UNCERTAINTY IN GAS TURBINE MEASUREMENTS

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Abstract

This paper presents a standard method of treating measurement error for gas turbine engine performance parameters. The lack of a standard method for estimating the errors associated with gas turbine performance data has made it impossible to compare measurement systems between facilities, and there has been confusion over the interpretation of error analysis. The mathematical uncertainty model presented is based on two components of measurement error: the bias error and the precision error. The uncertainty estimate is the interval about the measurement that is expected to encompass the true value. The propagation of error from basic measurements through calculated performance parameters is presented. Traceability of measurement back to the National Bureau of Standards is reviewed. Both precision and bias errors are determined in part by their traceability to the standards of the National Bureau of Standards. Performance parameter errors are further propagated from the measurement errors through functional relationships. Methods for handling traceability and the propagation of error are described in the paper.

Introduction

This paper is based on the authors' larger work, Handbook--Uncertainty in Gas Turbine Measurements, AEDC-TR-73-5 (AD-755356). The work reported in the handbook was sponsored by the Arnold Engineering Development Center, Air Force Systems Command, United States Air Force. The results presented were compiled by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), Arnold Air Force Station, Tennessee, under Contract F40600-73-C-0004. The preparation of the text was accomplished by Dr. R. B. Abernethy, Senior Project Engineer, Billy D. Powell, David L. Colbert, and Daniel G. Sanders, Pratt & Whitney Aircraft under subcontract to ARO, Inc. The contracted work consisted of a revision to the material in the "Interagency Chemical Rocket Propulsion Group (ICRPG) Handbook for Estimating the Uncertainty in Measurements made with Liquid Propellant Rocket Engine Systems," CPIA Publication No. 180 (same authors as above), substituting treatment of gas turbine measurement errors for rocket engine treatment and writing additional material applicable to gas turbine measurement errors.

Measurement Error

Measurements are always subject to errors. These errors may be caused by slight differences in construction of identical measurement instruments or they may be caused by environment variations and the methods that we use in handling the instruments. Still other errors are inherent in the design of the instruments themselves. It is difficult to conceive of a measurement that is free of error.

The basis for the uncertainty model lies in the nature of measurement error. We view error as the difference between what we see and what is truth (Figure 1). All we ever see is the measurement, yet if that measurement is to be useful, we must define an associated interval that includes the truth. Most measurement error models agree to this point. The great diversity between them is in defining the size of the interval for any given measurement.

A little reflection on measurement error will show us that all errors have two components: a fixed error and a random error.

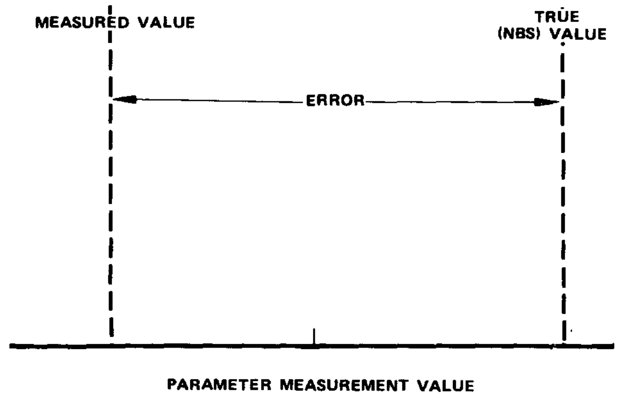

Figure 1. Measurement error.

Precision

Random error is seen in repeated measurements. The measurements do not agree exactly; we do not expect them to. There are always numerous small effects which cause disagreements. This random error between repeated measurements is called precision error. We use the standard deviation as a measure of precision error. A large standard deviation means a lot of scatter in the measurements. A smaller standard deviation indicates relatively less scatter. A statistic, s, is calculated from data to estimate the precision error and is called the precision index (Figure 2).

$$s = \sqrt{\frac{\sum (X_i - \overline{X})^2}{N - 1}}$$

where N is the number of measurements (X_i) that we have made and $\overline{X}$ is the average of the measurements.

Bias

The second component, bias, is the constant or systematic error (Figure 3). In repeated measurements, each measurement has the same bias. To determine the magnitude of bias in a given measurement situation, we must define the true value of the quantity being measured. This true value is usually unknown and unknowable. It is unknown because we cannot use the delicate NBS standard in making day-to-day measurements and unknowable because we cannot make perfect comparisons even when standards are available. Therefore, the bias is not easily determined. There is no nice statistic to estimate bias from data. We must, instead, rely on the best information available. Usually we rely on the engineering judgment of instrumentation and measurement engineers to provide an upper limit or bound on the bias. In this country we define the true value as the value defined by the National Bureau of Standards.

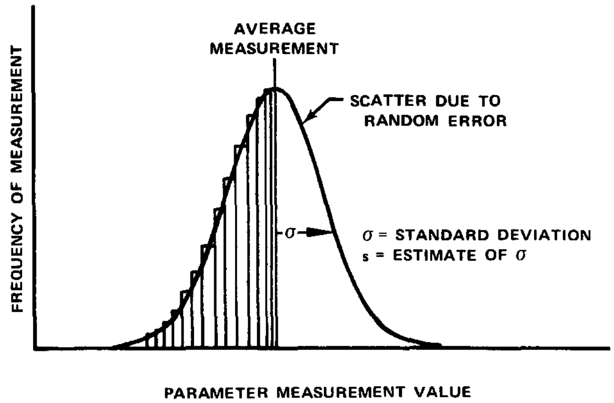

Figure 2. Precision error.

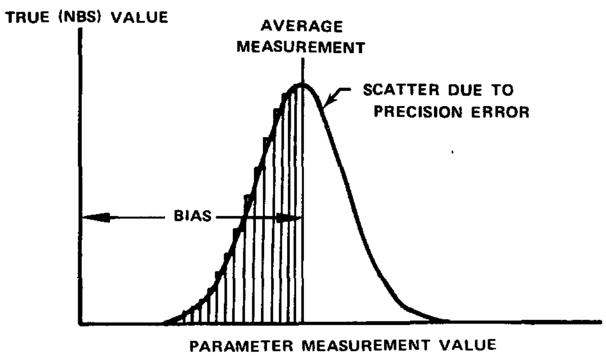

Figure 3. Bias error.

We may categorize bias into five classes (Figure 4): large known biases, small known biases, large unknown biases and small unknown biases that may have unknown sign (±) or known sign. The large known biases are eliminated by comparing the instrument to a standard instrument and obtaining a correction. This process is called <u>calibration</u>. Small known biases may or may not be corrected depending on the difficulty of the correction and the magnitude of the bias. The unknown biases are not correctable. That is, we know that they may exist but we do not know the sign or magnitude of the bias. Small unknown biases stem from errors introduced from the hierarchy of calibrations that relate the NBS standard to the working instrument.

Every effort must be made to eliminate all large unknown biases. The introduction of such errors converts the controlled measurement process into an uncontrolled worthless effort. Large unknown biases usually come from human errors in data processing, incorrect handling and installation of instrumentation, and unexpected environmental disturbances such as shock and bad flow profiles. We must assume that in a well controlled measurement process there are no large unknown biases. To ensure that a controlled measurement process exists, all measurements should be monitored with statistical quality control charts. Drifts, trends, and movements leading to out-of-control situations should be identified and investigated. Histories of data from calibrations are required for effective control. It is assumed throughout this paper that these precautions are observed and that the measurement process is in control; if not, the methods contained herein are invalid.

	KNOWN SIGN AND MAGNITUDE	UNKNOWN MAGNITUDE	
LARGE	(1) CALIBRATED OUT	(3) ASSUMED TO BE ELIMINATED	
SMALL	(2) NEGLIGIBLE, CONTRIBUTES TO BIAS LIMIT	(4) UNKNOWN SIGN	(5) KNOWN SIGN
		CONTRIBUTES TO BIAS LIMIT	

Figure 4. Five types of bias errors.

In summary, measurement systems are subject to two types of errors, bias and precision error (Figure 5). One sample standard deviation is used as the precision index. The bias limit is estimated as an upper limit on the maximum fixed error.

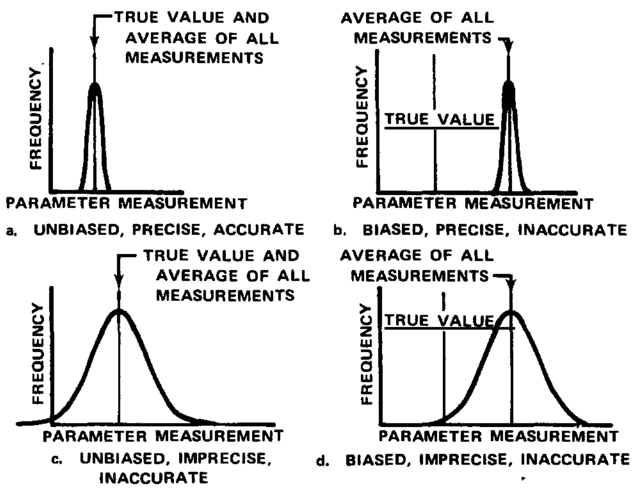

Figure 5. Measurement error (bias, precision, and accuracy).

<u>Uncertainty</u>

For simplicity and for comparisons, we need a single number to express a reasonable limit for error, some combination of bias and precision. It is impossible to define a rigorous statistic because the bias is an upper limit based on judgment which has unknown characteristics. Any function of these two numbers must be a hybrid combination of an unknown quantity (bias) and a statistic (precision). However, the need for a single number to measure error is so great that we are forced to adopt an arbitrary standard. The one most widely used is the bias limit plus a multiple of the precision error index. This formulation is recognized and recommended by the National Bureau of Standards and has been widely used in industry.

Uncertainty (Figure 6) may be centered about the measurement and is defined as:

$$U = \pm(B + t_{95}S)$$

where

B is the bias limit

S is the precision error index

t_{95} is the 95th percentile point for the two-tailed Student's "t" distribution. We have arbitrarily selected t = 2 for sample sizes greater than 30.

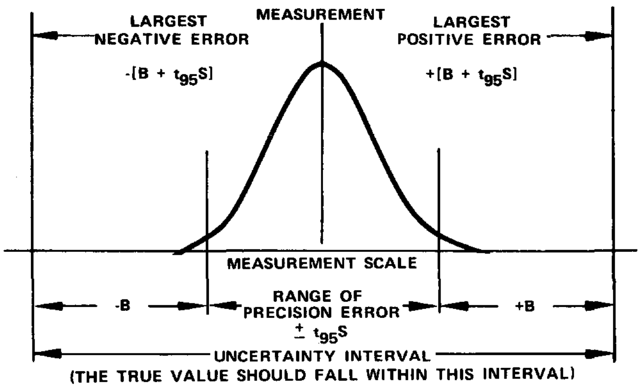

Figure 6. Measurement uncertainty, symmetrical bias.

If there is a nonsymmetrical bias limit (Figure 7), the uncertainty U is no longer symmetrical about the measurement. The upper limit of the interval is defined by the upper limit of the bias interval (B^+). The lower limit is defined by the lower limit of the bias interval (B^-). The uncertainty interval U is $U^- = B^- - t_{95}S$ to $U^+ = B^+ + t_{95}S$.

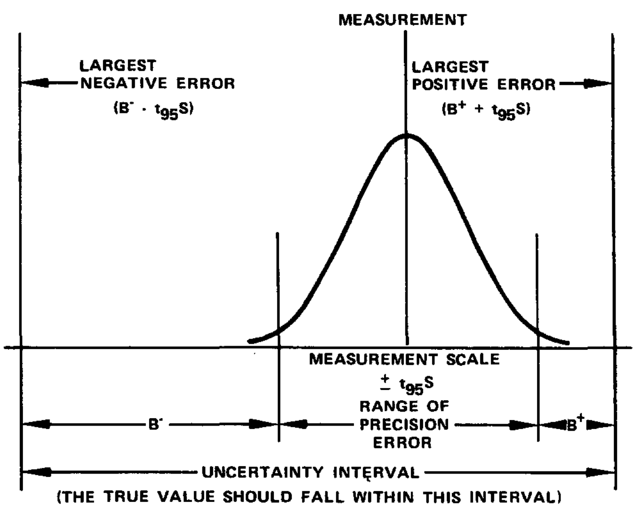

Figure 7. Measurement uncertainty, nonsymmetrical bias.

Reporting Format

The definition of the two components, bias and precision error and the limit, U, suggests a format for reporting the measurement error. The format will describe the components of error, which are necessary to estimate further propagation of the errors and a single value (U) which is an arbitrary upper limit of the size of the combined errors. The additional information, degrees of freedom for the estimate of S, is required to use the precision index. These four numbers provide all the information necessary to describe the measurement error. The reporting format is:

(1) S, the estimate of the precision index calculated from data.

(2) df, the degrees of freedom associated with the estimate of the precision index (s).

(3) B, the upper limit of the bias error of the measurement process or B^- and B^+ if the bias limit is nonsymmetrical.

(4) $U = \pm(B + t_{95}S)$, the uncertainty limit, beyond which measurement errors would not reasonably fall. The t value is the 95th percentile of the two-tailed Student's "t" distribution.

Alternatively, if the bias limit is nonsymmetrical, U is the interval between $U^- = B^- - t_{95}S$ and $U^+ = B^+ + t_{95}S$.

Uncertainty (U) should never be reported without the model components; bias, precision index and degrees of freedom. These components are required for further treatment of error such as the propagation from an engine to a propulsion system. It should be noted that uncertainty, U, can never be propagated. Although uncertainty is not a statistical confidence interval, it is an arbitrary substitute that is probably best interpreted as the largest error we might expect. Under any reasonable assumption for the distribution of bias, the coverage of U is greater than 95% but this cannot be proved as the distribution of bias is both unknown and unknowable.

Measurement Process

Uncertainty statements are based on a measurement process that must be defined. The process that we will discuss here is the measurement of thrust specific fuel consumption (TSFC) for a particular engine model at a given engine manufacturer's facility. The uncertainty will contain precision errors because of variations between installations and calibrations of many measurement instruments for each parameter. This uncertainty will be greater than the uncertainty for comparative tests to measure TSFC on a single stand for a single run. The single stand, single run model would assume that most installation-to-installation and calibration-to-calibration errors would be biases rather than precision errors. Biases are ignored in comparative tests.

The definition of the measurement system is prerequisite to defining the mathematical model. We must list all the elemental bias and precision error sources that are being estimated and how they are related to the engine performance parameter. We categorize errors into three groups: calibration-hierarchy errors, data acquisition, and data reduction errors.

Calibration-Hierarchy Errors

In recent years the demanding requirements of military and commercial aircraft have led to the establishment of extensive hierarchies of standards laboratories within the military and the aerospace industry. The National Bureau of Standards is at the apex of these hierarchies, providing the ultimate reference for each standards laboratory. It has become commonplace for government contracting agencies to require contractors to establish and prove traceability of their measurements to the NBS. This requirement has created even more extensive hierarchies of standards within the individual standards laboratories. At each level of these hierarchies, formal calibration procedures are used. These procedures not only define calibration methods and intervals but also specify just what information must be recorded during a calibration, i.e., meter model, serial number, calibration date, etc., in addition to actual measurement data.

A typical example of the traceability chain is the calibration hierarchy for a force measuring instrument, a load cell (Figure 8). The load cell is calibrated with a portable weigh kit while installed in the thrust stand. The weigh kit is calibrated with a force calibrator that is periodically calibrated in the company laboratory against a proving ring. At infrequent intervals, the ring is recalibrated at the National Bureau of Standards.

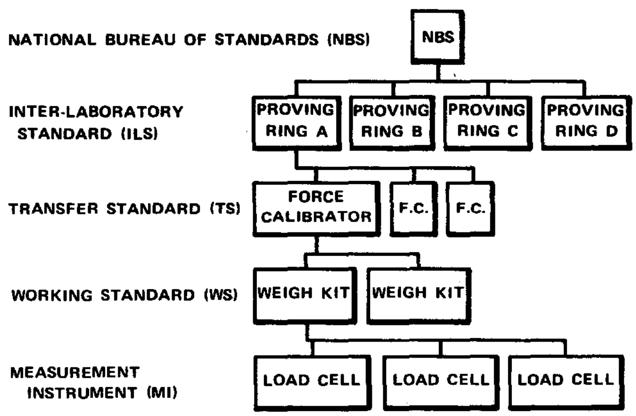

Figure 8. Force measurement calibration hierarchy.

The five levels in the hierarchy require four comparisons to calibrate the load cell. In each comparison, a precision error and a bias may be involved (Table 1).

Table 1. Calibration Hierarchy Error Sources

Comparison	Bias	Precision Error	Degrees of Freedom
NBS - Interlab Standard	b_{11}	s_{11}	df_{11}
Interlab Standard - Transfer Standard	b_{21}	s_{21}	df_{21}
Transfer Standard - Working Standard	b_{31}	s_{31}	df_{31}
Working Standard - Meas Load Cell	b_{41}	s_{41}	df_{41}

The measurement process takes place over a long period of time. During this period, many calibrations occur at each level. We view the precision errors of each comparison as precision errors affecting the measurement process we have defined. The overall effect on the measurement of force is a random (precision error) one. Therefore, the resultant overall precision error is the root sum square of the individual precision errors. For each comparison, the resultant calibration value is usually the average of several readings. The associated precision error would be a standard error of the mean for that number of readings. The precision error for the hierarchy is:

$$S_1 = \sqrt{s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2}$$

for four steps in the calibration process.

The degrees of freedom for each estimate of precision error may be combined using the Welch-Satterthwaite formula to provide an estimate of the degrees of freedom for the combined precision error.

$$df_1 = \frac{\left(s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2\right)^2}{\frac{s_{11}^4}{df_{11}} + \frac{s_{21}^4}{df_{21}} + \frac{s_{31}^4}{df_{31}} + \frac{s_{41}^4}{df_{41}}}$$

The Welch-Satterthwaite technique provides the best known estimate of the equivalent degrees of freedom.

The unknown bias error limit for the end instrument is usually a function of many elemental bias limits, perhaps ten or twenty. It is unreasonable to assume that all of these biases are cumulative. There must be a cancelling effect because some are positive and some are negative. For this reason, we have adopted the arbitrary rule that the bias limit B will be the root-sum-square of the elemental bias limit estimates:

$$B_1 = \sqrt{b_{11}^2 + b_{21}^2 + b_{31}^2 + b_{41}^2}$$

The uncertainty in the measurement instrument due to calibration is calculated using the uncertainty formula:

$$U_1 = \pm(B_1 + t_{95}\, S_1)$$

Data Acquisition Errors

Data are acquired by measuring the electrical output resulting from force applied to a strain gage type force transducer. Figure 9 illustrates some of the error sources associated with data acquisition. Other error sources such as electrical simulation, thrust bed mechanics and environmental effects are also present.

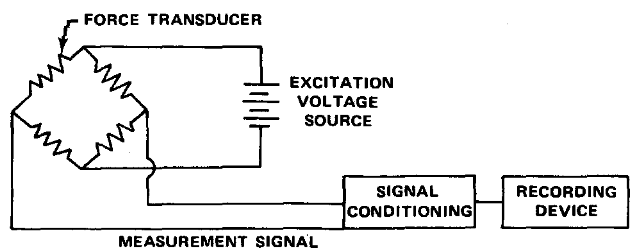

Figure 9. Data acquisition system.

All the data acquisition error sources are listed in Table 2. Symbols for the elemental bias and precision errors, and for the degrees of freedom are shown.

Table 2. Data Acquisition Error Sources

Error Source	Bias	Precision Index	Degrees of Freedom
Excitation Voltage	b_{12}	s_{12}	df_{12}
Electrical Simulation	b_{22}	s_{22}	df_{22}
Signal Conditioning	b_{32}	s_{32}	df_{32}
Recording Device	b_{42}	s_{42}	df_{42}
Force Transducer	b_{52}	s_{52}	df_{52}
Thrust Bed Mechanics	b_{62}	s_{62}	df_{62}
Environmental Effects	b_{72}	s_{72}	df_{72}

B_2 and S_2 represent the root sum square of the bias and precision error columns, respectively.

Data Reduction Errors

Computers operate on the raw data to produce output in engineering units. The errors in this process stem from the calibration curve fits (Figure 10), and the computer resolution.

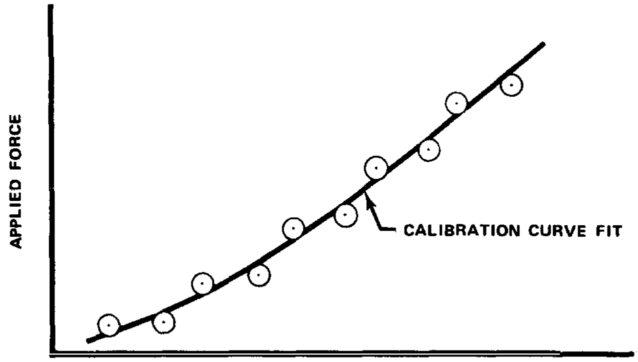

Figure 10. Calibration curve.

Symbols for the data reduction error sources are listed in Table 3. These errors are often negligible in each process.

Table 3. Data Reduction Error Sources

Error Source	Bias	Precision Error	Degrees of Freedom
Calibration Curve Fit	b_{13}	s_{13}	df_{13}
Computer Resolution	b_{23}	s_{23}	df_{23}

B_3 and S_3 represent the root sum square of the bias and precision error columns, respectively.

Measurement Uncertainty Model

The calibration-hierarchy errors, the data acquisition errors, and the data reduction errors are combined to obtain the precision index, bias limit, and uncertainty for the measurement.

$$S = \sqrt{S_1^2 + S_2^2 + S_3^2}$$

$$df = \frac{\left(S_1^2 + S_2^2 + S_3^2\right)^2}{\frac{S_1^4}{df_1} + \frac{S_2^4}{df_2} + \frac{S_3^4}{df_3}}$$

$$B = \sqrt{B_1^2 + B_2^2 + B_3^2}$$

$$U = \pm \left\{B + t_{95}\, S\right\}$$

By assuming completely hypothetical numbers for the elemental error terms for the calibration hierarchy, data acquisition, and data reduction processes, Table 4 tabulates values for all elemental bias and precision error terms and includes sample sizes for the calibration processes.

The errors associated with the calibration hierarchy, data acquisition, and data reduction stages in the measurement process are calculated below and are identified by S_1, S_2, and S_3, respectively, for precision indices and B_1, B_2, and B_3, respectively, for bias limits and U_1, U_2, and U_3, respectively, for uncertainty intervals.

1. Calibration bias limit for the force transducer is

$$B_1 = \pm \sqrt{\sum_i b_i^2}$$

$$B_1 = \pm \sqrt{(0.2)^2 + (0.2)^2 + (0.4)^2 + (0.8)^2}$$

$$B_1 = \pm 0.94 \text{ lb}$$

2. Calibration precision index estimate for the force transducer is

$$S_1 = \pm \sqrt{\sum_i s_i^2}$$

$$S_1 = \pm \sqrt{(10)^2 + (10)^2 + (14.1)^2 + (20)^2}$$

$$S_1 = \pm 28.3 \text{ lb}$$

Table 4. Force Measurement Elemental Error Values

Calibration Errors, lb			Data Acquisition Errors, lb		Data Reduction Errors, lb	
Bias	Precision	Sample Size	Bias	Precision	Bias	Precision
$b_{11} = \pm 0.2$	$s_{11} = \pm 10.0$	6	$b_{12} = \pm 5.0$	$s_{12} = \pm 5.0$	$b_{13} = \pm 10.0$	s_{13} = negligible
$b_{21} = \pm 0.2$	$s_{21} = \pm 10.0$	11	$b_{22} = \pm 5.0$	$s_{22} = \pm 5.0$	b_{23} = negligible	s_{23} = negligible
$b_{31} = \pm 0.4$	$s_{31} = \pm 14.1$	5	$b_{32} = \pm 5.0$	$s_{32} = \pm 5.0$		
$b_{41} = \pm 0.8$	$s_{41} = \pm 20.0$	17	$b_{42} = \pm 5.0$	$s_{42} = \pm 5.0$		
			$b_{52} = \pm 0.4$	$s_{52} = \pm 20.0$		
			$b_{62} = \pm 10.0$	$s_{62} = \pm 10.0$		
			$b_{72} = \pm 5.0$	$s_{72} = \pm 5.0$		
			(df = 31 for all elemental precision errors)			

3. To demonstrate use of the Welch-Satterthwaite method for determining degrees of freedom (df), small sample sizes for the calibration processes in the force transducer calibration hierarchy have been assumed. Sample sizes are included along with the elemental errors in Table 4.

$$df = \frac{\left(s_1^2 + s_2^2 + .. + s_n^2\right)^2}{\left(\frac{s_1^4}{df_1} + \frac{s_2^4}{df_2} + \ldots + \frac{s_n^4}{df_n}\right)}$$

where df_n = sample size minus one for the n^{th} calibration

$$df_1 = \frac{\left[(10)^2 + (10)^2 + (14.1)^2 + (20)^2\right]^2}{\frac{(10)^4}{5} + \frac{(10)^4}{10} + \frac{(14.1)^4}{4} + \frac{(20)^4}{16}}$$

$$= \frac{640 \times 10^3}{23 \times 10^3}$$

$$= 27.8$$

Under the "t" column in Table E-1 in the Handbook, Appendix E, t is 2.052 for 27 degrees of freedom and 2.048 for 28 degrees of freedom. Interpolating linearly gives a t of 2.049 for 27.8 degrees of freedom.

The calculation of calibration uncertainty (U_1) for the force transducer is then

$$U_1 = \pm(B_1 + t_{95} S_1)$$

$$= \pm(0.94 + 2.049 \times 28.3)$$

$$= \pm 58.9 \text{ lb}$$

4. Data acquisition bias limit is

$$B_2 = \pm \sqrt{\sum_i b_i^2}$$

$$= \pm \sqrt{(5)^2 + (5)^2 + (5)^2 + (5)^2 + (0.4)^2 + (10)^2 + (5)^2}$$

$$= \pm 15.0 \text{ lb}$$

5. Data acquisition precision index estimate is

$$S_2 = \pm \sqrt{\sum_i s_i^2}$$

$$= \pm \sqrt{(5)^2 + (5)^2 + (5)^2 + (5)^2 + (20)^2 + (10)^2 + (5)^2}$$

$$= \pm 25.0 \text{ lb}$$

6. Data acquisition uncertainty is

$$U_2 = \pm(B_2 + t_{95} S_2)$$

$$= \pm(15.0 + 2 \times 25.0)$$

$$= 65.0 \text{ lb}$$

$t_{95} = 2.00$ because df > 30 for S_2

7. Data reduction bias limit is

$$B_3 = \pm \sqrt{\sum_i b_i^2}$$

$$= \pm 10.0 \text{ lb}$$

8. Data reduction precision index estimate is

$$S_3 = \pm \sqrt{\sum_i s_i^2}$$

$$= 0$$

9. Data reduction uncertainty is

$$U_3 = \pm(B_3 + t_{95} S_3)$$

$$= \pm(10 + 0.0)$$

$$= \pm 10.0 \text{ lb}$$

10. Force measurement bias limit is

$$B_F = \pm \sqrt{B_1^2 + B_2^2 + B_3^2}$$

$$= \pm \sqrt{(0.9)^2 + (15)^2 + (10)^2}$$

$$= \pm 18.1 \text{ lb}$$

11. Force measurement precision index estimate is

$$S_F = \pm \sqrt{S_1^2 + S_2^2 + S_3^2}$$

$$= \pm \sqrt{(28.3)^2 + (25)^2 + 0^2}$$

$$= \pm 37.8 \text{ lb}$$

12. Degrees of freedom for force measurement are

$$df_F = \frac{\left(S_1^2 + S_2^2 + S_3^2\right)^2}{\frac{S_1^4}{df_1} + \frac{S_2^4}{df_2} + \frac{S_3^4}{df_3}}$$

$$= \frac{\left[(28.3)^2 + (25)^2\right]^2}{\frac{(28.3)^4}{27.8} + \frac{(25)^4}{31}}$$

$$= 57$$

13. Force measurement uncertainty is

$$U_F = \pm(B_F + t_{95} S_F)$$

$$= \pm(18.1 + 2.00 \times 37.8) \quad df_F = 57, \ t_{95} = 2.00$$

$$= \pm 93.7 \text{ lb}$$

Propagation of Measurement Error

Rarely are performance parameters measured directly; usually more basic quantities such as temperature, force, pressure, and fuel flow are measured, and the performance parameter is calculated as a function of the measurements. Error in the measurements is propagated to the parameter through the function. The effect of the propagation may be approximated with the Taylor's series methods.

The goal of any analysis of measurement system errors is to determine the resulting errors in the reduced parameters, for example TSFC, which is calculated as the ratio of fuel flow (W_f) to net thrust (F_N); $TSFC = W_f/F_N$. The technique for relating the errors of measurement to the errors in the reduced parameters is based on a Taylor's Series expansion from the calculus. The Taylor's expression for errors in thrust specific fuel consumption is

$$\Delta TSFC = \frac{\partial TSFC}{\partial W_f}\Delta W_f + \frac{\partial TSFC}{\partial F_N}\Delta F_N = \frac{1}{F_N}\Delta W_f - \frac{W_f}{F_N^2}\Delta F_N$$

Where $\partial TSFC/\partial W_f$ and $\partial TSFC/\partial F_N$ are the partial derivatives of thrust specific fuel consumption with respect to fuel flow and net thrust. The precision index is approximated by

$$S_{TSFC} = \sqrt{\left(\frac{\partial TSFC}{\partial W_f} S_{W_f}\right)^2 + \left(\frac{\partial TSFC}{\partial F} S_F\right)^2}$$

$$= \sqrt{\left(\frac{1}{F} S_{W_f}\right)^2 + \left(\frac{-W_f}{F^2} S_F\right)^2}$$

For example, the following hypothetical data were used to estimate thrust specific fuel consumption uncertainty:

Parameter	Nominal	Bias Limit	Precision Index	Degrees of Freedom	Uncertainty Limit
Thrust (F_N)	10,000	18.1 lb_f	37.8 lb_f	57	93.7 lb_f
Fuel Flow (W_f)	10,000	50 lb_m/hr	50 lb_m/hr	60	150 lb_m/hr

The nominal thrust specific fuel consumption is calculated from W_f/F_N:

$$\frac{W_f}{F_N} = \frac{10,000}{10,000}\frac{lb_m/hr}{lb_f} = 1.0\ lb_m/lb_f\text{-hr}$$

The precision index of thrust specific fuel consumption is

$$S_{TSFC} = \sqrt{\left(\frac{1}{F_N} S_{W_f}\right)^2 + \left(\frac{-W_f}{F_N^2} S_{F_N}\right)^2}$$

$$= \sqrt{\left(\frac{50}{10,000}\right)^2 + \left(\frac{-10,000}{10,000^2} \times 37.8\right)^2}$$

$$= \pm 0.0063\ lb_m/lb_f\text{-hr}$$

The propagation formula is similar for bias

$$B_{TSFC} = \sqrt{\left(\frac{\partial TSFC}{\partial W_f} B_{W_f}\right)^2 + \left(\frac{\partial TSFC}{\partial F_N} B_{F_N}\right)^2}$$

$$B_{TSFC} = \sqrt{\left(\frac{1}{F_N} B_{W_f}\right)^2 + \left(\frac{-W_f}{F_N^2} B_{F_N}\right)^2}$$

$$B_{TSFC} = \sqrt{\left(\frac{50}{10,000}\right)^2 + \left(\frac{-10,000}{10,000^2} 18.1\right)^2}$$

$$= \pm 0.0053\ lb_m/lb_f\text{-hr}$$

The degrees of freedom for the TSFC precision index can be found using the Welch-Satterthwaite technique. In this situation, the partial derivative weighting factors, which are used in the calculation of the precision index, must also be used in the Welch-Satterthwaite formula. Note: The calculation is carried out to illustrate the use of the partial derivatives with the Welch-Satterthwaite. It is not necessary to calculate the degrees of freedom for TSFC since the degrees of freedom for thrust and fuel flow are 57 and 60, respectively. The expected minimum result would be 57. The t multiple is essentially 2.0 for degrees of freedom greater than 30. When the degrees of freedom for each component are greater than 30, the Welch-Satterthwaite procedure can be omitted and t = 2.0 can be used.

$$df_{TSFC} = \frac{\left[\left(\frac{\partial TSFC}{\partial W_f} S_{W_f}\right)^2 + \left(\frac{\partial TSFC}{\partial F_N} S_{F_N}\right)^2\right]^2}{\frac{\left(\frac{\partial TSFC}{\partial W_f} S_{W_f}\right)^4}{df_{W_f}} + \frac{\left(\frac{\partial TSFC}{\partial F_N} S_{F_N}\right)^4}{df_{F_N}}}$$

$$= \frac{\left[\left(\frac{1}{F_N} S_{W_f}\right)^2 + \left(\frac{-W_f}{F_N^2} S_{F_N}\right)^2\right]^2}{\frac{\left(\frac{1}{F_N} S_{W_f}\right)^4}{df_{W_f}} + \frac{\left(\frac{-W_f}{F_N^2} S_{F_N}\right)^4}{df_{F_N}}}$$

$$= \frac{\left[\left(\frac{1}{10,000} \times 50\right)^2 + \left(\frac{-10,000}{10,000^2} \times 37.8\right)^2\right]^2}{\frac{\left(\frac{1}{10,000} \times 50\right)^4}{60} + \frac{\left(\frac{-10,000}{10,000^2} \times 37.8\right)^4}{57}}$$

$$= 110$$

The t value is 2, and the uncertainty is

$$U = \pm(B + t_{95} S) = \pm[0.0053 + (2.0)(0.0063)]$$

$$= \pm 0.0179\ lb_m/lb_f\text{-hr}$$

The results of the error analysis are presented in Table 5.

The uncertainty limit as a percentage of the nominal value may be calculated by dividing the uncertainty limit in engineering units by the corresponding nominal value and then multiplying by 100.

The handbook illustrates the uncertainty in several other turbine engine measured and performance parameters such as fuel flow, pressure, temperature, airflow, and net thrust. The Handbook also contains a Special Methods section treating special situations or conditions and an Appendix containing information on precision index for uniform distribution of error, propagation of errors by Taylor's Series, estimates of the precision index from multiple measurements, outlier detection schemes, and some statistical tables.

We believe that the methods we have presented represent the best technology available. The uncertainty model is the product of years of research in that the model has as its beginning the ICRPG work. The model represents the efforts of many organizations. We hope that you will respond with constructive criticism for the continued improvement of this model. We are indebted to the many engineers and statisticians who have contributed to the work. A few must be noted for their particular contributions, Dr. Joan Rosenblatt, Dr. H. H. Ku, and J. M. Cameron of the National Bureau of Standards for their helpful discussions and comments on both this handbook and CPIA 180, and similarly, R. E. Smith, Jr., Dep. Dir. of Engine Test Facility, as well as T. C. Austin, C. R. Bartlett, W. O. Boals, Jr., and T. J. Gillard of ARO, Inc., at the Arnold Engineering Development Center. Engineers at Pratt & Whitney Aircraft, Florida and Connecticut facilities, provided the authors with constructive and spirited criticism in every section. Various technical committees under the American Society of Mechanical Engineers (ASME), The American Institute of Aeronautics and Astronautics, and the International Standards Organizations expressed interest and comments.

Logic Decision Diagram

We have summarized the measurement uncertainty model in a logic decision diagram shown in Table 6.

Summary

We have briefly described the gas turbine engine measurement uncertainty model; the Handbook on Uncertainty in Gas Turbine Measurements is recommended to you for a more detailed and comprehensive treatment.

Table 5. Uncertainty Components

Parameter	Nominal Value	Bias Limit	Precision Error	Degrees of Freedom	Uncertainty
Thrust, F_N	10,000 lb_f	18.1 lb_f	37.8 lb_f	57	93.7 lb_f
Fuel Flow, W_f	10,000 lb_m/hr	50 lb_m/hr	50 lb_m/hr	60	150 lb_m/hr
Thrust Specific Fuel Consumption	1.0 lb_m/lb_f-hr	0.0053 lb_m/lb_f-hr	0.0063 lb_m/lb_f-hr	110	0.018 lb_m/lb_f-hr

Table 6. Logic Decision Diagram

To Estimate	Use	Formula
Bias Limit		
1. Elemental (b)	Judgment Supported by Special Test Data	Estimate a Reasonable Limit for Each Bias Error
2. Measurement Bias	Estimated Elementals	$B_j = \sqrt{\sum_i b_i^2}$
3. Performance Parameter Bias	Measurement Bias and the Propagation of Error (Taylor's Series)	$B_F = \sqrt{\sum_j \left(\frac{\partial F}{\partial j} B_j\right)^2}$ F Denotes Performance Parameter Function
Precision Index		
1. Elemental s_i	Data from Multiple Measurements	$S_i = \sqrt{\frac{\sum_L (X_L - X)^2}{N_L - 1}}$, $df = N_L - 1$
2. Measurement Precision Index	Calculated Elementals and Data	$S_j = \sqrt{\sum_i S_i^2}$, $df = \frac{\left[\sum_i S_i^2\right]^2}{\sum_i \left[\frac{S_i^4}{df_i}\right]}$
3. Performance Parameter Precision Index	Measurement Precision Indices and the Propagation of Error (Taylor's Series)	$S_F = \sqrt{\sum_j \left(\frac{\partial F}{\partial j}\right) S_j{}^2}$ $df_F = \frac{\left[\sum_i \left(\frac{\partial F}{\partial j}\right) S_j{}^2\right]^2}{\sum_j \left[\frac{\left(\left(\frac{\partial F}{\partial j}\right) S_j\right)^4}{df_j}\right]}$

Table 6. Logic Decision Diagram (continued)

To Estimate	Use	Formula
t_{95} Value	Degrees of Freedom Less Than 30 (df < 30)	Interpolate in Two-Tailed Student's "t" Table for t
	Degrees of Freedom Greater Than or Equal to 30 (df $\geq$ 30)	Use t = 2.0
<u>Uncertainty</u>		
1. Elemental	Elemental Bias Limit and Precision Index	$U_i = \pm\left[B_i + t_{95} S_i\right]$
2. Measurement	Measurement Bias Limits and Precision Indices	$U_j = \pm\left[B_j + t_{95} S_j\right]$
3. Performance Parameter	Performance Parameter Bias Limit and Precision Index	$U_F = \pm\left[B_F + t_{95} S_F\right]$