New Methods for Weibull & Log Normal Analysis

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Abstract In the decade since the U.S. Air Force Weibull Analysis Handbook[1] was published, many new methods and applications have developed in different industries. The Weibull method was first applied in the aerospace and automotive industries. Today it is employed in electric power, nuclear, medical, dental, and components such as bearings, compressors, cables, wheels, etc. Chemical, oil, instrumentation, electronics, and railroads are the newest converts to Weibull. The drudgery of laborious Weibull hand calculations and plotting is eliminated by the availability of friendly software[2]. The purpose of this paper to illustrate a few of these new methods and applications. For a more complete treatment, an A.S.M.E Weibull Workshop is recommended, as well as the New Weibull Handbook. Both will be available in 1993. Engineers prefer graphical methods that are emphasized herein. The new methods described herein include:

- Graphical Methods & Goodness-of-Fit
- Special Methods for Inspection Data
- Prediction of Future Failures- Weibull Risk
- Instrumentation Calibration Intervals
- Machine Tool and Robotic Accuracy
- WeiBayes to Reduce Testing Costs
- Monte Carlo Simulation

Graphical Methods Engineers look for trends in life data in many types of graphs. The data is often a simple tally sheet of events as a function of age. The tally data provides the engineer with insight when plotted on probability paper. There are specific probability papers which rectify different distributions. For life data the most popular by far is the Weibull distribution, followed by the log normal. The method converts the simple tally into a plot showing probability of failure versus age, such as time, milage, cycles, etc. If the data fits a straight line on the probability paper, there is evidence that the selected distribution is correct. The question is how good is the fit?

Simple Measures of Goodness-of-Fit Engineers are familiar with fitting curves with the regression method that produce correlation coefficients. The correlation coefficient is a simple measure of the strength of a linear relationship. It would be ideal for goodness of fit for probability plots except that correlation coefficients are artificially increased by assigning probability plotting positions to the ranked data. The correlation coefficients are too high. All probability plots such as Weibull, normal, & log normal, suffer this problem with the correlation coefficient, but it is the most widely used measure of fit. Engineers ask what correlation indicates a good fit? The problem is to find a critical value of the correlation coefficient for probability plots such that higher correlation means a good fit and lower correlations indicate marginal or poor fits.

The solution for the critical correlation coefficient employed new software[2] to do Monte Carlo simulation. A thousand random Weibull data sets were produced for each sample size. The correlation coefficients were calculated and ranked from smallest to largest. The results are used to estimate the lower 90% confidence bound for the correlation coefficient. (See Figure 1). To use these results, compare your correlation coefficient with the value from the curve. If your correlation coefficient equals or exceeds the critical value on the curve, you have a "good" Weibull fit.

Other analysts previously applied the same technique for normal probability paper[3]. Their 90th percentile is plotted on Figure 1 for the normal. The authors believe the normal critical coefficients also apply to the log normal. This has been approximately verified with the software[2]. The normal results employed 120,000 trials and used Blom's plotting position instead of median ranks but that should be a small difference. On going research includes three parameter Weibulls and two parameter normals, & log normals based on median rank plotting positions. Suspended data does not appear to affect the function except to increase the uncertainty, so the plot ordinate is the number of failures, which may be less than the sample size.

Special Methods for Inspection Interval Data All inspection data have deficiencies. The most common form represents benign failures that are only found when the system is shut down and inspected. This is interval data. The failures found at an inspection actually occurred earlier, before the inspection. This increases the uncertainties of fitting a Weibull. The plot will show vertical columns of points.(Figure 2). The vertical columns of plotted data infers the lower level data points are associated with earlier failure times. If the exact times were recorded, the data would have fallen to the left of the column and scattered about on the trend line. Some experts like Sherwin[4], suggest a better fit is obtained by ignoring the lower points in each column as the uppermost point is the most accurately located. The dashed line on Figure 2 represents this solution. The authors have had excellent success with this method. It is Inspection Option 1 in the software[2].

Another application for the same technique is coarse data. Perhaps failures recorded by month should have been recorded by day or hour. This will appear as interval data and the same technique may be employed. See for example, Figure 6.

Special Methods for Inspection Probit Data Another form of inspection data may involve destructive inspection. For example, each year the U.S. Navy takes samples of bombs, missiles, and projectiles from the fleet representing each production lot and fires them to determine shelf life. Duds and misfires are failures. Here the uncertainties on the Weibull plot are larger in the % Failure, (Y) direction than in the time of the inspection, (X). In the usual Weibull application to service failures, the uncertainties are greater in the time to failure, (X), than the % Failure, (Y), and the regression is done X on Y. With destructive inspection data, the regression should be Y on X. This kind of data may also be generated by non-destructive evaluations, such as fluorescent penetrate, eddy current and ultrasonic methods. Here again the uncertainties are greater in the (Y) direction. (Figure 3) Median ranks are not used to plot this data. Instead the % failure is calculated as the ratio of cumulative failures to number inspected. The data for each point has three elements, inspection time, number inspected, and cumulative number failed. As each inspection represents binomial trials, 95% confidence intervals are calculated from two binomial standard deviations. The plot and confidence intervals is obtained from the software[2] as Inspection Option 2.

Prediction of Future Failures-Weibull Risk Even a few failures of heart pacemakers, heavy truck steering links, aircraft engines, or aircraft fuselages is enough to cause great apprehension[5]. Responsible management immediately wants to know how many failures will occur in the next few months or year. Weibull risk analysis can provide accurate forecasts of future failures. In the past spreadsheet calculations were necessary but now this capability is in the software[2]. For example, Figure 4 shows a Weibull plot for 10 steering failures of wheel heavy trucks in a fleet of 14510 trucks. If the mileage on each truck is known or can be approximated, how many failures may be expected in the next years? The Weibull risk is shown in Figure 4A as output from the software[2].

The risk calculations are made as follows: If a unit has accumulated t miles to date without failure, and will accumulate u additional miles in a future period, the unit's contribution to the future risk is:

$$\frac{F(t+u)-F(t)}{1-F(t)}$$

where F(t) is the probability of the unit failing in the first t miles of service. This "cumulative hazard" is computed for each unfailed unit in the fleet. The sum of the cumulative hazards is an estimate of the number of failures that will occur in u additional miles, perhaps one month's usage.

If the failures have serious results, management will consider corrective action plans to reduce the risk. The reduction in risk for different corrective action plans may be compared using these Weibull methods. The technique is also used to predict safety hazards, spare parts requirements, maintenance labor, warranty and support costs.

Cost Effective Calibration Intervals In the past many organizations calibrated all instrumentation at fixed intervals, perhaps monthly or quarterly. In the mid-seventies some types of instrumentation were found to be much more stable than others. Some instrumentation should be recalibrated daily and other types yearly or longer. If instrumentation that is out-of-calibration is identified when returned to the laboratory, a Weibull plot is ideal for determining optimal calibration intervals[5]. Here we define failure as out-of-calibration and suspensions as in-calibration. Both types of data are used to determine the Weibull plot.

For critical instrumentation, the B1 life (life to 1% failure) might be employed as the calibration interval. This provides that 99% of the instrumentation in service is in-calibration. For less important instrumentation, B5 (95% in service in-calibration) or even B10 is used. The Weibull in Figure 5 is based on ten failures (out-of-cal units) and 90 suspensions (in-cal units). The calibration intervals is read from the Weibull plot. (B1=21 months, B5=33, B10=40).

All of the usual Weibull slope interpretations apply for calibration data. For example, shallow slopes show infant mortality, perhaps poor quality in the calibration process or installation problems. Slopes of one indicate random events are causing the out-of-cal failures. Random events probably relate to abusive practices such as over pressure, over voltage, dropping the instrument, contamination, etc. Slopes greater than one imply wear out modes.

If the instrumentation is returned for recalibration at fixed intervals, the software[2] inspection options should be employed because the data will pile-up at discrete intervals as mentioned above.

One of the newest calibration interval Weibull methods employs the absolute differences between redundant instruments[7]. The first few differences after the pair of instruments has been recalibrated are used to plot the Weibull. The B95 difference, 95% cumulative failure, is recommended as a signal to recalibrate again. This procedure is easily computerized as part of the data reduction to automatically produce out-of-calibration signals. Figure 6 illustrates the graphical Weibull calibration technique for showing data trends plus the zone of gage inaccuracy and gage unreliability. Differences larger than allowed (110 microinches) fall into the zone of gage inaccuracy on the X-axis while values above a cumulative occurrence fall into a zone of gage unreliability on the Y-axis. In this case, a 10:1 gage ratio, a measurement tolerance of ±0.0015 inches, and a requirement that 95% of all gages returned must be within tolerance at the end of the calibration period are combined with variables data spanning a six year period. The results justify extending micrometer standard bar recalibration periods far beyond the current one year calibration period which provides substantial cost reductions.

Machine Tool and Robotic Accuracy In an X-Y plane the distribution of the miss distances, the radii from the bullseye to the impact, is a Rayleigh distribution. The Weibull with a slope (Beta) of 2.0 is identical to the Rayleigh. Mr. Harvey Purchise of the Wheeltek Corporation uses the Weibull for calibrating his machine tools. For example, Figure 7 shows two datasets generated from normal random numbers. The use of Weibull allows a significant reduction in the number of calibration points.

With the Rayleigh there is an assumption that the random errors in X & Y are normal and equal. If the random errors in X & Y are not equal the miss distance is not Rayleigh but still appears to be Weibull. In three dimensions the miss distance is related to the Maxwell distribution, and is called the Generalized Rayleigh distribution. Empirically a Weibull with a Beta of 3.0 fits the 3D data. (Figure 8).

If you are confident that the X & Y errors are of the same size or have a supporting Weibull history, WeiBayes with Beta = 2.0, offers further reductions in sample size for the same accuracy. The significant advantages of WeiBayes over Weibull will be discussed next.

WeiBayes Applications to Reduce Testing Costs Prior knowledge of the slope, beta, can significantly improve the precision of small sample Weibulls[7,8,9]. The WeiBayes method[1] assumes the slope is known. WeiBayes is a one-parameter (η) Weibull. It will have smaller uncertainties that the two parameter Weibull. Similarly, the two parameter (η & β) Weibull has smaller uncertainties than the three parameter (η, β, & t_0) Weibull. WeiBayes uncertainties are reduced by the prior knowledge of the slope, β. The authors recommend a Weibull library or databank for establishing the Weibull slope histories to take advantage of the WeiBayes technique.

For example, compare 90% confidence bounds for B1 life using Weibull three parameter (t_0), Weibull two parameter, and one parameter (WeiBayes) analysis. For N=4, beta=3, and eta=100, The software[2] was used to generate 1000 trials, ranking B1. The SUMMARY report option produced the 5 and 95% bounds with t_0 on, t_0 off, and WeiBayes on, shown in Table 1.

WeiBayes is about 100 times more precise than the three parameter Weibull and about six times more precise than the two parameter Weibull. This benefit is available **if** β is known. Weibull libraries are very cost effective.

Sudden Death Testing with WeiBayes Many component manufacturers use Sudden Death accelerated testing to demonstrate the life and reliability of their components. The method tests the units in sets of 3, 4, or 5, each set in a single rig. The test is over when the first unit fails in each set, hence the name...Sudden Death. For 10 sets of four, this provides 10 failures with 30 suspensions and a Weibull that is almost as accurate as running all 40 to failure, but with significantly less test effort. If WeiBayes can be employed with Sudden Death, even greater reductions in test effort are obtained by reducing the number of Sudden Death sets. Alternatively, WeiBayes will provide increased precision for the same number of sets.

As example, compare Sudden Death, with and without WeiBayes, to the conventional approach of testing all 40 to failure. Assuming a Weibull η=100 hours, & β=3, the software[2] produced the results shown in Table 2.

The test time to run 40 to failure is 40 times the MTBF. For Beta=3, the ratio of Eta to MTBF is 0.892. For Eta=100, MTBF=89.2 and test time for 40 units is 3568. For the Sudden Death tests, the mean rank for the first failure in a set is 1/(N+1). 1/5 corresponds to a B20 life of 60.7. Ten Sudden Death rigs would require 607 hours of testing. The reduction in test times is (1-607/3568) = 0.83 or 83%. Sudden Death with WeiBayes provides a 68% improvement in B1 uncertainty with an 83% reduction in test time compared to testing all 40 to failure.

Monte Carlo Simulation There are many applications for this technique in Weibull analysis. The simplest is the frequent problem of a "weird" data set. Is the odd characteristic just statistical scatter or real variation? One solution is to generate a number of Weibull data sets with the same sample size, η and β, and look at the resulting plots. This capability is built into the software[2] as a sampling option. Samples may be generated from Weibulls, normals and log normals. This option is also useful for looking at Log Normal data on a Weibull plot or vice versa, the characteristics of mixtures of failure modes, and the variation in uncertainties with different sample sizes, and suspension arrays.

Weibull plots analyze one failure mode at a time. If component or system reliability is studied, many modes must be combined. This can be an analytical nightmare. Monte Carlo is an easy solution and is used by many organizations for this purpose[11,12,13]. The availability of fast, high capacity computers with special software for simulation has dramatically increased the use of Monte Carlo. Optimal inspection and replacement periods may be determined using analytical methods[14,15] or with simulation, (Figure 9). Multiple modes, NDE inspection reliabilities, retrofit and recall programs require Monte Carlo simulation.

The software[2] was used for several of the studies described herein. The new Monte Carlo simulation software provides unique capabilities for the Weibull analyst. Confidence intervals and median estimates are produced for β, η, B lives, reliability, r, r^2, t_0, and the Weibull line. Both median rank regression and maximum likelihood estimates may be simulated. Data sets that have random or progressive suspensions may be modeled. Similarly, uncertainties and confidence bounds associated with the three parameter Weibull (t zero), the normal, the log normal, and the WeiBayes method may be determined for the first time. The efficiencies of zero-failure and Sudden Weibull tests may be estimated. A computer tutorial is included with the software[2] illustrating all of the methods needed with illustrated problems.

Software All the analysis and Figures 2-8 in this paper were produced with WeibullSMITH™, MonteCarloSMITH™, and VisualSMITH™ software[2]. This friendly software implements all of the methods in the original Weibull Handbook[1], plus those more recently developed. The word friendly means that in most cases the software may be employed by a novice without reference to the user's manual, and that it is difficult to "bomb out", (prematurely exit from the program). Figures 1 & 9 were produced with Harvard Graphics, Version 3.0, from SPC Software with input from the software[2].

Comments: The authors invite comments and questions.

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Critical Correlation Coefficient

Normal (Blom) & Weibull (Johnson) Probability Plotting - 90% Confidence

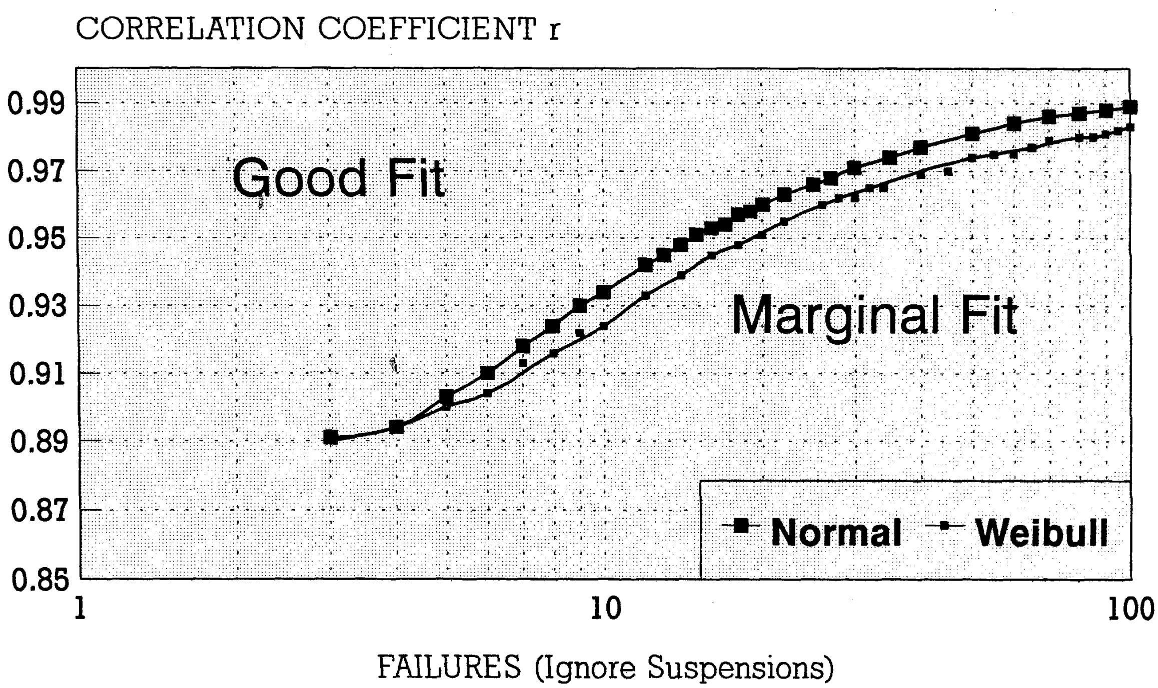

Figure 1.

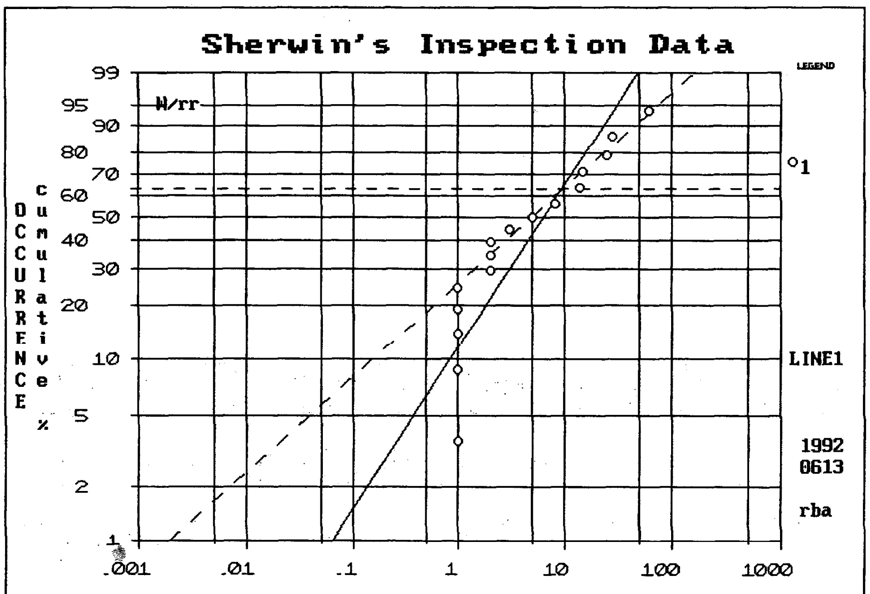

Figure 2 Inspection Time (Cycles)

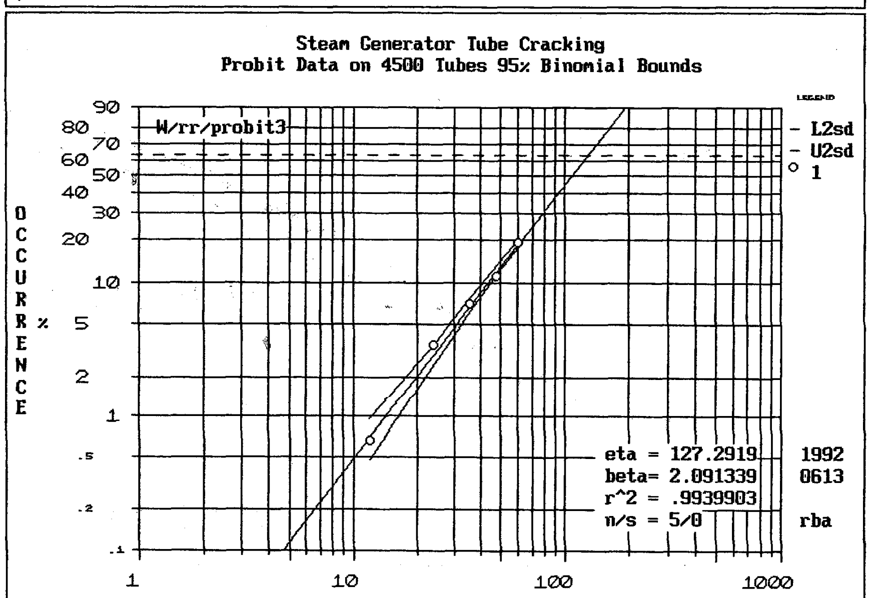

Figure 3 Inspection Times (months)

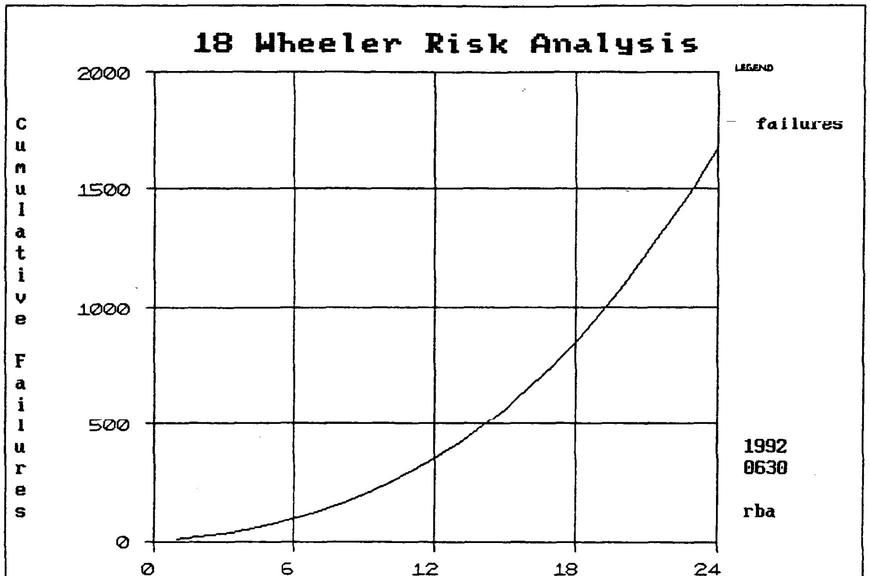

Figure 4A.......Calendar Time (months)

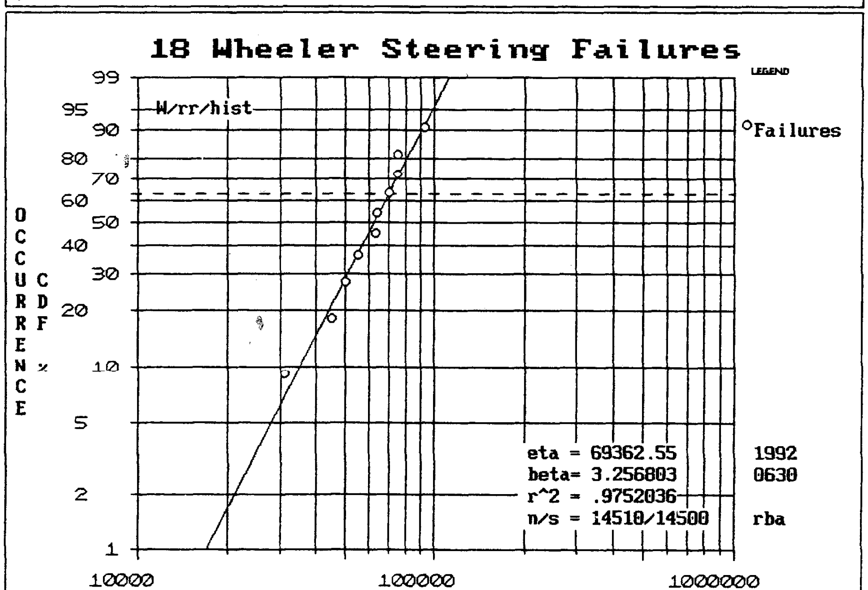

Figure 4.......Milage (miles)

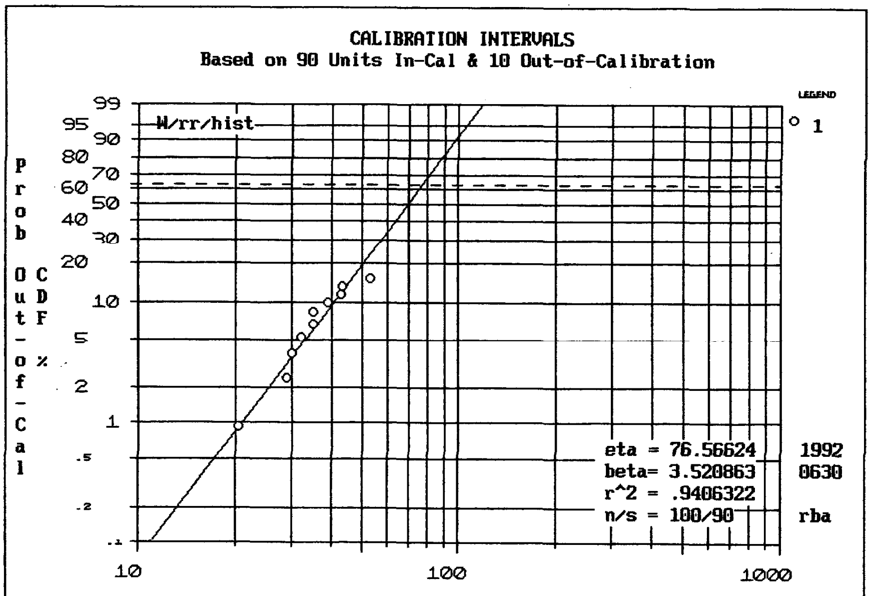

Figure 5 ....Time Since Last Calibration (months)

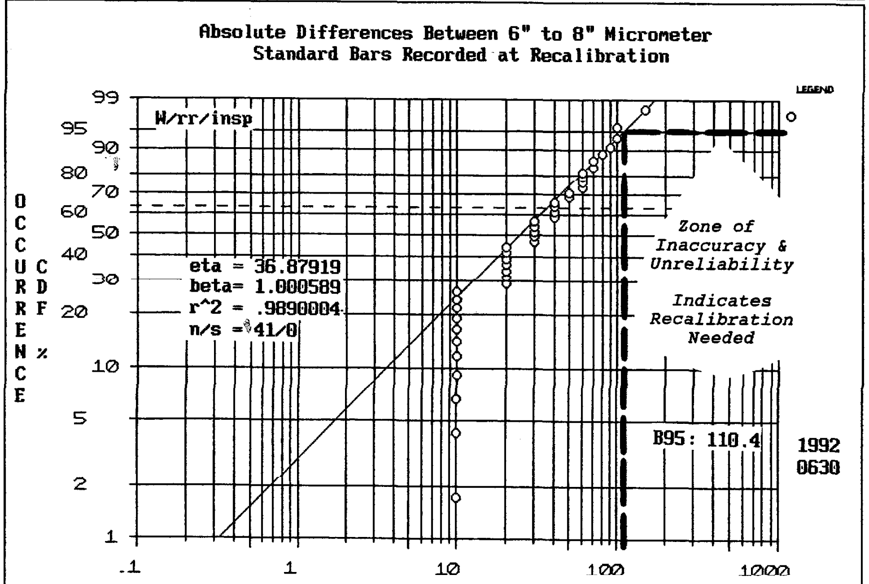

Figure 6......Absolute Difference (microinches)

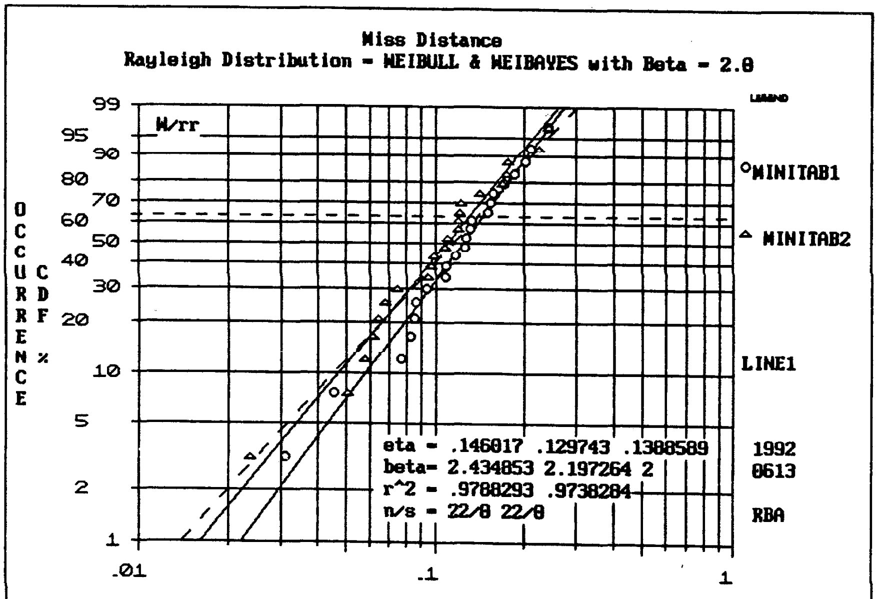

Figure 7. Radii of Mislocation (mm)

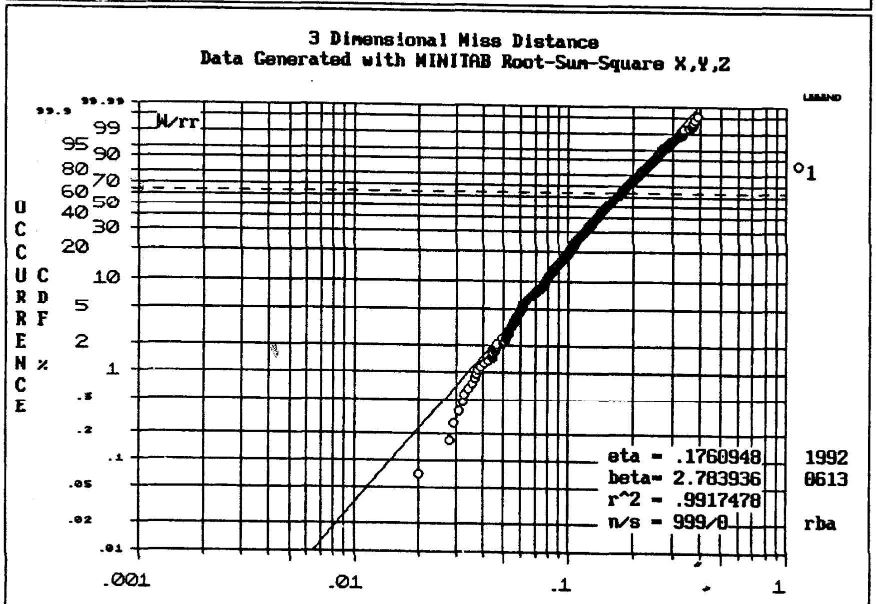

Figure 8. Miss Distance (units)

Total Cost = Failures + Replacements

Costs:1 Failure=$1000, 10 Replacements=$1000
Ten Systems, Ten Years, One Weibull Failure Mode

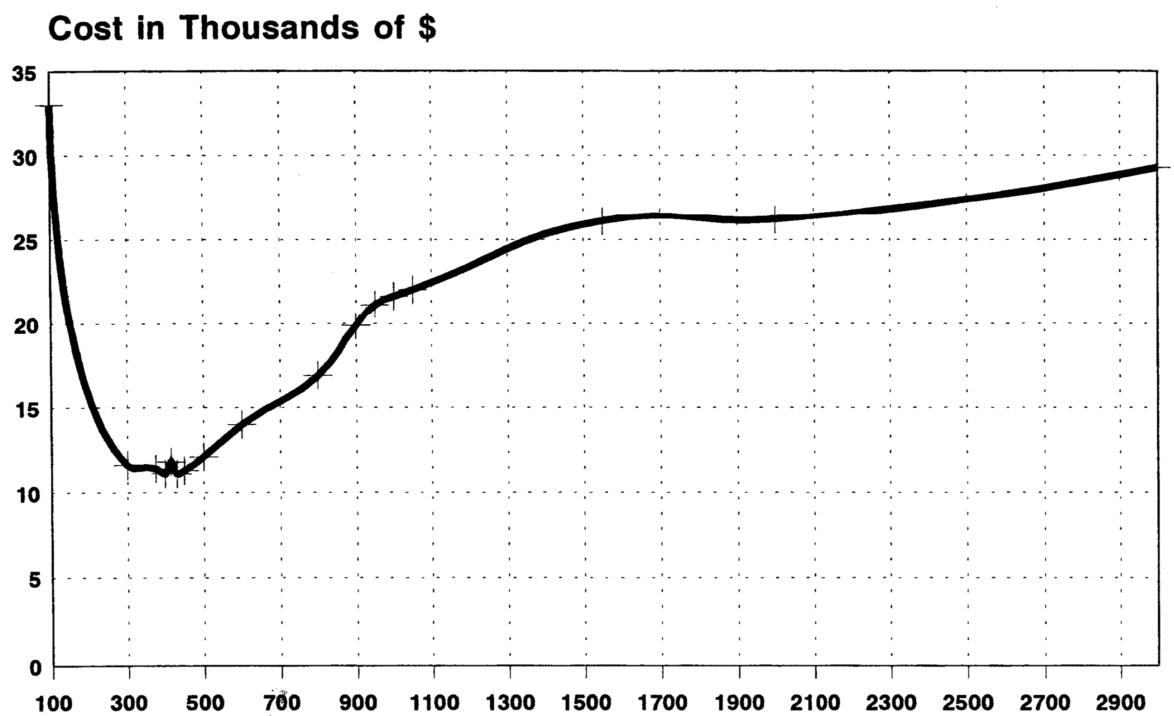

Figure 9 Inspection Interval

Table 1. Precision of One, Two and Three Parameter Weibulls

	Weibull t_0	Weibull	WeiBayes
B1 90% Bounds	1078-0	64.3-3.5	26.8-15.1
Range	1078	60.8	11.7

Table 2. Sudden Death with and without WeiBayes

	Weibull-40 Fail	Sudden Death-10 Fail	WeiBayes Sudden Death-10 Fail
B1 90% Bounds	31.4-13.3	50.0-19.3	30.0-24.2
Range	18.1	31.7	5.8
Test Time	3568	607	607