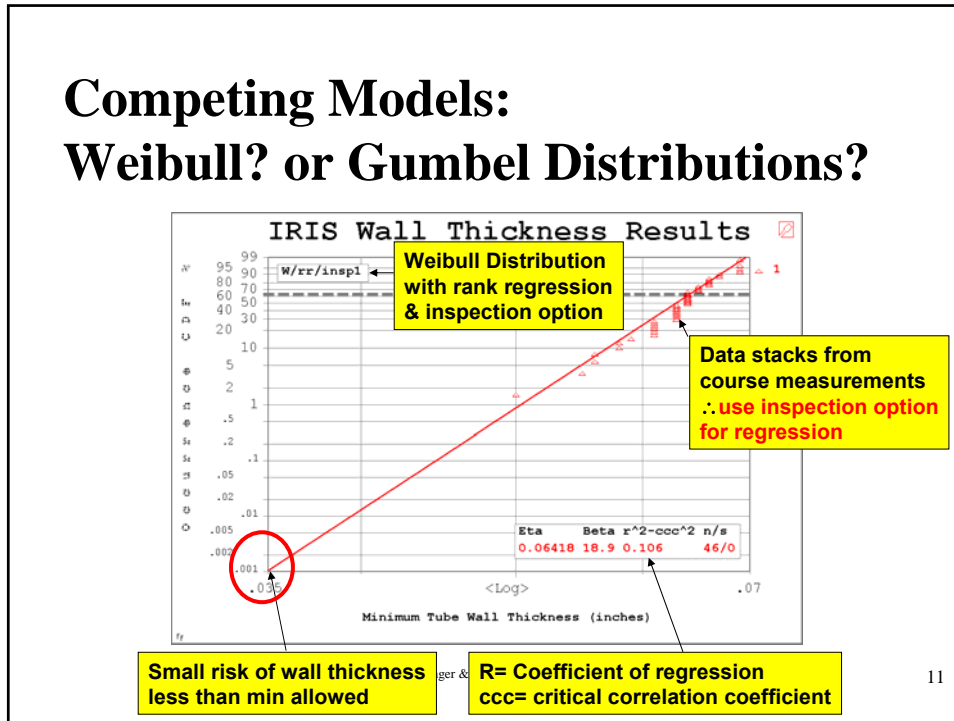


We have **stacks of data** from the heat exchanger inspection because the IRIS data have been rounded to three significant digits.

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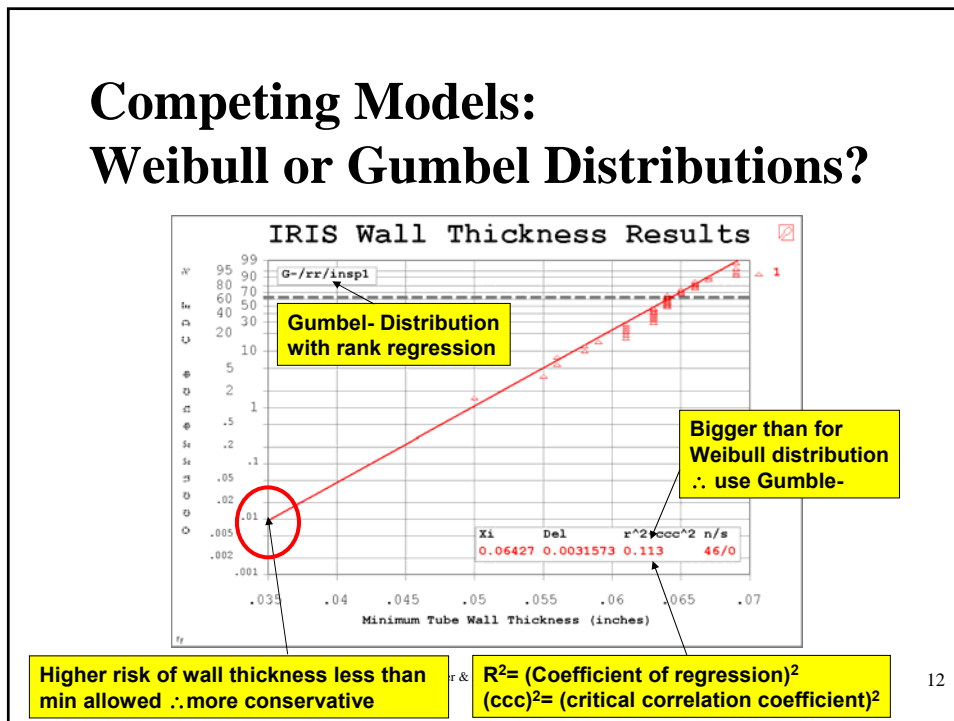
10

## Competing Models: Weibull? or Gumbel Distributions?



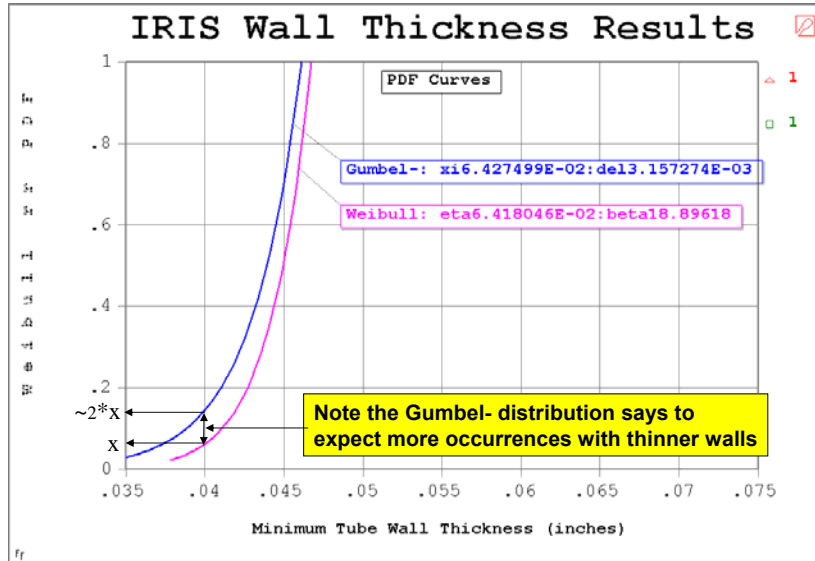
11

## Competing Models: Weibull or Gumbel Distributions?



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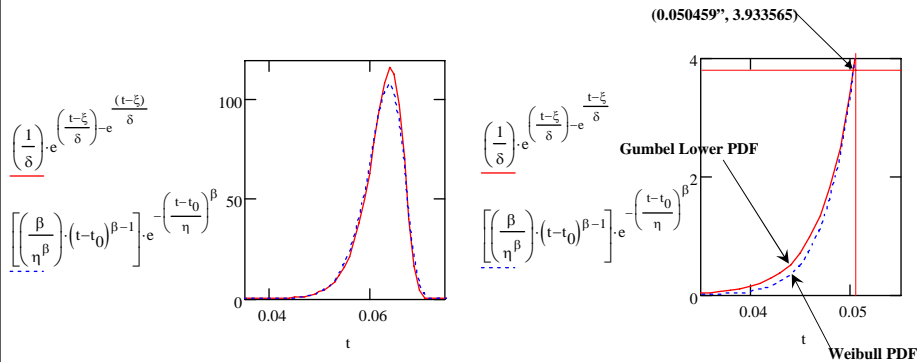
# PDF Curves



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# PDF Details



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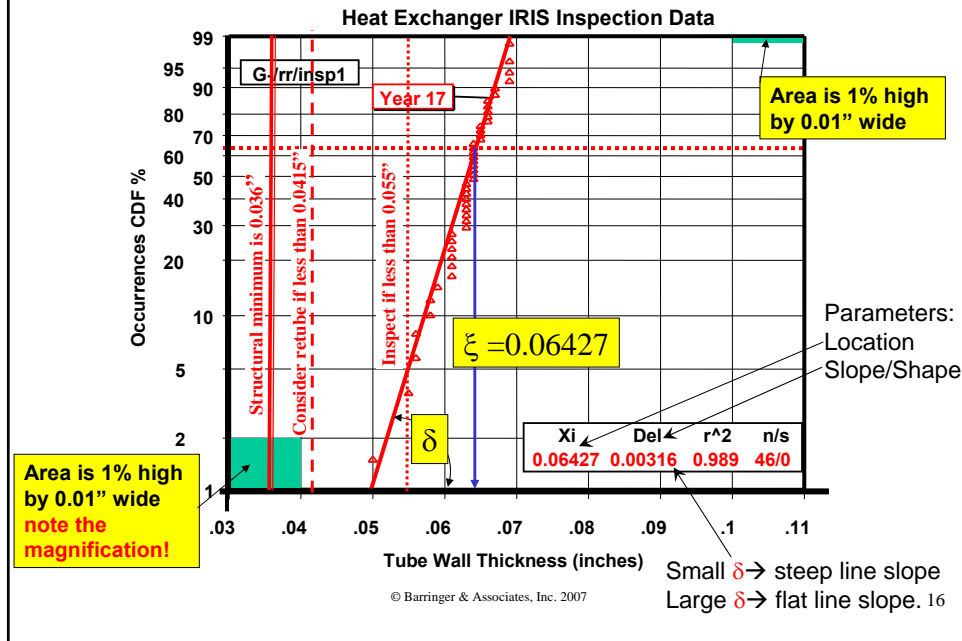
## Why Gumbel Lower Distribution?

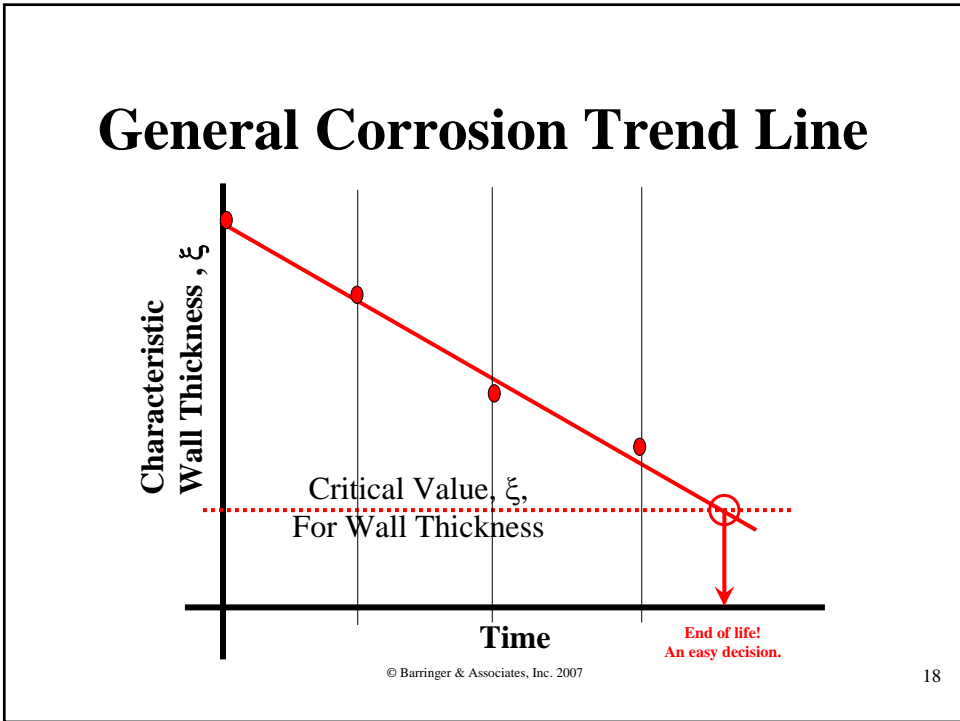
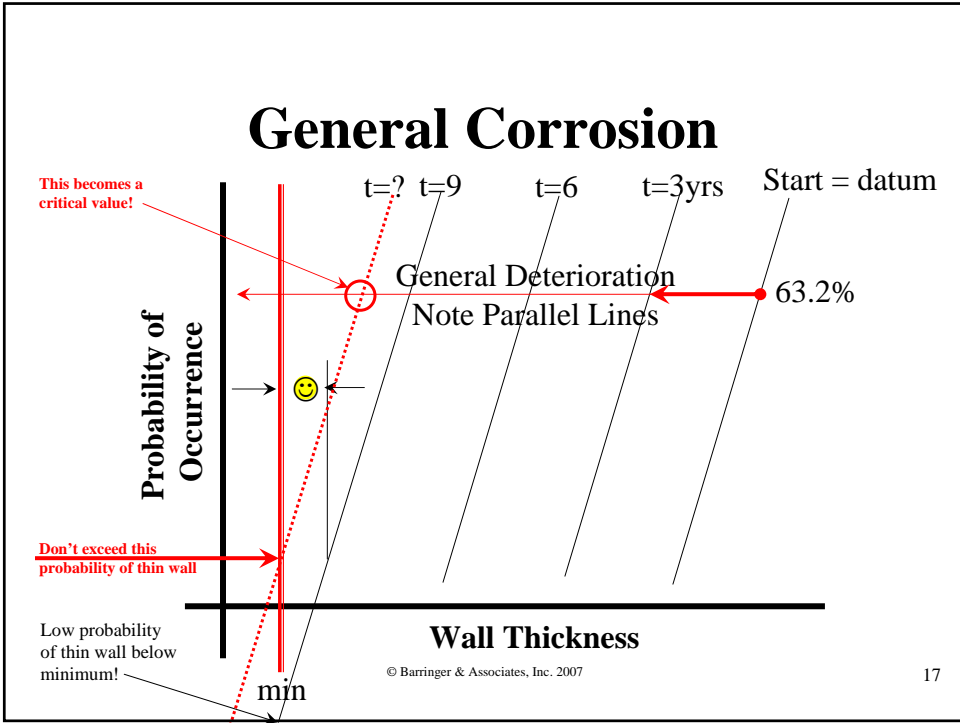
- The **SMALLEST** value is recorded for each tube thickness which motivates use of the **Gumbel smallest distribution**. Just as for flood data (the largest yearly value) motivates the use of the Gumbel largest distribution.
- The Gumbel smallest distribution is a better curve fit and shows greater % potential failure than Weibull, thus more conservative.

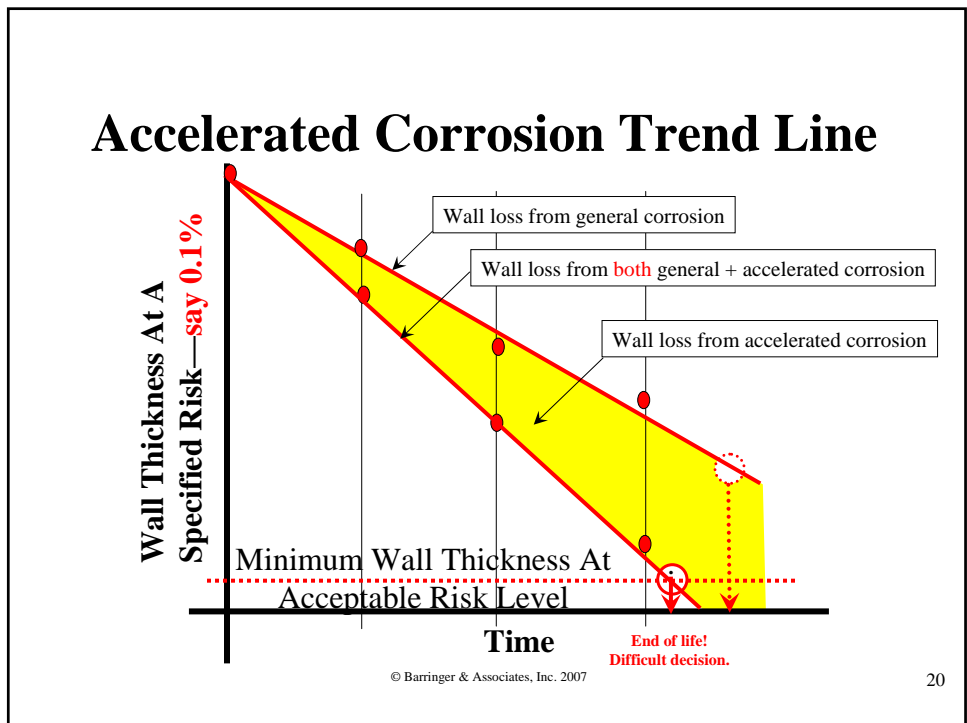
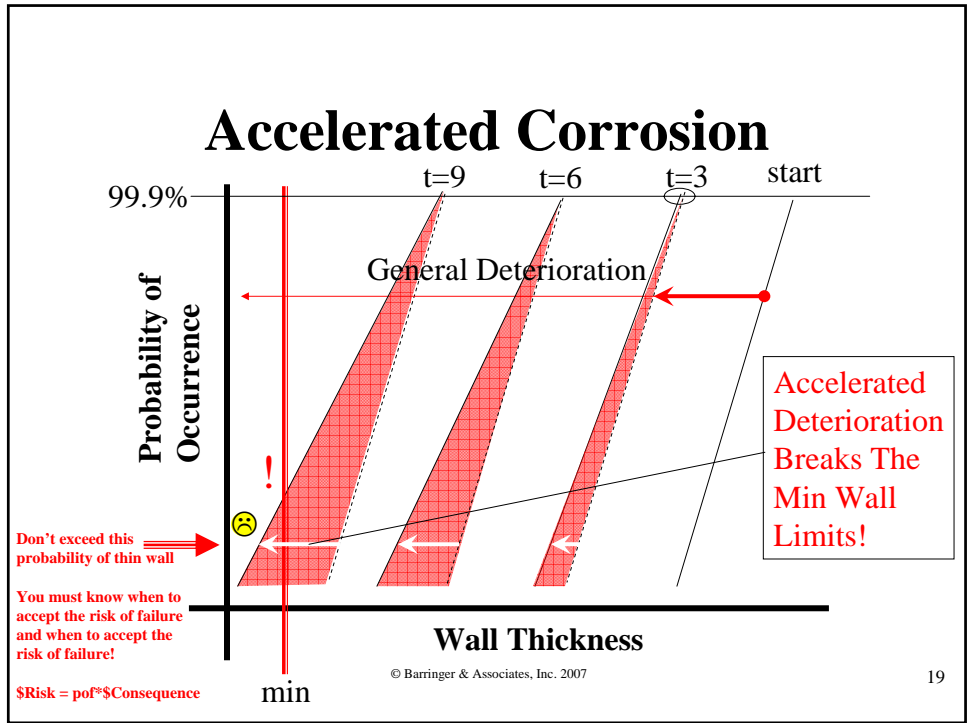
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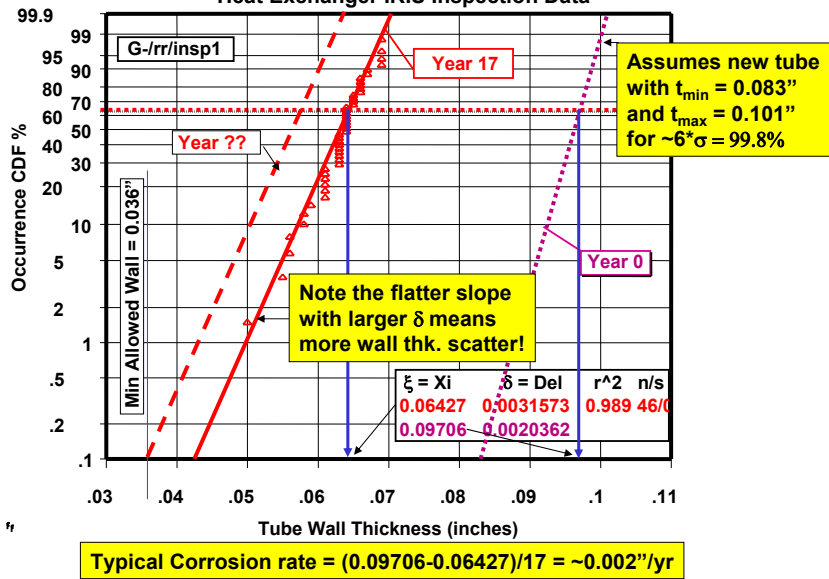
### Here's Where We Are At Year 17. Can We Make Year 20?





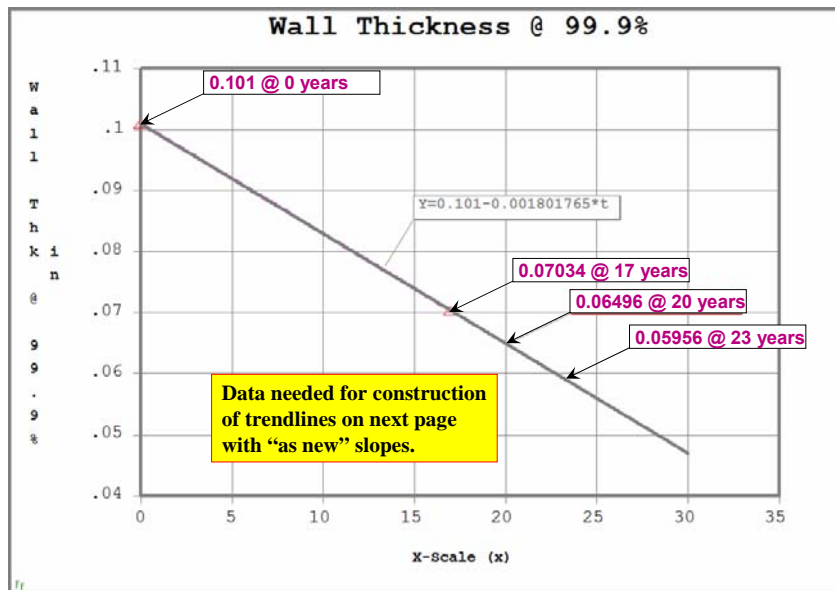


## You Must Know Wall Thickness At Time Zero Heat Exchanger IRIS Inspection Data

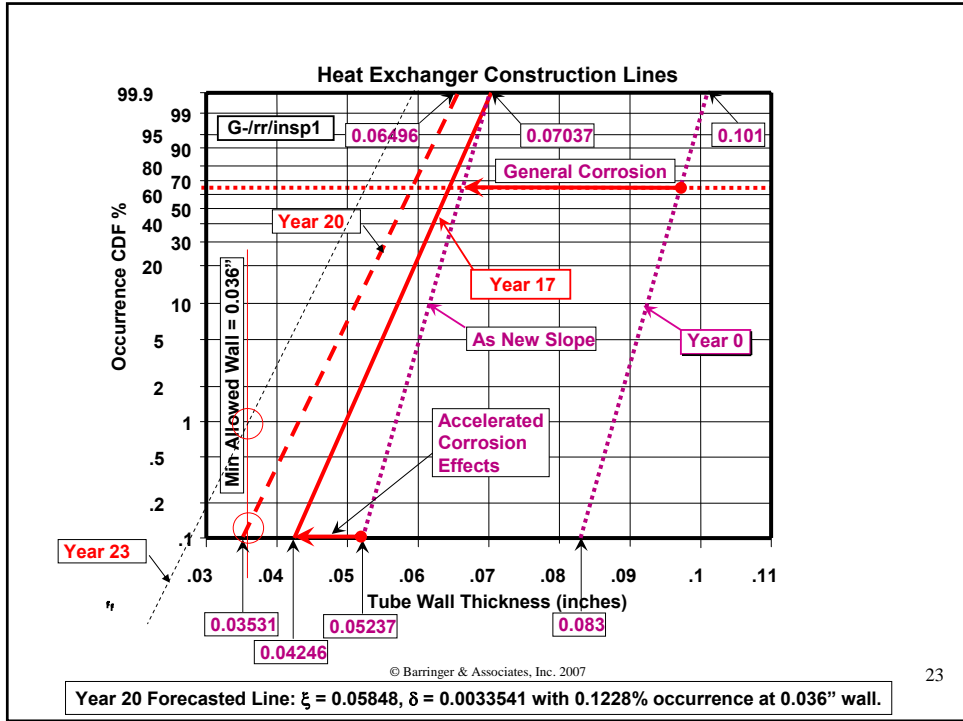


21

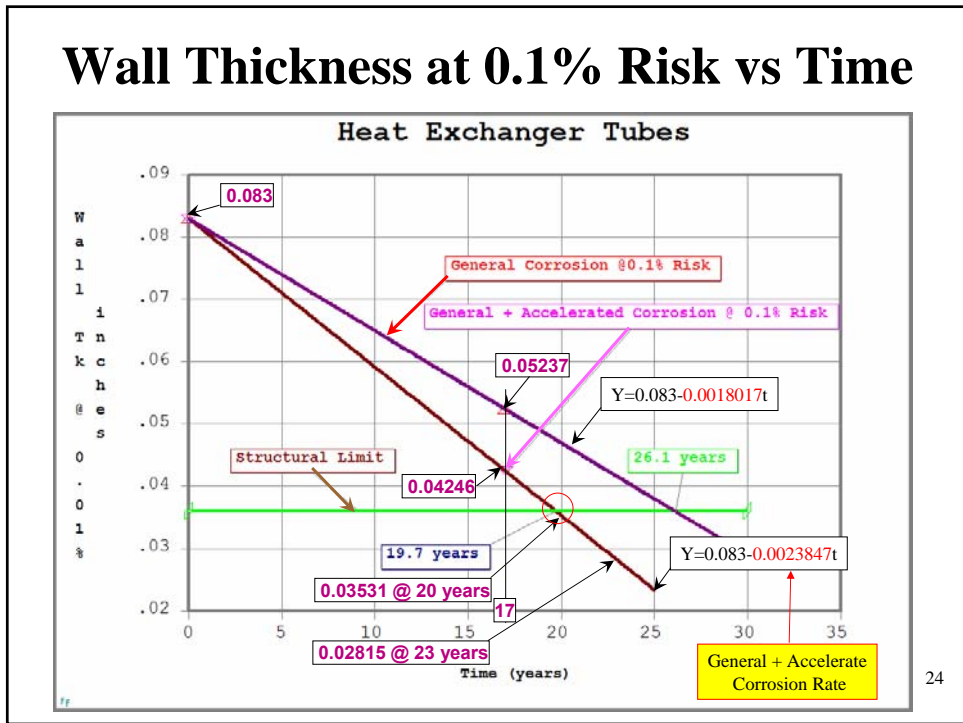
## Wall Thickness @ 99.9%



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## Retube Or Not Retube Now?

- At year 20 (next turnaround) the minimum wall thickness will decline to just under 0.036”
- The risk for falling below 0.036” min wall is 0.1228%
- **Time & Money Issues Converge**
- $\$risk = (\text{prob. of failure}) * \$Consequence, \therefore$   
 $\$risk\ exposure = 0.1228\% * \$750,000 = \$921$
- $\therefore$  take the risk for running 3 more years  
**—Do not retube now. Run to TA at yr 20.**

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## Tube Exchanger Summary

- Avoided the recently discovered and recently expected turnaround delay for accelerated delivery of heat exchanger (**\$750,000 expenditure avoided**) based on use of one day analysis of data.
- Pressing on toward the next turnaround three years into the future
- **At year 20, install a new tube bundle.**
- **What’s the risk for continuing to year 23?**  
 $0.91\% * \$750,000 = \$6,825$ —if risk adverse, reject.  
If risk accepting—**maybe, but very doubtful.**

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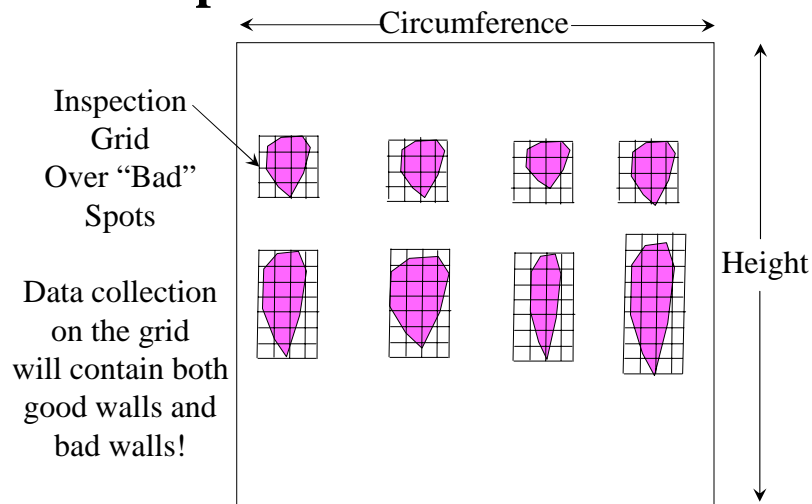
## Problem 2: Column Corrosion

- A column is rapidly losing wall thickness.
- Fluids/gasses within the column are violent.
- Frequent Inspections—data is all over the map!
- Loss of containment will impact personnel and environment issues with **big \$'s**
- What should we do:
  - Run?**—if so, for how long?
  - Shut down?**—if so, how to persuade the management team?

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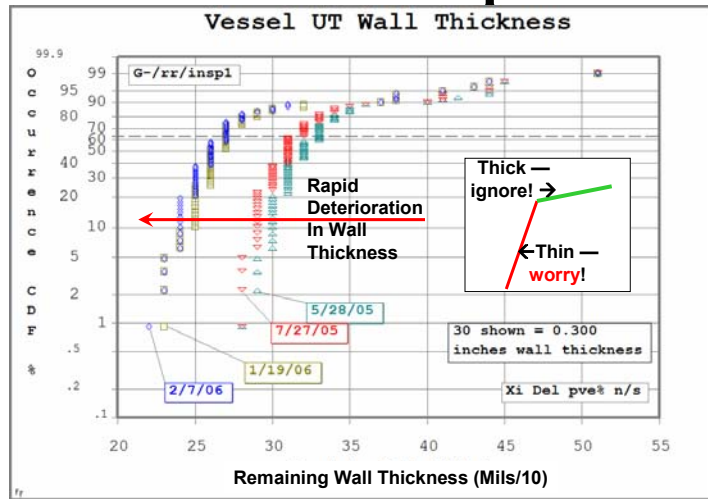
## Developed Outer Surface Of Tower



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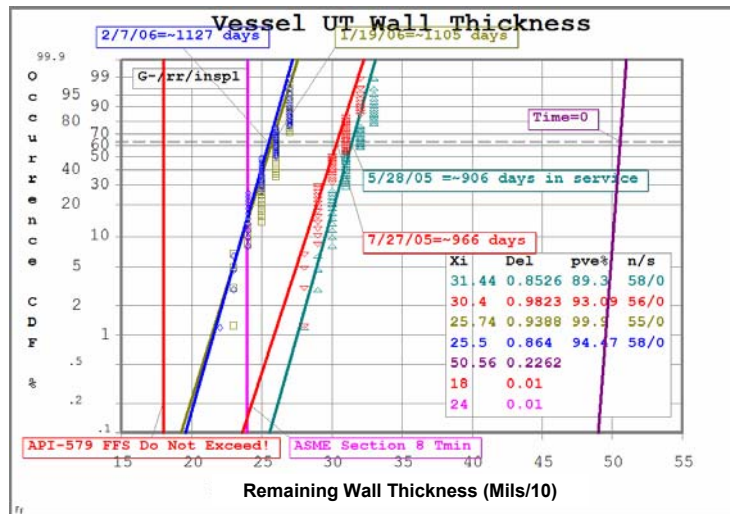
# Raw Data UT Inspections



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# Truncated Data—Thin Data Only



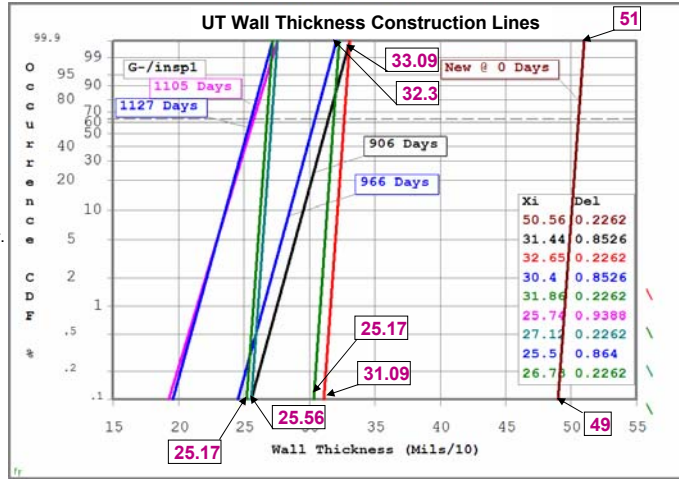
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# End Points For Corrosion Curve

General Corros.  
@ 0.1%  
Days Thickness  
0 49  
906 31.09  
966 25.17  
1105 25.56  
1127 25.17

Gen + Accel Cor.  
@ 0.1%  
Days Thickness  
0 49  
906 25.56  
966 23.61  
1105 19.26  
1127 19.53

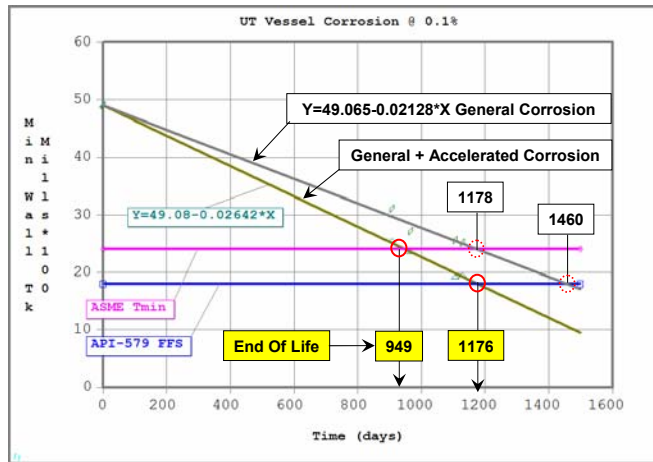


Gen + Accel Cor.  
@ 99.9%  
Days Thickness  
0 51  
906 33.09  
966 32.3  
1105 27.56  
1127 27.17

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# End Of Life Clearly Shown



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## Summary

- ASME minimum wall was violated at 949 days
- API fitness for service will be violated at 1176 days and we are 1127 days into service
- Plan an immediate orderly shutdown for replacement
- Outage + planned replacement = \$10,000,000
- Emergency outage + emergency replacement = \$20,000,000 because of safety hazards
- Risk is too high!  $0.1\% * \$10,000,000 = \$10,000$  and climbing toward \$20,000,000. **Take action now!**

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## Now, For Grins

- Consider the Gumbel **larger distribution**
- Houston flood
- Aircraft gust loads
- Space shuttle rocket motor O-ring burns

Discussions about the Gumbel lower distribution always raise questions about the Gumbel upper distribution

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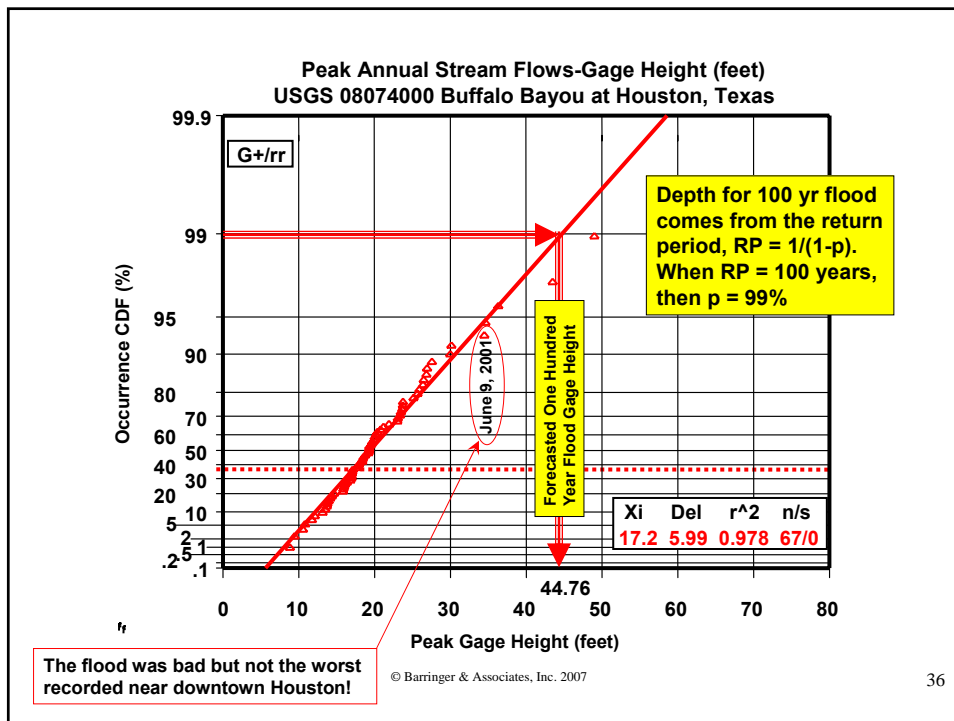
34

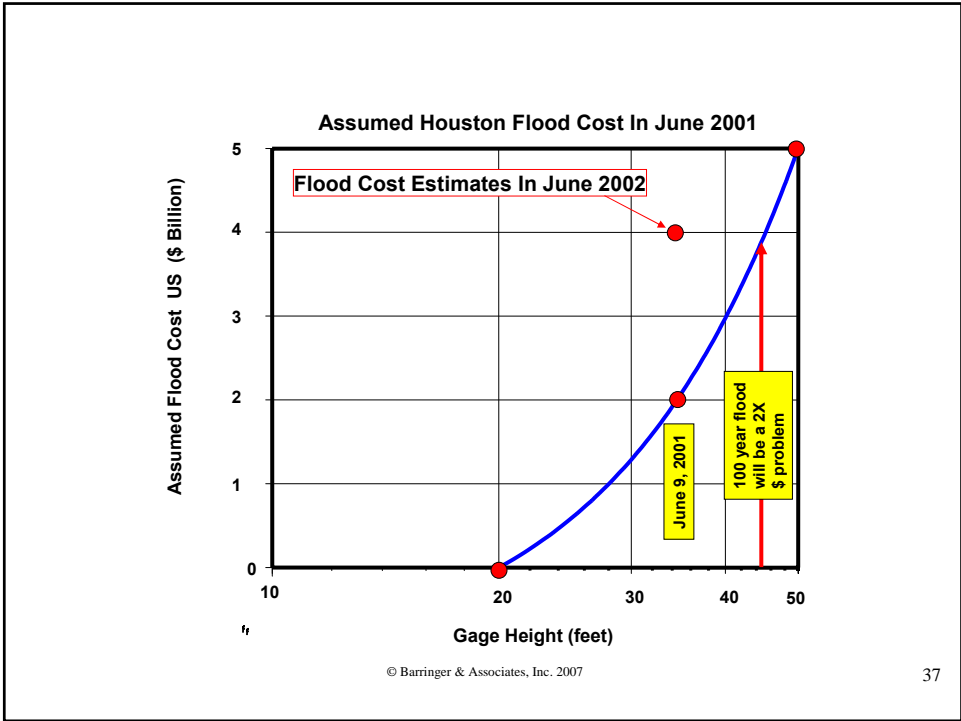
# It Rained A Little On June 9, 2001—23 Inches!



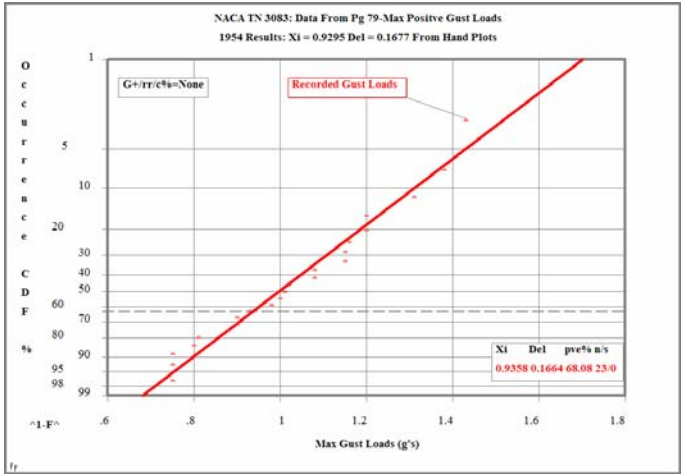
Cars are submerged on US 59 highway!

35

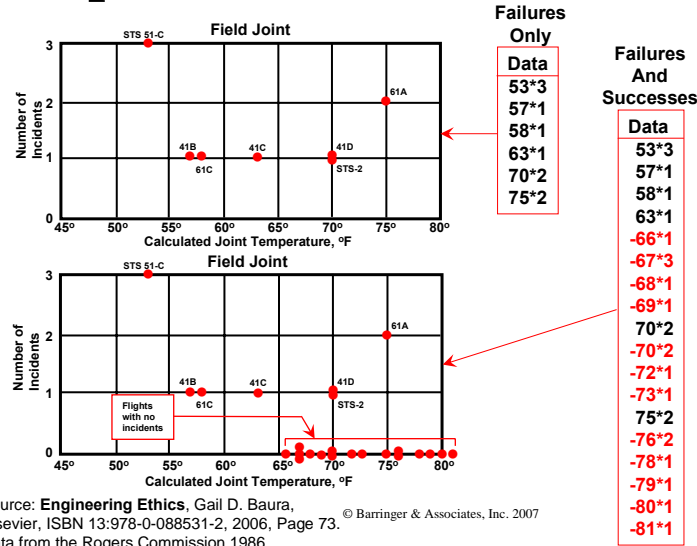




# Aircraft Positive Gust Loads



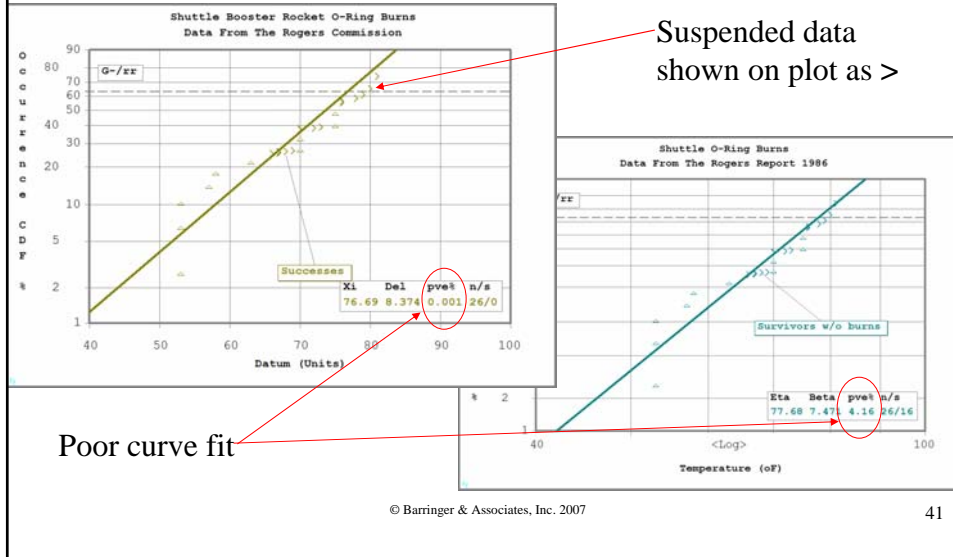
# Space Shuttle Burned O-Rings



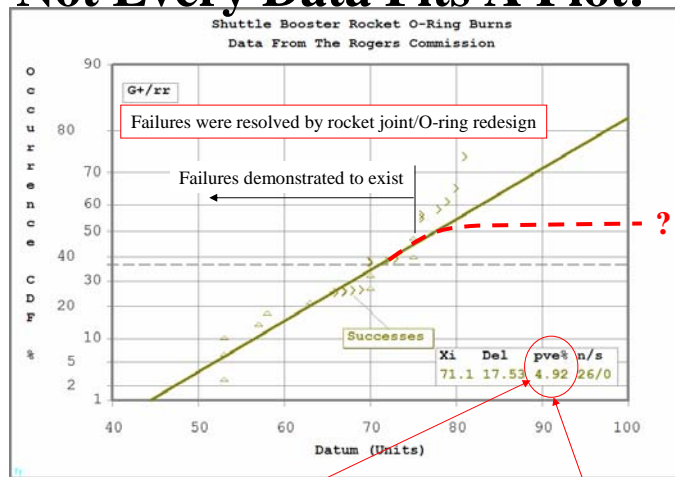
## Good Practice Advice—Watch Out!

- Gumbel upper & lower distributions **allow the use of negative numbers on the X-axis**
- When using suspensions (as a – sign) **make sure you turn on display of the suspensions** (under magnifying glass) so you can view they are in the correct locations **AND** (under the Method icon) **make sure to turn the negative sign to indicate suspension!**
- Else, you'll get misleading results.

# Which Plot?



# Gumble Upper Slightly Better-But Not Every Data Fits A Plot!



Better but not good curve fit

If you fail to turn on "c." is a suspension you will conclude this is a good fit!!



## Gumbel Upper Summary

- Works well when you have the largest recorded data such as **flood data, fatigue data**, etc.
- Watch for traps with suspensions when used without good practices can result in bad conclusions.
- If Weibull, Lognormal, etc. don't work then don't expect automatic success with all data by use of the Gumbel upper distribution.

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## Want More Details?

- Got to <http://www.barringer1.com/problem.htm>
- Look at **WinSMITH Weibull software** (which also includes Gumbel large and small distributions)
- See biographies at <http://www.barringer1.com> of **Dr. Weibull** and **Dr. Abernethy** who is the world's leading expert in Weibull analysis
- Dr. Weibull got many of his ideas on extreme values while working at Bofors Steel in Sweden—you can see Bofors antiaircraft guns at the **Museum of the Pacific** in Fredricksburg, TX.
- See Gumbel, E. J., **Statistics of Extremes**, Columbia University Press, New York, 1958
- See **Statistical Theory of Extreme Values And Some Practical Applications**, A Series of Lectures, PB 175818, 12 Feb 1954 by Emil J. Gumbel, National Bureau of Standards, U.S. Dept of Commerce, NTIS 
- See **A New Method Of Analyzing Extreme-Value Data**, NACA TN 3053, Jan 1954, U.S. Dept of Commerce, NTIS, by Julius Lieblein National Bureau of Standards 

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