Abstract

This paper presents a standard method of treating measurement error for gas turbine engine performance parameters. The lack of a standard method for estimating the errors associated with gas turbine performance data has made it impossible to compare measurement systems between facilities, and there has been confusion over the interpretation of error analysis. The mathematical uncertainty model presented is based on two components of measurement error: the bias error and the precision error. The uncertainty estimate is the interval about the measurement that is expected to encompass the true value. The propagation of error from basic measurements through calculated performance parameters is presented. Traceability of measurement back to the National Bureau of Standards is reviewed. Both precision and bias errors are determined in part by their traceability to the standards of the National Bureau of Standards. Performance parameter errors are further propagated from the measurement errors through functional relationships. Methods for handling traceability and the propagation of error are described in the paper.

Introduction

This paper is based on the authors' larger work, Handbook—Uncertainty in Gas Turbine Measurements; AEDC-TR-73-5 (AD-755356). The work reported in the handbook was sponsored by the Arnold Engineering Development Center, Air Force Systems Command, United States Air Force. The results presented were compiled by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), Arnold Air Force Station, Tennessee, under Contract F40600-73-C-0004. The preparation of the text was accomplished by Dr. R. B. Abernethy, Senior Project Engineer, Billy D. Powell, David L. Colbert, and Daniel G. Sanders, Pratt & Whitney Aircraft under subcontract to ARO, Inc. The contracted work consisted of a revision to the material in the "Interagency Chemical Rocket Propulsion Group (ICRPG) Handbook for Estimating the Uncertainty in Measurements made with Liquid Propellant Rocket Engine Systems," CPIA Publication No. 180 (same authors as above), substituting treatment of gas turbine measurement errors for rocket engine treatment and writing additional material applicable to gas turbine measurement errors.

Measurement Error

Measurements are always subject to errors. These errors may be caused by slight differences in construction of identical measurement instruments or they may be caused by environment variations and the methods that we use in handling the instruments. Still other errors are inherent in the design of the instruments themselves. It is difficult to conceive of a measurement that is free of error.

The basis for the uncertainty model lies in the nature of measurement error. We view error as the difference between what we see and what is truth (Figure 1). All we ever see is the measurement, yet if that measurement is to be useful, we must define an associated interval that includes the truth. Most measurement error models agree to this point. The great diversity between them is in defining the size of the interval for any given measurement.

A little reflection on measurement error will show us that all errors have two components: a fixed error and a random error.

Precision

Random error is seen in repeated measurements. The measurements do not agree exactly; we do not expect them to. There are always numerous small effects which cause disagreements. This random error between repeated measurements is called precision error. We use the standard deviation as a measure of precision error. A large standard deviation means a lot of scatter in the measurements. A smaller standard deviation indicates relatively less scatter. A statistic, $s$, is calculated from data to estimate the precision error and is called the precision index (Figure 2).

$$ s = \sqrt{\frac{\sum (X_i - \bar{X})^2}{N - 1}} $$

where $N$ is the number of measurements ($X_i$) that we have made and $\bar{X}$ is the average of the measurements.

Bias

The second component, bias, is the constant or systematic error (Figure 3). In repeated measurements, each measurement has the same bias. To determine the magnitude of bias in a given measurement situation, we must define the true value of the quantity being measured. This true value is usually unknown and unknowable. It
is unknown because we cannot use the delicate NBS standard in making day-to-day measurements and unknowable because we cannot make perfect comparisons even when standards are available. Therefore, the bias is not easily determined. There is no nice statistic to estimate bias from data. We must, instead, rely on the best information available. Usually we rely on the engineering judgment of instrumentation and measurement engineers to provide an upper limit or bound on the bias. In this country we define the true value as the value defined by the National Bureau of Standards.

Uncertainty

For simplicity and for comparisons, we need a single number to express a reasonable limit for error, some combination of bias and precision. It is impossible to define a rigorous statistic because the bias is an upper limit based on judgment which has unknown characteristics. Any function of these two numbers must be a hybrid combination of an unknown quantity (bias) and a statistic (precision). However, the need for a single number to measure error is so great that we are forced to adopt an arbitrary standard. The one most widely used is the bias limit plus a multiple of the precision error index. This formulation is recognized and recommended by the National Bureau of Standards and has been widely used in industry.

Uncertainty (Figure 6) may be centered about the measurement and is defined as:

\[ U = \pm (B + t_{95}S) \]
If there is a nonsymmetrical bias limit (Figure 7), the uncertainty $U$ is no longer symmetrical about the measurement. The upper limit of the interval is defined by the upper limit of the bias interval ($B''$). The lower limit is defined by the lower limit of the bias interval ($B$). The uncertainty interval $U$ is $U^-=B^-+t_{95}s$ to $U^+=B^++t_{95}s$.

Figure 6. Measurement uncertainty, symmetrical bias.

If there is a nonsymmetrical bias limit (Figure 7), the uncertainty $U$ is no longer symmetrical about the measurement. The upper limit of the interval is defined by the upper limit of the bias interval ($B''$). The lower limit is defined by the lower limit of the bias interval ($B$). The uncertainty interval $U$ is $U^-=B^-+t_{95}s$ to $U^+=B^++t_{95}s$.

Figure 7. Measurement uncertainty, nonsymmetrical bias.

The definition of the two components, bias and precision error and the limit, $U$, suggests a format for reporting the measurement error. The format will describe the components of error, which are necessary to estimate further propagation of the errors and a single value ($U$) which is an arbitrary upper limit of the size of the combined errors. The additional information, degrees of freedom for the estimate of $S$, is required to use the precision index. These four numbers provide all the information necessary to describe the measurement error. The reporting format is:

1. $S$, the estimate of the precision index calculated from data.
2. $df$, the degrees of freedom associated with the estimate of the precision index ($s$).
3. $B$, the upper limit of the bias error of the measurement process or $B''$ and $B^+$ if the bias limit is nonsymmetrical.
4. $U = s(B + t_{95}s)$, the uncertainty limit, beyond which measurement errors would not reasonably fall. The $t$ value is the 95th percentile of the two-tailed Student's "t" distribution.

Alternatively, if the bias limit is nonsymmetrical, $U$ is the interval between $U^-=B^-+t_{95}s$ and $U^+=B^++t_{95}s$.

Uncertainty ($U$) should never be reported without the model components; bias, precision index and degrees of freedom. These components are required for further treatment of error such as the propagation from an engine to a propulsion system. It should be noted that uncertainty, $U$, can never be propagated. Although uncertainty is not a statistical confidence interval, it is an arbitrary substitute that is probably best interpreted as the largest error we might expect. Under any reasonable assumption for the distribution of bias, the coverage of $U$ is greater than 95% but this cannot be proved as the distribution of bias is both unknown and unknowable.

Measurement Process

Uncertainty statements are based on a measurement process that must be defined. The process that we will discuss here is the measurement of thrust specific fuel consumption (TSFC) for a particular engine model at a given engine manufacturer's facility. The uncertainty will contain precision errors because of variations between installations and calibrations of many measurement instruments for each parameter. This uncertainty will be greater than the uncertainty for comparative tests to measure TSFC on a single stand. The single stand, single run model would assume that most installation-to-installation and calibration-to-calibration errors would be biases rather than precision errors. Biases are ignored in comparative tests.

The definition of the measurement system is prerequisite to defining the mathematical model. We must list all the elemental bias and precision error sources that are being estimated and how they are related to the engine performance parameter. We categorize errors into three groups: calibration-hierarchy errors, data acquisition, and data reduction errors.

Calibration-Hierarchy Errors

In recent years the demanding requirements of military and commercial aircraft have led to the establishment of extensive hierarchies of standards laboratories within the military and the aerospace industry. The National Bureau of Standards is at the apex of these hierarchies, providing the ultimate reference for each standards laboratory. It has become commonplace for government contracting agencies to require contractors to establish and prove tracability of their measurements to the NBS. This requirement has created even more extensive hierarchies of standards within the individual standards laboratories. At each level of these
Hierarchies, formal calibration procedures are used. These procedures not only define calibration methods and intervals but also specify just what information must be recorded during a calibration, i.e., meter model, serial number, calibration date, etc., in addition to actual measurement data.

A typical example of the traceability chain is the calibration hierarchy for a force measuring instrument, a load cell (Figure 8). The load cell is calibrated with a portable weigh kit while installed in the thrust stand. The weigh kit is calibrated with a force calibrator that is periodically calibrated in the company laboratory against a proving ring. At infrequent intervals, the ring is recalibrated at the National Bureau of Standards.

The five levels in the hierarchy require four comparisons to calibrate the load cell. In each comparison, a precision error and a bias may be involved (Table 1).

Table 1. Calibration Hierarchy Error Sources

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Bias</th>
<th>Precision Error</th>
<th>Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>NBS - Interlab Standard</td>
<td>b11</td>
<td>s11</td>
<td>df11</td>
</tr>
<tr>
<td>Interlab Standard - Transfer Standard</td>
<td>b21</td>
<td>s21</td>
<td>df21</td>
</tr>
<tr>
<td>Transfer Standard - Working Standard</td>
<td>b31</td>
<td>s31</td>
<td>df31</td>
</tr>
<tr>
<td>Working Standard - Meas Load Cell</td>
<td>b41</td>
<td>s41</td>
<td>df41</td>
</tr>
</tbody>
</table>

The measurement process takes place over a long period of time. During this period, many calibrations occur at each level. We view the precision errors of each comparison as precision errors affecting the measurement process we have defined. The overall effect on the measurement of force is a random (precision error) one. Therefore, the resultant overall precision error is the root sum square of the individual precision errors. For each comparison, the resultant calibration value is usually the average of several readings. The associated precision error would be a standard error of the mean for that number of readings. The precision error for the hierarchy is:

\[ s_1 = \sqrt{s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2} \]

for four steps in the calibration process.

The degrees of freedom for each estimate of precision error may be combined using the Welch-Satterthwaite formula to provide an estimate of the degrees of freedom for the combined precision error.

\[ df_1 = \frac{\left( \frac{s_{11}^2}{df_{11}} + \frac{s_{21}^2}{df_{21}} + \frac{s_{31}^2}{df_{31}} + \frac{s_{41}^2}{df_{41}} \right)^2}{\left( \frac{1}{df_{11}} + \frac{1}{df_{21}} + \frac{1}{df_{31}} + \frac{1}{df_{41}} \right)} \]

The Welch-Satterthwaite technique provides the best known estimate of the equivalent degrees of freedom.

The unknown bias error limit for the end instrument is usually a function of many elemental bias limits, perhaps ten or twenty. It is unreasonable to assume that all of these biases are cumulative. There must be a cancelling effect because some are positive and some are negative. For this reason, we have adopted the arbitrary rule that the bias limit B will be the root-sum-square of the elemental bias limit estimates:

\[ B = \sqrt{b_{11}^2 + b_{21}^2 + b_{31}^2 + b_{41}^2} \]

The uncertainty in the measurement instrument due to calibration is calculated using the uncertainty formula:

\[ U = \left( B + t_{95} s_1 \right) \]

Data Acquisition Errors

Data are acquired by measuring the electrical output resulting from force applied to a strain gage type force transducer. Figure 9 illustrates some of the error sources associated with data acquisition. Other error sources such as electrical simulation, thrust bed mechanics and environmental effects are also present.

Table 2. Data Acquisition Error Sources

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Bias</th>
<th>Precision Index</th>
<th>Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation Voltage</td>
<td>b12</td>
<td>s12</td>
<td>df12</td>
</tr>
<tr>
<td>Electrical Simulation</td>
<td>b22</td>
<td>s22</td>
<td>df22</td>
</tr>
<tr>
<td>Signal Conditioning</td>
<td>b32</td>
<td>s32</td>
<td>df32</td>
</tr>
<tr>
<td>Recording Device</td>
<td>b42</td>
<td>s42</td>
<td>df42</td>
</tr>
<tr>
<td>Force Transducer</td>
<td>b52</td>
<td>s52</td>
<td>df52</td>
</tr>
<tr>
<td>Thrust Bed Mechanics</td>
<td>b62</td>
<td>s62</td>
<td>df62</td>
</tr>
<tr>
<td>Environmental Effects</td>
<td>b72</td>
<td>s72</td>
<td>df72</td>
</tr>
</tbody>
</table>

\[ B_2 \text{ and } S_2 \text{ represent the root sum square of the bias and precision error columns, respectively.} \]
Data Reduction Errors

Computers operate on the raw data to produce output in engineering units. The errors in this process stem from the calibration curve fits (Figure 10), and the computer resolution.

Figure 10. Calibration curve.

Symbols for the data reduction error sources are listed in Table 3. These errors are often negligible in each process.

Table 3. Data Reduction Error Sources

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Bias</th>
<th>Precision</th>
<th>Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration Curve Fit</td>
<td>( b_{13} )</td>
<td>( s_{13} )</td>
<td>( df_{13} )</td>
</tr>
<tr>
<td>Computer Resolution</td>
<td>( b_{23} )</td>
<td>( s_{23} )</td>
<td>( df_{23} )</td>
</tr>
</tbody>
</table>

\( B_3 \) and \( S_3 \) represent the root sum square of the bias and precision error columns, respectively.

Measurement Uncertainty Model

The calibration-hierarchy errors, the data acquisition errors, and the data reduction errors are combined to obtain the precision index, bias limit, and uncertainty for the measurement.

By assuming completely hypothetical numbers for the elemental error terms for the calibration hierarchy, data acquisition, and data reduction processes, Table 4 tabulates values for all elemental bias and precision error terms and includes sample sizes for the calibration processes.

The errors associated with the calibration hierarchy, data acquisition, and data reduction stages in the measurement process are calculated below and are identified by \( S_1 \), \( S_2 \), and \( S_3 \), respectively, for precision indices and \( B_1 \), \( B_2 \), and \( B_3 \), respectively, for bias limits and \( U_1 \), \( U_2 \), and \( U_3 \), respectively, for uncertainty intervals.

1. Calibration bias limit for the force transducer is

\[
B_1 = \pm \sqrt{\sum b_i^2}
\]

\[
B_1 = \pm \sqrt{(0.2)^2 + (0.2)^2 + (0.4)^2 + (0.8)^2}
\]

\[
B_1 = \pm 0.94 \text{ lb}
\]

2. Calibration precision index estimate for the force transducer is

\[
S_1 = \pm \sqrt{\sum s_i^2}
\]

\[
S_1 = \pm \sqrt{(10)^2 + (10)^2 + (14.1)^2 + (20)^2}
\]

\[
S_1 = \pm 28.3 \text{ lb}
\]

Table 4. Force Measurement Elemental Error Values

<table>
<thead>
<tr>
<th>Calibration Errors, lb</th>
<th>Data Acquisition Errors, lb</th>
<th>Data Reduction Errors, lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bias ( b_{11} )</td>
<td>Precision ( s_{11} )</td>
<td>Sample Size</td>
</tr>
<tr>
<td>( \pm 0.2 )</td>
<td>( \pm 10.0 )</td>
<td>6</td>
</tr>
<tr>
<td>( \pm 0.2 )</td>
<td>( \pm 10.0 )</td>
<td>11</td>
</tr>
<tr>
<td>( \pm 0.4 )</td>
<td>( \pm 14.1 )</td>
<td>5</td>
</tr>
<tr>
<td>( \pm 0.8 )</td>
<td>( \pm 20.0 )</td>
<td>17</td>
</tr>
</tbody>
</table>

(df = 31 for all elemental precision errors)
3. To demonstrate use of the Welch—Satterthwaite method for determining degrees of freedom (df), small sample sizes for the calibration processes in the force transducer calibration hierarchy have been assumed. Sample sizes are included along with the elemental errors in Table 4.

\[
\text{df} = \frac{\left(\frac{s_1^2 + s_2^2 + \ldots + s_n^2}{df_1} + \frac{s_2^2 + \ldots + s_n^2}{df_2} + \ldots + \frac{s_n^2}{df_n}\right)^2}{\left(\frac{s_1^2}{df_1}\right) + \left(\frac{s_2^2}{df_2}\right) + \ldots + \left(\frac{s_n^2}{df_n}\right)}
\]

where \(df_n = \text{sample size minus one for the } n\text{th calibration}\)

\[
\text{df}_1 = \frac{(10)^2 + (10)^2 + (14.1)^2 + (20)^2}{10} = \frac{640 \times 10^3}{23 \times 10^3} = 27.8
\]

Under the "t" column in Table E-1 in the Handbook, Appendix E, \(t\) is 2.052 for 27 degrees of freedom and 2.048 for 28 degrees of freedom. Interpolating linearly gives a \(t\) of 2.049 for 27.8 degrees of freedom.

The calculation of calibration uncertainty \((U_1)\) for the force transducer is then

\[
U_1 = \pm (B_1 + t_{95}S_1) = \pm (0.94 + 2.049 \times 28.3) = \pm 58.9 \text{ lb}
\]

4. Data acquisition bias limit is

\[
B_2 = \pm \sqrt{\sum_i b_i^2} = \pm \sqrt{(5)^2 + (5)^2 + (5)^2 + (0.4)^2 + (10)^2 + (5)^2} = \pm 15.0 \text{ lb}
\]

5. Data acquisition precision index estimate is

\[
S_2 = \pm \sqrt{\sum_i s_i^2} = \pm \sqrt{(5)^2 + (5)^2 + (5)^2 + (10)^2 + (5)^2} = \pm 25.0 \text{ lb}
\]

6. Data acquisition uncertainty is

\[
U_2 = \pm (B_2 + t_{95}S_2) = \pm (15.0 + 2 \times 25.0) = 65.0 \text{ lb}
\]

\(t_{95} = 2.00\) because \(df > 30\) for \(S_2\)

7. Data reduction bias limit is

\[
B_3 = \pm \sqrt{\sum_i b_i^2} = \pm 10.0 \text{ lb}
\]

8. Data reduction precision index estimate is

\[
S_3 = \pm \sqrt{\sum_i s_i^2} = \pm 0
\]

9. Data reduction uncertainty is

\[
U_3 = \pm (B_3 + t_{95}S_3) = \pm (10 + 0.0) = \pm 10.0 \text{ lb}
\]

10. Force measurement bias limit is

\[
B_F = \pm \sqrt{B_1^2 + B_2^2 + B_3^2} = \pm \sqrt{(0.9)^2 + (15)^2 + (10)^2} = \pm 18.1 \text{ lb}
\]

11. Force measurement precision index estimate is

\[
S_F = \pm \sqrt{S_1^2 + S_2^2 + S_3^2} = \pm \sqrt{(28.3)^2 + (25)^2 + 0^2} = \pm 37.8 \text{ lb}
\]

12. Degrees of freedom for force measurement are

\[
df_F = \frac{(S_1^2 + S_2^2 + S_3^2)^2}{S_1^2 + S_2^2 + S_3^2} = \frac{57}{27.8 + 25^2} = 57
\]

13. Force measurement uncertainty is

\[
U_F = \pm (B_F + t_{95}S_F) = \pm (18.1 + 2.00 \times 37.8) \quad df_F = 57, \quad t_{95} = 2.00 = \pm 93.7 \text{ lb}
\]

**Propagation of Measurement Error**

Rarely are performance parameters measured directly; usually more basic quantities such as temperature, force, pressure, and fuel flow are measured, and
the performance parameter is calculated as a function of the measurements. Error in the measurements is propagated to the parameter through the function. The effect of the propagation may be approximated with the Taylor’s series methods.

The goal of any analysis of measurement system errors is to determine the resulting errors in the reduced parameters, for example TSFC, which is calculated as the ratio of fuel flow ($W_f$) to net thrust ($F_N$): 

$$TSFC = \frac{W_f}{F_N}$$

The technique for relating the errors of measurement to the errors in the reduced parameters is based on a Taylor’s Series expansion from the calculus. The Taylor’s expression for errors in thrust specific fuel consumption is

$$\Delta TSFC = \left( \frac{\partial TSFC}{\partial W_f} \Delta W_f + \frac{\partial TSFC}{\partial F_N} \Delta F_N \right)$$

Where $\frac{\partial TSFC}{\partial W_f}$ and $\frac{\partial TSFC}{\partial F_N}$ are the partial derivatives of thrust specific fuel consumption with respect to fuel flow and net thrust. The precision index is approximated by

$$S_{TSFC} = \sqrt{\frac{\partial TSFC}{\partial W_f}^2 \left( S_{W_f} \right)^2 + \frac{\partial TSFC}{\partial F_N}^2 \left( S_{F_N} \right)^2}$$

For example, the following hypothetical data were used to estimate thrust specific fuel consumption uncertainty:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal Limit</th>
<th>Bias Limit</th>
<th>Precision Index</th>
<th>Degrees of Freedom Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thrust ($F_N$)</td>
<td>10,000 lb</td>
<td>1.1 lb</td>
<td>37.8 lb</td>
<td>57</td>
</tr>
<tr>
<td>Fuel Flow ($W_f$)</td>
<td>10,000 lbm/hr</td>
<td>50 lbm/hr</td>
<td>50 lbm/hr</td>
<td>60</td>
</tr>
</tbody>
</table>

The nominal thrust specific fuel consumption is calculated from $W_f/F_N$:

$$\frac{W_f}{F_N} = 10,000 \text{ lbm/hr} = 1.0 \text{ lbm/lb- hr}$$

The precision index of thrust specific fuel consumption is

$$S_{TSFC} = \sqrt{\frac{1}{F_N^2} \left( S_{W_f} \right)^2 + \left( \frac{-W_f}{F_N^2} \right)^2 \left( S_{F_N} \right)^2}$$

$$= \sqrt{\left( \frac{50}{10,000} \right)^2 + \left( \frac{-10,000}{10,000^2} \times 37.8 \right)^2}$$

$$= \pm 0.0063 \text{ lbm/lb- hr}$$

The propagation formula is similar for bias

$$B_{TSFC} = \sqrt{\left( \frac{\partial TSFC}{\partial W_f} B_{W_f} \right)^2 + \left( \frac{\partial TSFC}{\partial F_N} B_{F_N} \right)^2}$$

The degrees of freedom for the TSFC precision index can be found using the Welch-Satterthwaite technique. In this situation, the partial derivative weighting factors, which are used in the calculation of the precision index, must also be used in the Welch-Satterthwaite formula. Note: The calculation is carried out to illustrate the use of the partial derivatives with the Welch-Satterthwaite. It is not necessary to calculate the degrees of freedom for TSFC since the degrees of freedom for thrust and fuel flow are 57 and 60, respectively. The expected minimum result would be 57. The t multiple is essentially 2.0 for degrees of freedom greater than 30.

When the degrees of freedom for each component are greater than 30, the Welch-Satterthwaite procedure can be omitted and $t = 2.0$ can be used.

The results of the error analysis are presented in Table 5.

The handbook illustrates the uncertainty in several other turbine engine measured and performance parameters such as fuel flow, pressure, temperature, airflow, and net thrust. The Handbook also contains a Special Methods section treating special situations or conditions and an Appendix containing information on precision index for uniform distribution of error, propagation of errors by Taylor’s Series, estimates of the precision index from multiple measurements, outlier detection schemes, and some statistical tables.

We believe that the methods we have presented represent the best technology available. The uncertainty model is the product of years of research in that the
model has as its beginning the ICRPG work. The model represents the efforts of many organizations. We hope that you will respond with constructive criticism for the continued improvement of this model. We are indebted to the many engineers and statisticians who have contributed to the work. A few must be noted for their particular contributions, Dr. Joan Rosenblatt, Dr. H. H. Ku, and J. M. Cameron of the National Bureau of Standards for their helpful discussions and comments on both this handbook and CPIA 180, and similarly, R. E. Smith, Jr., Dep. Dir., of Engine Test Facility, as well as T. C. Austin, C. R. Bartlett, W. O. Boals, Jr., and T. J. Gillard of ARO, Inc., at the Arnold Engineering Development Center. Engineers at Pratt & Whitney Aircraft, Florida and Connecticut facilities, provided the authors with constructive and spirited criticism in every section. Various technical committees under the American Society of Mechanical Engineers (ASME), The American Institute of Aeronautics and Astronautics, and the International Standards Organizations expressed interest and comments.

**Logic Decision Diagram**

We have summarized the measurement uncertainty model in a logic decision diagram shown in Table 6.

**Summary**

We have briefly described the gas turbine engine measurement uncertainty model; the Handbook on Uncertainty in Gas Turbine Measurements is recommended to you for a more detailed and comprehensive treatment.

### Table 5. Uncertainty Components

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal Value</th>
<th>Bias Limit</th>
<th>Precision Error</th>
<th>Degrees of Freedom</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thrust, $F_N$</td>
<td>10,000 lbf</td>
<td>18.1 lbf</td>
<td>37.8 lbf</td>
<td>57</td>
<td>93.7 lbf</td>
</tr>
<tr>
<td>Fuel Flow, $W_f$</td>
<td>10,000 lbm/hr</td>
<td>50 lbm/hr</td>
<td>50 lbm/hr</td>
<td>60</td>
<td>150 lbm/hr</td>
</tr>
<tr>
<td>Thrust Specific</td>
<td>1.0 lbm/lbf</td>
<td>0.0053 lbm/lbf</td>
<td>0.0063 lbm/lbf/hr</td>
<td>110</td>
<td>0.018 lbm/lbf/hr</td>
</tr>
</tbody>
</table>

### Table 6. Logic Decision Diagram

<table>
<thead>
<tr>
<th>To Estimate</th>
<th>Use</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bias Limit</td>
<td>1. Elemental (b)</td>
<td>Estimate a Reasonable Limit for Each Bias Error</td>
</tr>
<tr>
<td></td>
<td>Judgment Supported by Special Test Data</td>
<td>$B_j = \sqrt{\sum b_i^2}$</td>
</tr>
<tr>
<td></td>
<td>2. Measurement Bias</td>
<td>Estimated Elementals</td>
</tr>
<tr>
<td></td>
<td>3. Performance Parameter Bias</td>
<td>Measurement Bias and the Propagation of Error (Taylor's Series)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$B_F = \sqrt{\sum \left( \frac{\partial F}{\partial X} B_i \right)^2}$</td>
</tr>
<tr>
<td>Precision Index</td>
<td>1. Elemental $s_i$</td>
<td>Data from Multiple Measurements</td>
</tr>
<tr>
<td></td>
<td>$S_i = \sqrt{\frac{\sum (X_j - \bar{X})^2}{N - 1}}$, df = $N - 1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Measurement Precision Index</td>
<td>Calculated Elementals and Data</td>
</tr>
<tr>
<td></td>
<td>$S_j = \sqrt{\sum \frac{s_i^2}{df_i}}$, df = $\sum \frac{s_i^2}{df_i}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Performance Parameter Precision Index</td>
<td>Measurement Precision Indices and the Propagation of Error (Taylor's Series)</td>
</tr>
<tr>
<td></td>
<td>$S_F = \sqrt{\sum \left( \frac{\partial F}{\partial X} S_i \right)^2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>df$_F = \sqrt{\sum \left( \frac{\partial F}{\partial S_i} S_i \right)^2}$</td>
<td></td>
</tr>
</tbody>
</table>

1. elemental (b) judgment supported by special test data
2. measurement bias estimated elementals
3. performance parameter bias measurement bias and the propagation of error (taylor's series)
Table 6. Logic Decision Diagram (continued)

<table>
<thead>
<tr>
<th>To Estimate</th>
<th>Use</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{95}$ Value</td>
<td>Degrees of Freedom Less Than 30 ($df &lt; 30$)</td>
<td>Interpolate in Two-Tailed Student's &quot;t&quot; Table for t</td>
</tr>
<tr>
<td></td>
<td>Degrees of Freedom Greater Than or Equal to 30 ($df \geq 30$)</td>
<td>Use $t = 2.0$</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>Elemental</td>
<td>Elemental Bias Limit and Precision Index</td>
</tr>
<tr>
<td></td>
<td>Measurement</td>
<td>Measurement Bias Limits and Precision Indices</td>
</tr>
<tr>
<td></td>
<td>Performance Parameter</td>
<td>Performance Parameter Bias Limit and Precision Index</td>
</tr>
</tbody>
</table>

Elemental Bias Limit and Precision Index:

$$U_i = \pm \left[ B_i + t_{95} S_i \right]$$

Measurement Bias Limit and Precision Indices:

$$U_j = \pm \left[ B_j + t_{95} S_j \right]$$

Performance Parameter Bias Limit and Precision Index:

$$U_F = \pm \left[ B_F + t_{95} S_F \right]$$