APPENDIX A – BACKGROUND

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APPENDIX A

Background

Before going into the specifics of devising reliability growth planning curves, it is useful to look at the history of this process to learn why the curves have the form that they do.

The earliest reference that we have found on this subject is An Analytical Model of Reliability Growth Through Testing by H. K. Weiss, Handbook No. 54 304, 17p, AD-035 767, May 1954, Northrop Aircraft Inc., Hawthrone, California. Also, a useful survey of some early reliability growth methods is Reliability Growth Modeling by Larry H. Crow, Technical Handbook No. 55, August 1972, U.S. Army Materiel Systems Analysis Agency, Aberdeen Proving Ground, Maryland.

The Duane Postulate.

James T. Duane, an engineer with General Electric's Motor and Generator Department, published a paper titled "Learning Curve Approach to Reliability Monitoring" in IEEE Transactions on Aerospace, Vol. 2, No. 2, 1964. This paper recorded his observation that if changes to improve reliability (which are now termed fixes) are incorporated into the design of a system under development, then on a log-log plot, the graph of cumulative failure rate vs. cumulative test time is linear. This observation has become known as the "Duane Postulate." This empirically derived statement is the key to the most commonly accepted growth model in use today (see Section 6.1.4). A graph given in Duane's paper is shown in Figure 6.1. The straight lines are based on a least squares fit of the data. The negative slope of each line is defined to be the growth rate, α, for that line.

Duane's Growth Model.

On a Log-Log Plot, the graph of *Cumulative Failure Rate* *Vs* *Cumulative Test Time* *is Linear*

Let N (t) = the total Number of failures by time t. Then the average failure rate, also called Cumulative failure rate C(t), can be found by dividing N(t) by t.

$$C(t) = \frac{N(t)}{t}$$

Let δ be the y-intercept on a log-log plot of the straight line that Duane postulated. The slope-intercept formula for this line then becomes:

$$Log\ C(t) \ = \ \delta - \alpha\ Log\, t$$

where log denotes the natural (base e) logarithm (although any base could be used).

The Duane Postulate:
$Log\ C(t) = \delta - \alpha Log\ t$

Taking anti-logs

$$C(t) \ = \ \lambda\, t^{-\alpha}$$

where

$$\delta \ = \ \ln \lambda$$

Multiplying C(t) by t gives N(t), and multiplying $t^{-\alpha}$ by t adds 1 to the exponent, $t^{1-\alpha}$.

So

$$N(t) \ = \ \lambda t^{1-\alpha}$$

Taking the first derivative of the number of failures with respect to time gives the instantaneous failure rate, r(t), at time t.

$$r(t) \ = \ \frac{d\, N(t)}{d\, t} \ = \ \lambda(1-\alpha)t^{-\alpha}$$

Duane's model thus has two parameters, α and λ. The first, α, determines the shape of the growth curve. The second, λ, is the size parameter for the curve. With these two parameters, the cumulative number of failures N(t), the average failure rate C(t), and the instantaneous failure rate r(t) can be calculated for any time t within the test. Further, given α and λ, it is possible to solve for t, the amount of testing time it will take to achieve a specific reliability. This assumes that the factors affecting reliability growth remain unchanged across the development.

Drawbacks to Duane's Method.

Duane stated that α could be universally treated as being .5, as that seemed to be the modal value within his database. This has since been shown to be unrealistic. It does not allow for different test environments causing failures to be surfaced at different rates, and for different levels of engineering effort causing different rates of fix insertion.

The reliability values calculated using his method are treated as being deterministic. That is, there is no allowance for the variation that is typically observed about an estimated value, and there is no way of judging whether the observed value, which rarely matches the estimated value, is close enough. Further, there is no way to check whether the model is valid for the current test situation.

All Duane growth curves pass through the origin of the graph. That is, the item under test is imputed to have zero reliability at the start of test.

The Crow/AMSAA Growth Model.

Larry H. Crow while at the U.S. Army Materiel Systems Analysis Activity's Reliability and Maintainability Division published Reliability Analysis for Complex, Repairable Systems, Technical Handbook No. 138, December 1975, U.S. AMSAA, Aberdeen Proving Ground, Maryland. In this report, Dr. Crow explored the advantages of using a Nonhomogeneous Poisson Process with a Weibull intensity function to model several phenomena, including reliability growth. If system failure times follow the Duane Postulate, then they can be modeled as a Nonhomogeneous Poisson Process with Weibull intensity function. To make the transition from Duane's formulae to the Weibull intensity functional forms, β has to be substituted for $1-\alpha$. Thus the parameters in the Crow model are λ and β, where β determines the shape of the curve. The physical interpretation of β (called the growth parameter) is the ratio of the current (instantaneous) MTBF to average (cumulative) MTBF at time t.

This stochastic interpretation immediately brings the benefits of Statistics to the formulae that Duane had derived. That is, the parameters λ and β can be determined using maximum likelihood estimators (mle's) rather than β being assumed to be fixed. Further, hypothesis tests and confidence limits can be determined for the parameters, and Goodness-of-Fit tests can be performed on the model. This eliminates the first two drawbacks of Duane's model. We will discuss later how Crow handles the problem of imputed zero reliability at the start of test.

One should take note that even though the growth rate estimate $\hat{\alpha}$ can be calculated from Crow's growth parameter estimate, $\hat{\beta}$, and it is still interpreted as the estimate of the negative slope of a straight line on a Log-Log plot, Crow's estimates of λ and β are somewhat different from the ones derived using Duane's procedures. This follows from the fact that the estimation procedure is mle, not least squares, thus each model's parameters correspond to different straight lines, respectively.