6 Estimation and presentation of elemental errors

6.1 Summary of procedure

Obtain an estimate of each error. If the data is available to estimate the experimental standard deviation, classify the error as a random error. Otherwise, classify it as a systematic error.

Review the test objective, test duration and number of calibrations that will affect the test result. Make the final classification of elemental errors for each measurement. If an error increases the scatter in the measurement result in the defined test, it is a random error; otherwise, it is a systematic error.

6.2 Calculate the experimental standard deviation

There are many ways to calculate the experimental standard deviation:

a) If the parameter to be measured can be held constant, a number of repeated measurements can be used to evaluate equation (1)

$$S = \sqrt{\sum_{i=1}^{N} \frac{(X_i - \overline{X})^2}{N - 1}} \tag{10}$$

b) If there are M redundant instruments or M redundant measurements and the parameter to be measured can be held constant to take N repeat readings, the following pooled estimate of the experimental standard deviation for individual readings can be used:

$$S_{pooled} = \sqrt{\frac{\sum_{i=1}^{N}\sum_{j=1}^{M} (X_{ji} - \overline{X}_i)^2}{M(N - 1)}} \tag{11}$$

For the experimental standard deviation of the average value of the parameter

$$S_{\bar{\bar{x}}} = \frac{S_{pooled}}{\sqrt{MN}} \tag{12}$$

c) If a pair of instruments (providing measurements X_{1i} and X_{2i}) which have the same experimental standard deviation are used to estimate a parameter that is not constant with time, the difference between the readings, Δ, may be used to estimate the experimental standard deviation of the individual instruments as follows:

$$S = \sqrt{\frac{\sum_{i=1}^{N} (\Delta_i - \bar{\Delta})^2}{2(N - 1)}} \tag{13}$$

where

$$\Delta_i = X_{1i} - X_{2i}$$

If the degrees of freedom are less than 30, the small sample methods shown in annex C are required.

6.3 Estimate the systematic error limit

In spite of applying all known corrections to overcome imperfections in calibration, data acquisition and data reduction processes, some systematic errors will probably remain. To determine the exact systematic error in a measurement, it would be necessary to compare the true value and the measurements. However, as the true value is unknown, it is necessary to carry out special tests or utilize existing data that will provide systematic error information. The following examples are given in order of preference.

a) Interlaboratory or interfacility tests make it possible to obtain the distribution of systematic errors between facilities (Reference ISO 5725).

b) Comparisons of standards with instruments in the actual test environment may be used.

c) Comparison of independent measurements that depend on different principles can provide systematic error information. For example, in a gas turbine test, airflow can be measured with (1) an orifice, (2) a bellmouth nozzle, (3) compressor speed-flow rig data, (4) turbine flow parameters and (5) jet nozzle calibrations.

d) When it is known that a systematic error results from a particular cause, calibrations may be performed allowing the cause to perturbate through its complete range to determine the range of systematic error.

e) If there is no source of data for systematic error, the estimate must be based on judgment. An estimate of an upper limit of the systematic error is needed. Instrumentation manufacturers' reports and other references may provide information. It is important to distinguish between the "estimate" of an upper limit on systematic error obtained by this method and the more reliable estimate of a random error arrived at by analyzing data. There is a general tendency to underestimate systematic uncertainties when a subjective approach is used, partly through human optimism and partly through the possibility of overlooking the existence of some sources of systematic error. Great care is therefore necessary when quoting systematic error limits.

Sometimes the physics of the measurement system provide knowledge of the sign but not the magnitude of the systematic error. For example, hot thermocouples radiate and conduct thermal energy away from the sensor to indicate lower temperatures. The systematic error limits in this case are non-symmetrical, i.e., not of the form $\pm B$. They are of the form B^+ for the upper limit and B^- for the lower limit. Thus, typical systematic error limits associated with a radiating thermocouple could be:

$B^+ = 0$ degrees
$B^- = -10$ degrees

For elemental error sources, the interval from B^+ to B^- must include zero.

6.4 Final error classification based on the defined measurement

Uncertainty statements must be related to a well defined measurement process. The final classification of errors into systematic (bias) and random (precision) depends on the definition of the measurement process. Some of these considerations are:

a) Long versus Short Term Testing (see 6.4.1)

b) Comparative versus Absolute Testing (see 6.4.2)

c) Averaging to Reduce Random Error (see 6.4.3)

6.4.1 Long versus short term testing

The calibration histories accumulated before or during the testing period may influence the uncertainty analysis.

1) When the instrumentation is calibrated only once, all the calibration error is frozen into systematic error. The error in the calibration correction is a constant and cannot increase the scatter in a test result. Thus, the calibration error, made up in general of systematic and fossilized random errors, is considered to be all systematic errors in this case.

2) If the test period is long enough that instrumentation may be calibrated several times or more and/or several test stands are involved, the random error in the calibration hierarchy (see 5.4) should be treated as contributing to the overall experimental standard deviation. The experimental standard deviations may be derived from calibration data.

6.4.2 Comparative versus absolute testing

The objective of a comparative test is to determine, with the smallest measurement uncertainty possible, the net effect of a design change. The first test is run with the standard or baseline configuration. The second test is run with the design change. The difference between the results of these tests is an indication of the effect of the design change. As long as only the difference or net effect between the two tests is considered, all systematic errors, being fixed, will cancel out. The measurement uncertainty will be composed of random errors only.

The uncertainty of the back-to-back tests can be considerably reduced by repeating them several times and averaging the differences.

All errors in a comparative test arise from random errors in data acquisition and data reduction. Systematic errors are effectively zero. Since calibration random errors have been considered systematic errors, they also are effectively zero.

The test result is the difference in flow between two test results, r_1 and r_2.

$$\Delta r = r_1 - r_2 \tag{14}$$

and

$$S_{\Delta r} = \sqrt{S_{r_1}^2 + S_{r_2}^2} = \sqrt{2}\, S_{r_1} \tag{15}$$

where S_{r_1} is the random error of the first test, the root sum-square of the experimental standard deviations from data acquisition and data reduction, and S_{r_2} is assumed to equal S_{r_1}.

6.4.3 Averaging to reduce random error

Averaging test results is often used to improve the random uncertainty. Careful consideration should be given to designing the test series to average as many causes of variation as possible within cost constraints. The design should be tailored to the specific situation. For example, if experience indicates time-to-time and rig-to-rig variations are significant, a design that averages multiple test measurement results on one rig on one day may produce optimistic random error estimates compared to testing several rigs, each mounted several times, over a period of weeks. The list of possibilities may include the above plus test stand-to-test stand, instrument-to-instrument, mount-to-mount and environmental, fuel, power and test crew variation. Historic data is invaluable for studying these effects.* If the pretest uncertainty analysis identifies unacceptably large error sources, special tests to measure the effects should be considered.

* A statistical technique, analysis of variance (ANOVA) is useful for partitioning total variance by cause.

6.5 Example: a calibration constant

Assume a test meter is to be compared or calibrated with a master meter at one flow level. The objective is to determine a correction, called a calibration constant, that will be added to the test meter observations when it is installed for test. This calibration constant correction will make the test meter "read like" the master meter. During the calibration, the master meter is used to set the flow level as it is usually more accurate than the test meter. To reduce the calibration random error, N=13 comparisons will be made and averaged. If the data were plotted, the data might look like figure 10.

If the master meter systematic error limit from its own calibration is judged to be no larger than B_M, what will the test meter uncertainty be after calibration?

Define Δ_i = Master Meter Reading$_i$ − Test Meter Reading$_i$

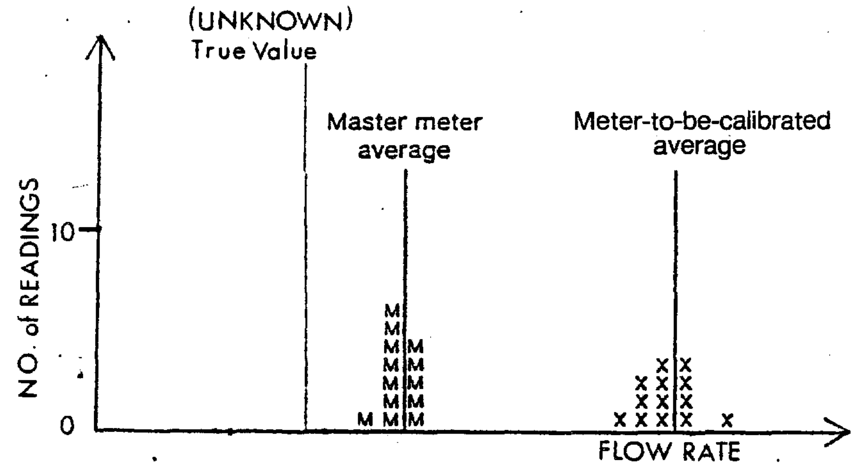
Calibration Constant equals the average

$$K = \bar{\Delta} = \frac{\Sigma \Delta_i}{13} \tag{16}$$

The sample standard deviation of the calibration constant K is:

$$S_K = \frac{S_\Delta}{\sqrt{13}} = \sqrt{\frac{\Sigma(\Delta - \bar{\Delta})^2}{13(12)}} \tag{17}$$

The test meter is later installed in a test stand. Each observation made on the test meter is corrected by adding K. By this process, the error in K from the calibration process is propagated to the corrected data from the test stand.



Master meter systematic error = B_M
Calibration random error = S_K

Figure 10 — Calibration should compensate for test meter systematic error

If the defined measurement process is short, involving a single calibration, K is constant and this error must be a constant or systematic error. It includes the systematic error in the master meter plus the random error in the calibration process. The random error is fossilized into systematic error. The fossilization is indicated by an asterisk. We can estimate an upper limit for this systematic error as:

$$B_K = \sqrt{B_M^2 + (t_{95}\, S_K)^{*2}} \qquad (18)$$

Where B_M is the systematic error limit of the master meter and t_{95} = 2.179 for 12 degrees of freedom (annex C).)

This calibration systematic error limit would be combined with systematic error limits from data acquisition and data reduction to obtain the measurement systematic error limit. There would also be random error from these last two processes.

If the uncalibrated test meter had a systematic error limit judged to be B_T, the calibration process improved the test accuracy if B_K is less than B_T. Note that the calibration process does not change the test meter random error which is included in the data acquisition random error. However, the test meter random error contributes to the calibration random error S_K. This contribution is reduced by averaging the calibration data.

If the test process is long and involves several calibrations, the calibration error contributes both systematic error (B_M) and random error (t_{95} S_K) to the final test result.

If the test process is comparative, the difference between two tests with a single calibration, the calibration error is all systematic error and cancels out when one result is subtracted from the other.

7 Combination and propagation errors

7.1 Summary of procedure

Root-sum-square the systematic error limits and experimental standard deviations for each measurement. Propagate the measurement systematic error and random error limits separately all the way to the final test result, either by sensitivity factors or by finitely incrementing the data reduction program. Work consistently in either absolute units or percentages.

7.2 Combining sample standard deviations

The experimental standard deviation (S) of the measurement is the root-sum-square of the elemental experimental standard deviations from all sources, that is;

$$S = \sqrt{\sum_{j=1}^{5}\sum_{i=1}^{k} S_{ij}^2} \tag{19}$$

where j defines the category: such as (1) calibration, (2) data acquisition, (3) data reduction, (4) errors of method and (5) subjective or personal, and i defines the sources within the categories.

For example: the experimental standard deviation for the calibration process in table 1 is:

$$S_1 = S_{Calibration} = \sqrt{\sum_{i=1}^{4} S_{i1}^2} = \sqrt{S_{11}^2 + s_{21}^2 + S_{31}^2 + S_{41}^2} \tag{20}$$

The measurement experimental standard deviation is the root-sum-square of all the elemental experimental standard deviations in the measurement system:

$$S = S_{Measurement} = \sqrt{\sum_{j=1}^{3} S_j^2} = \sqrt{S_1^2 + S_2^2 + S_3^2} \tag{21}$$

Categories (4) or (5) are optional and may or may not be employed.

7.3 Combining elemental systematic error limits

If there were only a few sources of elemental systematic errors, it might be reasonable to add them together to obtain the overall systematic error limits. For example, if there were three sources, the probability that they would all be plus (or minus) would be one-half raised to the third power or one eighth. However, the probability that all three will have the same sign and be at the systematic error limit is extremely small. In actual practice, most measurements will have ten, twenty or more sources of systematic error. The probability that they would all be plus (or minus) and be at their limit is close to zero, and therefore, it is more appropriate to combine them by root-sum-square.

If a measurement uncertainty analysis identifies four or less sources of systematic error, there should be some concern that some sources have been overlooked. The analysis should be redone and expert help should be recruited to examine the calibration hierarchy, the data acquisition process and the data reduction procedure for additional sources.

Therefore, the systematic error limit will be used herein as the root-sum-square of the elemental errors from all sources.

$$B = \sqrt{\sum_j \sum_i B_{ij}^2} \tag{22}$$

For example: the systematic error limit for the calibration hierarchy (table 1) is

$$B_1 = B_{Cal} = \sqrt{B_{11}^2 + B_{21}^2 + B_{31}^2 + B_{41}^2} \tag{23}$$

The systematic error limit for the basic measurement process is

$$B = \sqrt{B_1^2 + B_2^2 + B_3^2} \tag{24}$$

If any of the elemental systematic error limits are non-symmetrical, separate root-sum-squares are used to obtain B^+ and B^-. For example, assume B_{21} and B_{23} are non-symmetrical, i.e. B_{21}^+, B_{21}^-, B_{23}^+ and B_{23}^- are available. Then

$$B^+ = \sqrt{B_{11}^2 + (B_{21}^+)^2 + B_{31}^2 + B_{41}^2 + B_2^2 + B_{13}^2 + (B_{23}^+)^2} \tag{25}$$

$$B^- = \sqrt{B_{11}^2 + (B_{21}^-)^2 + B_{31}^2 + B_{41}^2 + B_2^2 + B_{13}^2 + (B_{23}^-)^2} \tag{26}$$

7.4 Propagation of measurement errors

Fluid flow parameters are rarely measured directly; usually more basic quantities such as temperature and pressure are measured, and the fluid flow parameter is calculated as a function of the measurements. Error in the measurements is propagated to the parameter through the function. The effect of the propagation may be approximated with the Taylor's series methods. It is convenient to introduce the concept of the sensitivity of a result to a measured quantity as the error propagated to the result due to unit error in the measurement. The "sensitivity coefficient" (also known as "influence coefficient") of each subsidiary quantity is most easily obtained in one of two ways.

a) Analytically

Where there is a known mathematical relationship between the result, R, and subsidiary quantities, Y_1, Y_2 . . . Y_K the dimensional sensitivity coefficient, θ_i of the quantity Y_i, is obtained by partial differentiation. Thus, if $R = f(Y_1, Y_2 \ldots Y_K)$, then

$$\theta_j = \frac{\partial R}{\partial Y_i} \qquad (27)$$

Analogously, the relative (nondimensional) sensitivity coefficient, θ_i', is

$$\theta_i' = \frac{\partial R/R}{\partial Y_i/Y_i} \qquad (28)$$

In this form, the sensitivity is expressed as "percent/percent." That is, θ_i' is the percentage change in R brought about by a 1% change in Y_i. This is the form to be used if the uncertainties to be combined are expressed as percentages of their associated variables rather than absolute values.

b) Numerically

Where no mathematical relationship is available or when differentiation is difficult, finite increments may be used to evaluate θ_i. This is a convenient method with computer calculations.

Here θ_i is given by

$$\theta_i = \frac{\Delta R}{\Delta Y_i} \qquad (29)$$

The result is calculated using Y_i to obtain R, and then recalculated using $(Y_i + \Delta Y)$ to obtain $(R + \Delta R)$. The value of ΔY used should be as small as practicable.

Care should be taken to ensure that the errors are independent. With complex parameters, the same measurement may be used more than once in the formula. This may increase or decrease the error depending on whether the sign of the measurement is the same or opposite. If the Taylor's series relates the most elementary mesurements to the ultimate parameter or result, these "linked" relationships will be properly accounted for.

This effect can be covered by calculating a modified θ by simultaneous perturbation of all the inputs likely to be affected, thus:

θ_{link} = (Change in output R due to simultaneous application of linked error in all inputs, y_i)

An example of this is barometric pressure which affects all pressure inputs simultaneously, in a "gauge-pressure" system. Another example is the use of a common working standard to calibrate all the pressure transducers.

Such linked errors can then be combined with independent ones, thus:

$$S(R) = \sqrt{[\theta_{link}\, S(y_{link})]^2 + \Sigma\, [\theta_i\, S(y_i)]^2} \qquad (30)$$

7.5 Airflow example

In this example, airflow is determined by the use of a sonic nozzle and measurements of upstream stagnation temperature and stagnation pressure (figure 11).

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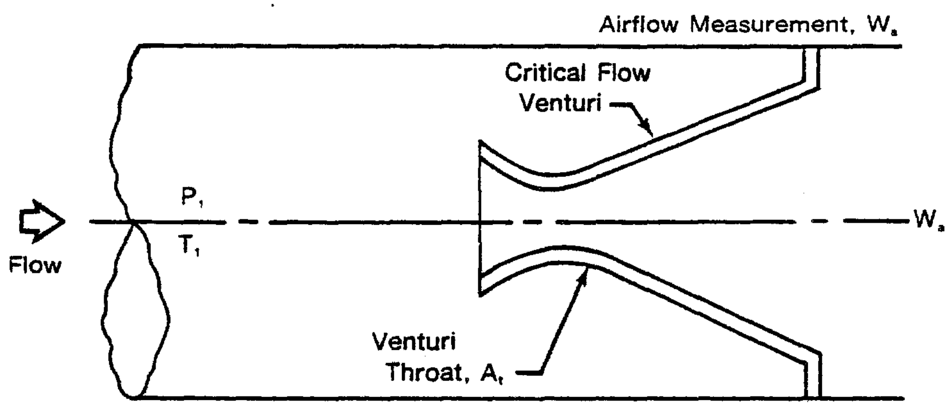

Figure 11 — Flow through a sonic nozzle

The flow is calculated from

$$W = CaF_a\varphi^* \frac{P_{1t}}{\sqrt{T_{1t}}} \qquad (31)$$

where

$\dot{W}$ is the mass flowrate of air
F_a is the factor to account for thermal expansion of the venturi
a is the venturi throat area
P_{1t} is the total (stagnation) pressure upstream
T_{1t} is the total temperature upstream
φ^* is the factor to account for the properties of the air (critical flow constant)
C is the discharge coefficient

The experimental standard deviation for the Flow (S_w) is calculated using the Taylor's series expansion.

Assuming C equals 1 and has negligible error

$$S_w = \left[(\theta_{F_a} S_{F_a})^2 + (\theta_{\varphi^*} S_{\varphi^*})^2 + (\theta_a S_a)^2 + (\theta_{P_{1t}} S_{P_{1t}})^2 + (\theta_{T_{1t}} S_{T_{1t}})^2 \right]^{1/2} \qquad (32)$$

where

$$\frac{\partial W}{\partial F_a}$$

denotes the partial derivative of W with respect to F_a.

$$S_w = C\left[\left(\frac{\varphi^* a P_{1t}}{\sqrt{T_{1t}}} S_{F_a}\right)^2 + \left(\frac{F_a a P_{1t}}{\sqrt{T_{1t}}} S_{\varphi^*}\right)^2 + \left(\frac{F_a\varphi^* P_{1t}}{\sqrt{T_{1t}}} S_a\right)^2 + \left(\frac{F_a\varphi^* a}{\sqrt{T_{1t}}} S_{P_{1t}}\right)^2 + \left(\frac{F_a\varphi^* a P_{1t}}{-2\sqrt{T_{1t}^3}} S_{T_{1t}}\right)^2 \right]^{1/2} \qquad (33)$$

By inserting the values and random errors from table 4 into equation (32), the random error of 0.17 kg/sec for airflow is obtained.

The systematic error in the flow calculation is propagated from the systematic error limits of the measured variables. Using the Taylor's series formula gives

$$B_w = \left[(\theta_{x_1} B_{x_1})^2 + (\theta_{x_2} B_{x_2})^2 + (\theta_{x_3} B_{x_3})^2 + \ldots + (\theta_{x_m} B_{x_m})^2 \right]^{1/2} \qquad (34)$$

For this example,

$$B_w = \left[(\theta_{F_a} B_{F_a})^2 + (\theta_{\varphi^*} B_{\varphi^*})^2 + (\theta_a B_a)^2 + (\theta_{P_{1t}} B_{P_{1t}})^2 + (\theta_{T_{1t}} B_{T_{1t}})^2 \right]^{1/2} \qquad (35)$$

Taking the necessary partial derivatives gives

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$$B_w = C\Big[\Big(\frac{\varphi^* a P_{1t}}{\sqrt{T_{1t}}} B_{F_a}\Big)^2 + \Big(\frac{F_a a P_{1t}}{\sqrt{T_{1t}}} B_{\varphi^*}\Big)^2 + \Big(\frac{F_a \varphi^* P_{1t}}{\sqrt{T_{1t}}} B_a\Big)^2 + \Big(\frac{F_a \varphi^* a}{\sqrt{T_{1t}}} B_{P_{1t}}\Big)^2 + \Big(\frac{F_a \varphi^* a P_{1t}}{-2\sqrt{T_{1t}^3}} B_{T_{1t}}\Big)^2\Big]^{1/2} \qquad (36)$$

By inserting the values and systematic error limits of the measured parameters from table 4 into equation (36), a systematic error limit of 0.32 kg/sec is obtained for a nominal airflow of 112.64 kg/sec.

Table 4 contains a summary of the measurement uncertainty analysis for this flow measurement. It should be noted the errors listed only apply to the nominal values.

Table 4 — Flow data

Parameter	*Units*	*Nominal value*	*Experimental standard deviation (one experimental standard deviation)*	*Systematic error*
F_a	—	1.00	0.0	0.001
C	—	1.0	0.0	0.0
φ^*	kg $K^{1/2}$ newton sec	0.0404	0.0	4.04×10^{-3}
a	m^2	0.191	9.55×10^{-5}	3.82×10^{-4}
P_{1t}	Pa	2.54×10^5	345.0	345.0
T_{1t}	K	303.0	0.17	0.17
∴ w	kg/sec	112.64	0.17	0.32

8 Calculation of uncertainty

8.1 Summary of procedure

Select U_{ADD} and/or U_{RSS} and combine the systematic and random errors of the test result to obtain the uncertainty. The test result plus and minus the uncertainty is the uncertainty interval that should contain the true value with high probability.

8.2 Uncertainty intervals

For simplicity of presentation, a single number (some combination of systematic and random errors) is needed to express a reasonable limit for error. The single number should have a simple interpretation (like the largest error reasonably expected) and be useful without complex explanation. It is usually impossible to define a single rigorous statistic because the systematic error is an upper limit based on judgment which has unknown characteristics.* This function is a hybrid combination of an unknown quantity (systematic error) and a statistic (random error). If both numbers were statistics, a confidence interval would be recommended. 95% or 99% confidence levels would be available at the discretion of the analyst. Although rigorous statistical confidence levels are not available, two uncertainty intervals, approximately analogous to 95% and 99% levels, are recommended. This analogy is discussed in Annex F.

8.3 Symmetrical intervals

Uncertainty (figure 12) for the symmetrical systematic error case is centered about the measurement and the uncertainty intervals are defined as:

R − U, R + U, where

$$U_{ADD} = U_{99} = (B + t_{95} S) \qquad (37)$$

$$U_{RSS} = U_{95} = \sqrt{B^2 + (t_{95} S)^2} \qquad (38)$$

If the sample standard deviation is based on small samples, the methods in annex C may be used to determine a value of Student's t_{95}. For large samples (>30), 2 may be substituted for t_{95} in equations (37) and (38).

If the test result is an average $(\bar{R})$ based on sample size N, instead of a single value (R), $S/\sqrt{N}$ should be substituted for S.

The uncertainty interval selected (equations (37) or (38)) should be provided in the presentation; the components (systematic error, random error, degrees of freedom) should be available in an appendix or in supporting documentation. These three components may be required to substantiate and explain the uncertainty value, to provide a sound technical base

* If information exists to justify the assumption that the systematic error limits have a random distribution, a rigorous statistic can be defined as shown in annex E.

for improved measurements, and to propagate the uncertainty from measured parameters to fluid flow parameters and from fluid flow parameters to other more complex performance parameters (i.e., fuel flow to Thrust Specific Fuel Consumption (TSFC), TSFC to aircraft range, etc).

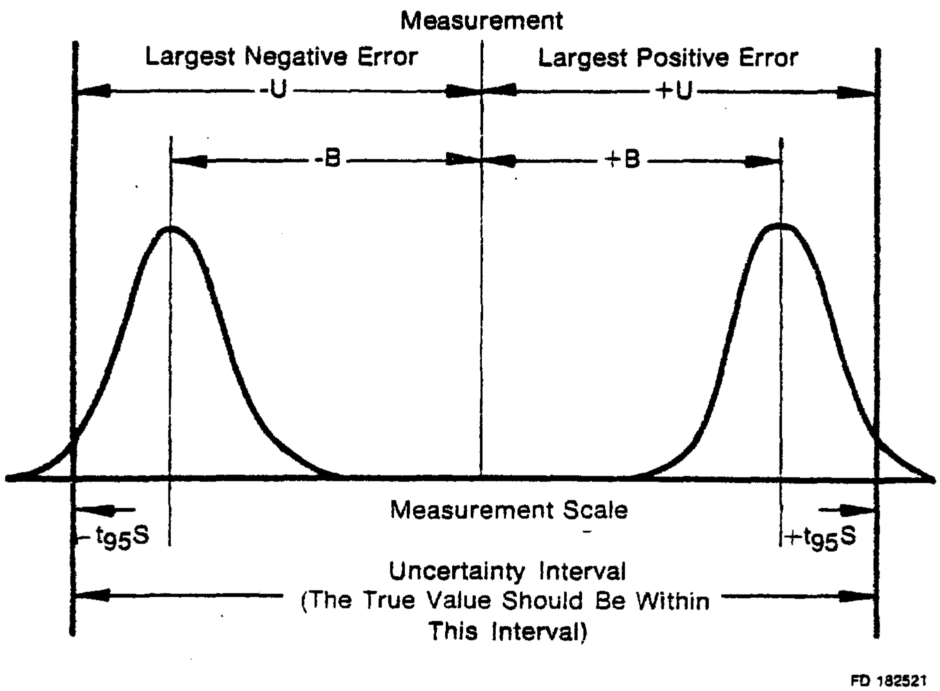

Figure 12 — Measurement uncertainty interval (U_{99}); symmetrical systematic error

9 Presentation of results

9.1 *Summary of requirement*

The summary report should contain the nominal level of the test result, the systematic error, the sample standard deviation, the degrees of freedom and the uncertainty. The equation used to calculate uncertainty, U_{ADD} or U_{RSS} should be stated. The summary should reference a table of the elemental errors considered and included in the uncertainty.

9.2 *Reporting error summary*

The definition of the components, systematic error limit, experimental standard deviation and the limit (U) suggests a summary format for reporting measurement error. The format will describe the components of error, which are necessary to estimate further propagation of the errors, and a single value (U) which is the largest error expected from the combined errors. Additional information, degrees of freedom for the estimate of S, is required to use the experimental standard deviation if small samples were used to calculate S. These summary numbers provide the information necessary to accept or reject the measurement error. The reporting format is:

a) S, the estimate of the experimental standard deviation, calculated from data.

b) For small samples, ν, the degrees of freedom associated with the estimate of the experimental standard deviation (S). The degrees of freedom for small samples (less than 30) is obtained from the Welch-Satterthwaite procedure illustrated in annex C.

c) B, the upper limit of the systematic error of the measurement process or B^- and B^+ if the systematic error limit is non-symmetrical.

d) The uncertainty formula should be stated. $U_{99} = (B + t_{95}\,S)$ or $U_{95} = \sqrt{B^2 + (t_{95}\,S)^2}$, the uncertainty interval, within which the error should fall. If the systematic error limit is non-symmetrical, $U_{99}^{-} = B^{-} - t_{95}S$ and $U_{99}^{+} = B^{+} + t_{95}\,S$. No more than two significant places should be reported. For small samples see annex C.

The model components, S, ν, B, and U, are required to report the error of any measurement process. The first three components, S, ν, and B, are necessary to: (1) indicate corrective action if the uncertainty is unacceptably large before the test, (2) to propagate the uncertainty to more complex parameters, and (3) to substantiate the uncertainty limit.

9.3 Reporting error — table of elemental sources

To support the measurement uncertainty summary, a table detailing the elemental error sources is needed for several purposes. If corrective action is needed to reduce the uncertainty or to identify data validity problems, the elemental contributions are required. Further, if the uncertainty quoted in the summary appears to be optimistically small, the list of sources considered should be reviewed to identify missing sources. For this reason, it is important to list all sources considered even if negligible.

Note that all errors in table 5 have been propagated from the basic measurement to the end result before listing and, therefore, they are expressed in units of the test result.

Table 5 — Elemental error sources

	ij subscript	*Source*	*Measurement nominal value*	*Experimental standard deviation* S_{ij}	*Degrees of freedom* ν_{ij}	*Systematic error limit* B_{ij}	*Source of systematic error*	
Calibration	11							
	21							
	31							
	—							
	—							
	—							
Data Acquisition	12							
	22							
	32							
	42							
	—							
	—							
	—							
Data Reduction	13							
	23							
	33							
	—							
	—							
	—							
	Results		Nominal Value ____	$S = \sqrt{\Sigma S_{ij}^{2}}$ ____	ν w/s ____	$B = \sqrt{\Sigma B^2}$ ____		$t_{95\%}$ ____

$$U_{99} = B + t_{95}\,(S) = \_\_\_$$

$$U_{95} = \sqrt{B^2 + (t_{95}S)^2} = \_\_\_$$

9.4 Pre-test analysis and corrective action

Uncertainty is a function of the measurement process. It provides an estimate of the largest error that may reasonably be expected for that measurement process. Errors larger than the uncertainty should rarely occur. If the difference to be detected in an experiment is of the same size or smaller than the projected uncertainty, corrective action should be taken to reduce the uncertainty. Therefore, it is recommended that an uncertainty analysis always be done before the test or experiment. The recommended corrective action depends on whether the systematic or the random error is too large as shown in table 6.

Table 6 — Recommended corrective action if the predicted pretest measurement accuracy is unacceptable

Systematic Error Limit Too Large:	*Random Error Too Large:*
• Improve calibration	• Larger test sample
• Independent calibrations for redundant meters	• More precise instrumentation
• Concommitant variable	• Redundant instrumentation
• In place calibration	• Data smoothing — Moving average — Filter — Regression
	• Improve design of experiment

9.5 Post-test analysis and data validity

Post-test analysis is required to confirm the pretest estimates or to identify data validity problems. Comparison of measurement test results with the pretest analysis is an excellent data validity check. The random error of the repeated points or redundant instruments should not be significantly larger than the pretest measurement estimates. When redundant instrumentation or calculation methods are available, the individual uncertainty intervals should be compared for consistency with each other and with the pretest measurement uncertainty analysis.

Three cases are illustrated in figure 13.

When there is no overlap between uncertainty intervals, as in Case I, a problem exists. The true value cannot be contained within both intervals. That is, there should be a very low probability that the true value lies outside any of the uncertainty intervals. Either the uncertainty analysis is wrong or a data validity problem exists. Investigation to identify bad readings, overlooked systematic error, etc., is necessary to resolve this discrepancy. Redundant and dissimilar instrumentation should be compared. Partial overlap of the uncertainty intervals, as in Case II, also signals that a problem may exist. The magnitude of the problem depends on the amount of overlap. The only situation when one can be confident that the data is valid and the uncertainty analysis is correct is Case III, when the uncertainty intervals completely overlap.

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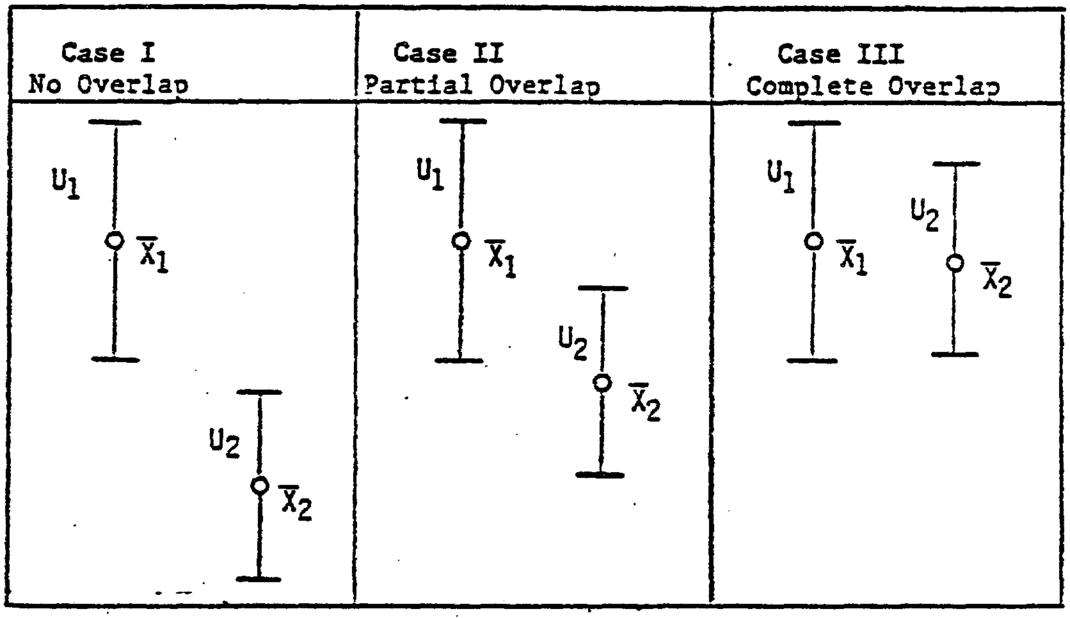

Figure 13 — Three post-test measurement uncertainty interval comparisons