Load-Strength Interference

Loads vary, strengths vary, and reliability usually declines for mechanical systems. The cause of failures is a load-strength interference problem frequently describing mechanical systems which go bump in the middle of the night. Failure of mechanical systems "made the same" as other systems, invokes the old Cornish prayer:

"From ghoulies and ghosties and long-leggety beasties and things that go bump in the night—Good Lord deliver us!"

Bumps in the night occur when loads are higher than strengths, or strengths are lower than the loads. So it might be a good idea to work out these details with facts rather than praying for help from the Lord to cover our ignorance.

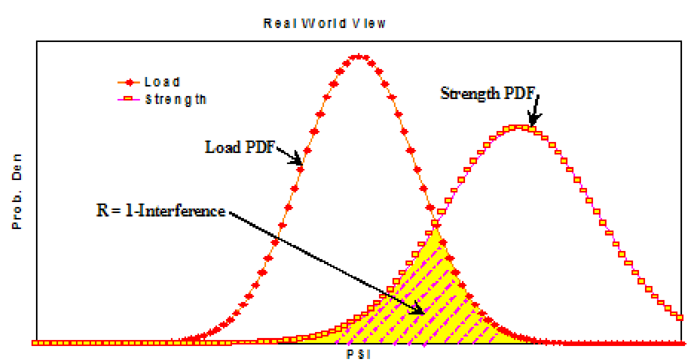
Things don't fail on the basis of averages (assuming average loads are widely separated from average strengths), parts fail on high loads/weak strengths. To get your thinking adjusted to this concept, people don't usually drown in average depths in a river, they drown in deep water!

Seldom is the load fixed. When loads vary to the low side everything is failure free. When loads vary to the high side, failures occur and reliability is lost.

Seldom is the strength fixed. When strength varies to the low side, often because of lack of homogenous substances, failures occur and thus reliability is lost. When strengths vary to the high side everything is failures free.

The condition of unreliability is described in Figure 1. Loads and strengths interfere in the overlap area and this is the area of concern for failures.

Figure 1—Load-Strength Interference



The overlap of load-strength in Figure 1 is not literally the calculated area. It's a joint probability of occurrence problem or roughly ½ of the literal area. The joint probabilities are described by a double integral for reliability. If large safety margins are used, the probability of incurring failure from load-strength interference is usually very low when both loads and strengths are well known. If low safety margins are used, ignorance abounds for of load distributions, or ignorance of strength distributions, it's another sad story. Failures occur where loads/strengths overlap.

O'Connor (see **Practical Reliability Engineering**, by Patrick D. T. O'Connor, John Wiley, 2002, ISBN 0-470-84462-0, ***Chapter 4 Load-strength Interference***, page 114-129) points out that where loading roughness is low (i.e. small standard deviation of loads) and strengths are well behaved (i.e., small standard deviations of strengths) and displaced widely to the right of the loads you can achieve intrinsic reliability where the probability of failure is low when large safety margins of 3-5 are used. If the load curve is skewed to the right and the strength curve is skewed to the left, then larger safety margins are required for high reliability. In short, you need to know the load curves and the strength curves and keep them widely separated to achieve high reliability by avoiding load-strength interference. Safety margins of 3-5 are suitable for pressure vessels but not airplanes—so keep in mind the class of equipment you're designing/maintaining and use the appropriate strategies.

Reliability of the component can be determined as the probability of load being less than the strength for all possible values of strength (see **Reliability-Based Design**, by S. S. Rao, McGraw-Hill, Inc., 1992, ISBN 0-07-051192-6, ***Chapter 8: Strength Based Reliability and Interference Theory,*** page 235-273). Rao's equations of importance are:

$$R(s) = \int_{-\infty}^{\infty} f_S(s) * [\int_{-\infty}^{s} f_L(l) dl] ds = \int_{-\infty}^{\infty} f_S(s) * F_L(s) ds$$

←Equivalent to Rao's Eq. 8.9

Where $f_S(s)$ is the probability density function of the strength, $f_L(l)$ is the probability density function of the load, and $F_L(s)$ is the cumulative distribution function of the load in units of the strength. The reliability statement is a statement of success.

The statement of **un**reliability UR = 1- R = pof is a statement of probability of failure expressed by Rao as:

$$P_f(s) = \int_{-\infty}^{\infty} f_L(s) * F_S(s) ds$$

←Equivalent to Rao's Equation 8.13

Where $f_L(s)$ is the probability density function of the load in strength units and $F_S(s)$ is the cumulative distribution function of the load in strength units.

You have three obvious ways to solve this complicated and convoluted problem:

1) Use Mathcad to solve the integral (download the mathcad ZIPPED file.
2) Use WinSMITH Weibull to solve the problem using Monte Carlo simulation--(download the demonstration program to solve the problem click on the calculator icon, click on load-strength interference and input the statistical data). The methodology for the simulation is described below for the Excel simulation.
3) Use Excel to solve the problem (download a Monte Carlo simulation). The

simulation draws a random load and a random strength from the described distribution. If the strength is greater than the load you have a success. If the random load exceeds the random strength, you have a failure. The calculation for reliability is R = (successes)/(successes + failures).

With the Mathcad file and the Excel simulation file, you can see the equations used for the probability density function and cumulative distribution function.

How do you find the correct statistical distributions to use for the load analysis?

1) Measure and record the loads over time. Treat the data as samples. Construct a probability distribution in WinSMITH Weibull. Use good common sense and good engineering judgment to select the appropriate distribution. The distribution will allow you to predict loads above/below the actual data recorded when you treat the data as a sample.

2) Measure the strengths for many samples as described above for the loads. If you have many data, you may find strength data displays a failure-free zone. The strength phenomena occurs with offset of the origin of the distribution. This is described in The New Weibull Handbook as a t_0 shift for a 3-parameter distribution. Dr. Abernethy in The New Weibull Handbook sets four requirements for use of a 3-parameter distribution:

1. More than 21 data points are required for a valid analysis
2. Raw data plotted on a 2-parameter probability plot will show a concave downward appearance.
3. The goodness of fit criteria (i.e., R^2 or PVE%) must show substantial improvement with use of a 3-parameter distribution compared to the 2-parameter curve fit.
4. A physical explanation of the reason for shifting the origin of the distribution must be obvious. For example, if the yield strength of the steel grade is 110,000 psi, then no steel is released by the steel mill if it's strength is less than 110,000 psi. Thus yield strengths when plotted on a 2-parameter probability plot display a concave downward appearance but will show a straight line on a 3-parameter plot when when 110,000 psi is subtracted from the raw data to reposition the data--be careful with data in the t_0 as the numbers can be misleading unless all details are kept in the datum as recorded.

By the way, many data collected often are recorded in convenient units. Convenient units often results in stacks of data on a probability plot. This requires the use of special regression techniques to achieve the correct statistics and the method can be selected in WinSMITH Weibull under the methods icon.

Loads and strengths are often described by the following common distributions:

1. Normal (Gaussian statistics of the bell shaped curve)
2. Weibull (Weak links in the chain failures)
3. Lognormal (Events accelerate with many small data and some large data)
4. Gumbel smaller distribution (Where **small data** are of concern and recorded)
5. Gumbel upper distribution (Where **large data** are of concern and recorded)

Other distributions can be envisioned and used when appropriate but these mentioned distributions will cover 95^{+}% of the situations. Use common sense and good engineering judgment in selecting the distributions. Make sure the distributions display reasonable graphics for comprehension by the engineering community. The comprehension criterion for engineers is simple: No graphics—No comprehension! The equations for each distribution are shown in Figure 2. The PDF is the probability density function which has an area under the curve of unity and shows the shape of the curve you would get if you made a tally sheet of occurrences on the Y-axis versus the unit of measure on the X-axis. The CDF is the cumulative distribution frequency and integrates the area under the PDF (which is then subtracted from unity to predict the % of the population that will occur on the Y-axis versus the unit of measure on the X-axis.

Figure 2: PDF and CDF Equations

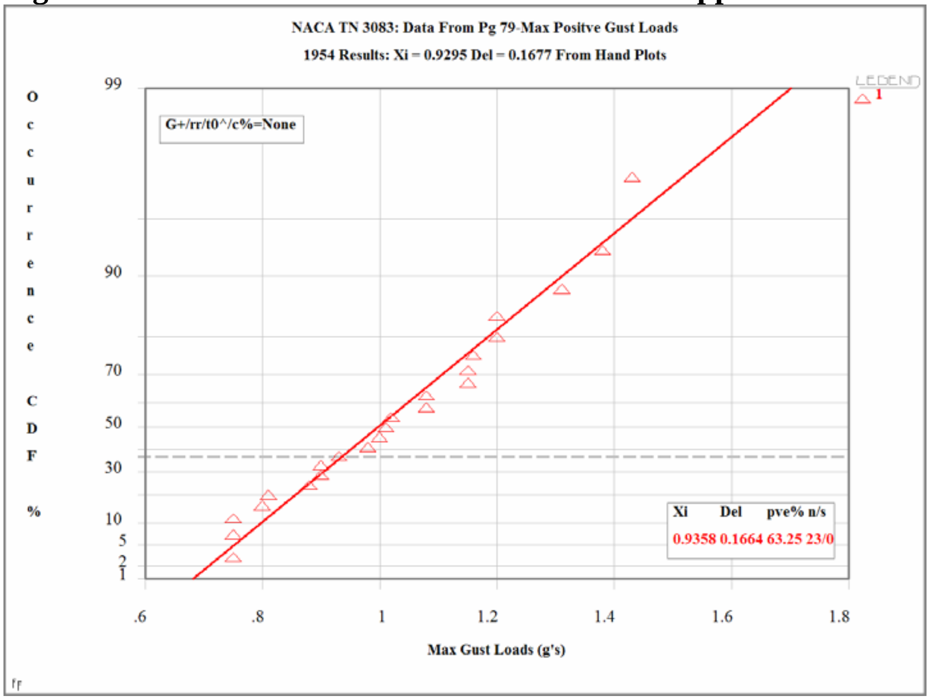
Equations For Different Load-Strength Combinations

	PDF	CDF
See **WinSMITH Weibull:** Mean = μ Std. deviation = σ **Normal**	$f_N(t) = \left(\frac{1}{\sigma\cdot\sqrt{2\cdot\pi}}\right)\cdot e^{\frac{-\left[(t-\mu)^2\right]}{2\cdot\sigma^2}}$	$F_N(t) = 1 - \int_t^{\infty}\left(\frac{1}{\sigma\cdot\sqrt{2\cdot\pi}}\right)\cdot e^{\frac{-\left[(t-\mu)^2\right]}{2\cdot\sigma^2}}\,dt$
See **WinSMITH Weibull** : MuAL = μ SigF = σ Displace Origin = t_0 **Lognormal**	$f_{LN}(t) = \left[\left[\frac{1}{(t-t_0)\cdot\ln(\sigma)\cdot\sqrt{2\cdot\pi}}\right]\cdot e^{-\frac{\ln\left(\frac{t-t_0}{\mu}\right)^2}{2\cdot\ln(\sigma)^2}}\right]$	$F_{LN}(t) = 1 - \int_t^{\infty}\left[\left[\frac{1}{(t-t_0)\cdot\ln(\sigma)\cdot\sqrt{2\cdot\pi}}\right]\cdot e^{-\frac{\ln\left(\frac{t-t_0}{\mu}\right)^2}{2\cdot\ln(\sigma)^2}}\right]dt$
See **WinSMITH Weibull:** Shape factor = β Characteristic Life = η Displace Origin = t_0 **Weibull**	$f_W(t) = \left[\left(\frac{\beta}{\eta^{\beta}}\right)\cdot(t-t_0)^{\beta-1}\right]\cdot e^{-\left(\frac{t-t_0}{\eta}\right)^{\beta}}$	$F_W(t) = 1 - e^{-\left(\frac{t-t_0}{\eta}\right)^{\beta}}$
See **WinSMITH Weibull:** Location = ξ (XI) Shape Factor = δ (Del) **Gumbel Lower**	$f_{GL}(t) = \left(\frac{1}{\delta}\right)\cdot e^{\left(\frac{t-\xi}{\delta}\right) - e^{\frac{t-\xi}{\delta}}}$	$F_{GL}(t) = 1 - e^{-e^{\frac{t-\xi}{\delta}}}$
See **WinSMITH Weibull:** Location = ξ (XI) Shape Factor = δ (Del) **Gumbel Upper**	$f_{GU}(t) = \left(\frac{1}{\delta}\right)\cdot e^{\left[\frac{-(t-\xi)}{\delta}\right] - e^{-\frac{t-\xi}{\delta}}}$	$F_{GU}(t) = e^{-e^{\frac{-(t-\xi)}{\delta}}}$

Sometimes the problems are difficult to solve in closed form solutions (where a failure free interval exists). Others are difficult to solve where the probability of failure is very low and a very large number of iterations must be run to get an answer that is not zero.

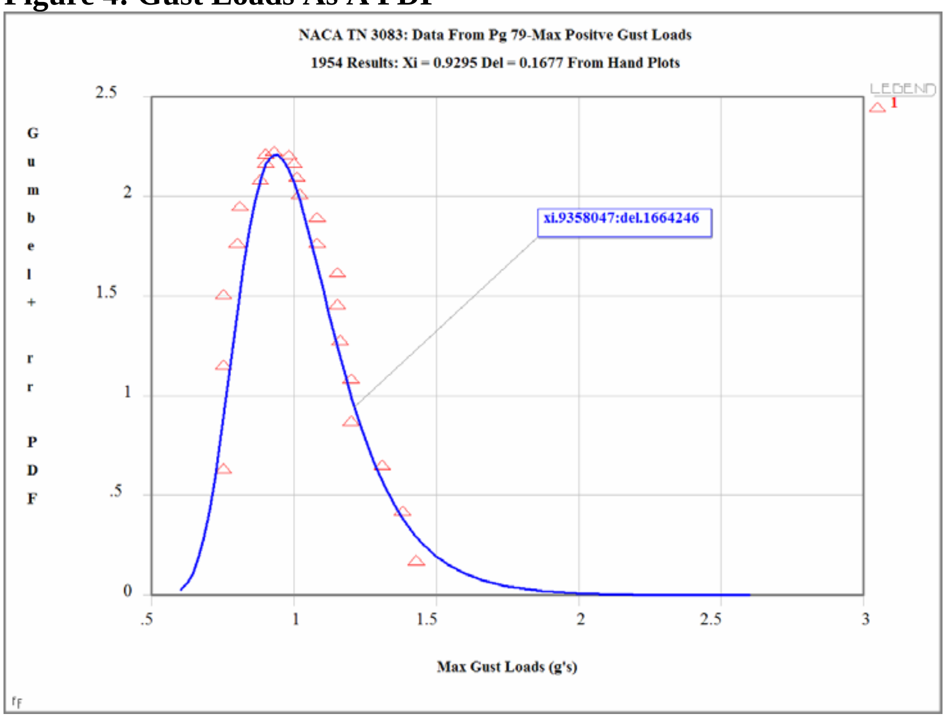
Consider the gust loads in Figure 3. The X-axis is give in g-loads. The data recorded was the maximum positive g-load from each of 23 flights. The plot as made in WinSMITH Weibull software.

Figure 3: Gust Loads Described With A Gumbel Upper Distribution



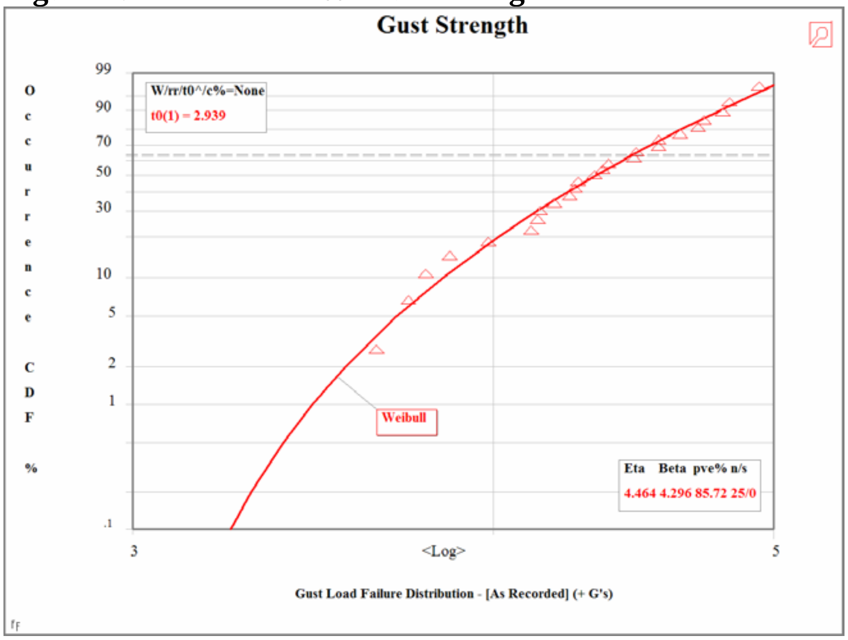
The PDF curve of the load is described in Figure 4. The data was generated from WinSMITH Weibull and plotted in WinSMITH Visual. Notice the long tail to the right with this actual data from a 1954 NACA document.

Figure 4: Gust Loads As A PDF



Strengths were obtained and plotted in a Weibull plot in Figure 5. Again, note the X-axis is shown in g-loads and the failure free interval is 2.939.

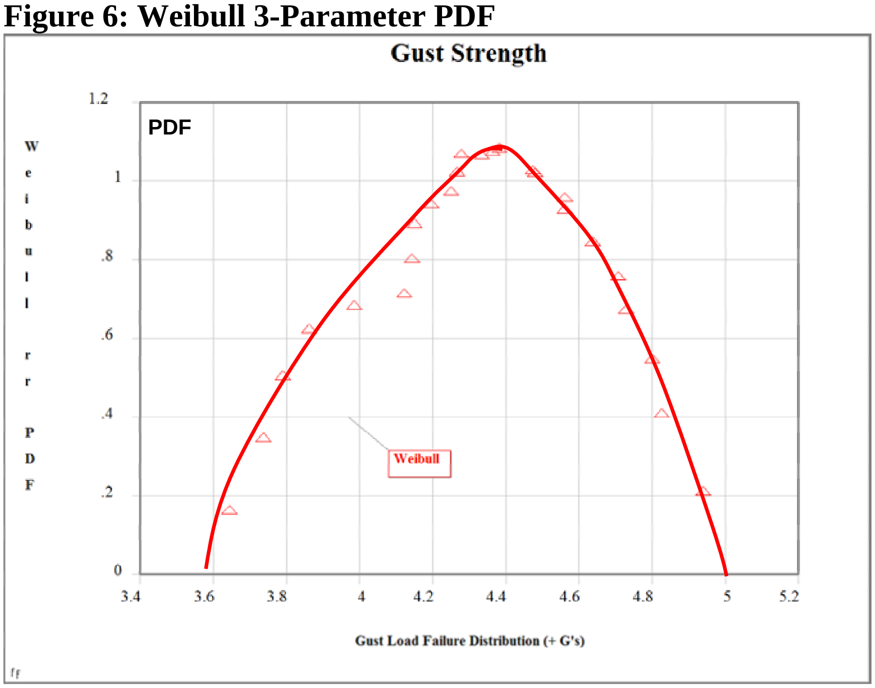
Figure 5: 3-Parameter Weibull Strength Plot



This strength curves is selected on purpose because of it's calculation difficulties.

The PDF for Figure 5 is shown in Figure 6.

Figure 6: Weibull 3-Parameter PDF



The calculations and difficulties are shown in Figure 7.

Figure 7: Mathcad Load-Strength Interference Calculations

Some times the problem is difficult in closed form solution. Consider this case where aircraft gust loads and airframe strength are widely separated with a failure free zone.

Load is Gumble Upper:

$\xi := 0.9358$ $\delta := 0.1664$ $g := 0, 0.1 .. 10$

Strength is Weibull 3-Parameter:

$\beta := 4.296$ $\eta := 4.464$ $t_0 := 2.939$

$$f_{GU}(g) := \left(\frac{1}{\delta}\right)\cdot e^{\left[\frac{-(g-\xi)}{\delta}\right] - e^{-\frac{g-\xi}{\delta}}}$$

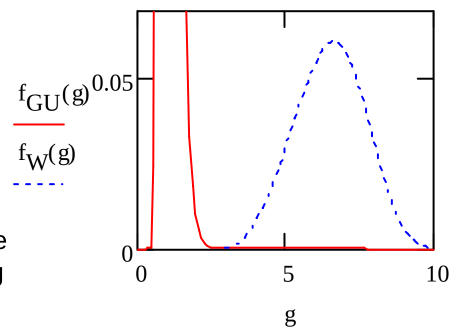
$$F_{GU}(g) := e^{-e^{\frac{-(g-\xi)}{\delta}}}$$

$$f_W(g) := \left[\left(\frac{\beta}{\eta^{\beta}}\right)\cdot(g-t_0)^{\beta_L-1}\right]\cdot e^{-\left(\frac{g-t_0}{\eta}\right)^{\beta}}$$

$$F_W(g) := 1 - e^{-\left(\frac{g-t_0}{\eta}\right)^{\beta}}$$

$$R(g) := \int_0^{10} f_W(g)\cdot F_{GU}(g)\,dg$$

$$P(g) := \int_0^{10} f_{GU}(g)\cdot F_W(g)\,dg$$



The analytical solution blows up! Why?--it's because of the Weibull strength and the failure free zone for g loads less than 3.

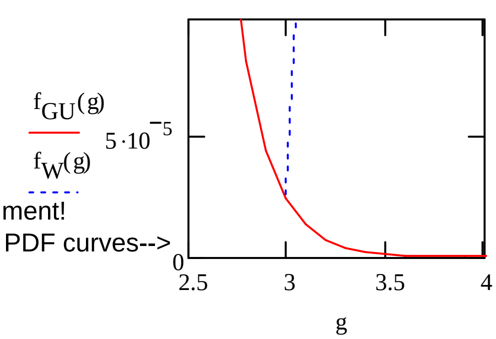
The Reliability and Probability Of Failure equations must be changed to avoid the failure free zone!!

$$R(g) := \int_3^{10} f_W(g)\cdot F_{GU}(g)\,dg \qquad P(g) := \int_3^{10} f_{GU}(g)\cdot F_W(g)\,dg$$

$R(10) = 0.187069$ <--This is obviously an error!

$P(10) = 1.629122\times 10^{-10}$ <--Looks better!

Don't forget to use good engineering judgment!
Notice the small amount of overlap in the PDF curves-->



Use good engineering judgment and practical experience in interpreting the answer, and take some of the results with a grain of salt. By the way, if the structure only has a 2-Parameter Weibull plot for strength, you will get a significantly different probability for failure so strength distributions are very important!

Return to the list of problems by **clicking here**.

Refer to the caveats on the **Problem Of The Month Page** about the limitations of the solution above. Maybe you have a better idea on how to solve the problem. Maybe you find where I've screwed-up the solution and you can point out my errors as you check my calculations. E-mail your comments, criticism, and corrections to: Paul Barringer by **clicking here**. Return to the top of this problem.

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