SECTION VIII
SPECIAL METHODS

8.1 GENERAL

This section treats several methods for special situations or conditions:

1. Measurement uncertainty[9] for multi-engine installation (similar engines).
2. Measurement uncertainty for the single stand, single engine process in comparison with the many stand, many engine process.
3. Confidence interval for uncertainty when biases are negligible.
4. Compressor efficiency error analysis.
5. How to interpret uncertainty.
6. Dynamic measurement uncertainty.

8.2 MEASUREMENT UNCERTAINTY FOR MULTI-ENGINE INSTALLATIONS (SIMILAR ENGINES)

8.2.1 General

The uncertainty in performance parameters for the multi-engine aircraft for similar engines is a function of bias limits and precision indices of the individual engines for those performance parameters. If, for example, the parameter of interest is thrust, the total thrust (F_T) for the aircraft is the sum of the thrust values for the individual engines. The precision index (S_{F_T}) for total thrust is the root-sum-square of the precision indices (S_i) for the individuals (assuming that the engine run-to-run variance is negligible) and where S_i is the same for all engines:

$$S_{F_T}^{\,2} = \sum_{i=1}^{K} S_i^2 = KS_i^2$$

or

$$S_{F_T} = S_i\sqrt{K} \qquad \text{(VIII-1)}$$

where K is the number of engines. The degrees of freedom (df) associated with a production engine facility would exceed 30 in almost every case. The df is calculated from the degrees of freedom (df_i) for each engine.

$$df = \frac{\left(\sum_{i=1}^{K} S_i^2\right)^2}{\sum_{i=1}^{K} \frac{S_i^4}{df_i}} \equiv \frac{\left(KS_i^2\right)^2}{\left(\frac{1}{df_i}\right)KS_i^4} = Kdf_i > 30 \qquad \text{(VIII-2)}$$

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[9]For a definition of terms used in this Handbook, see the Glossary in Section IX.

The bias limit (B) for the installation is the sum of the bias limits (B_i) for each of the engines:

$$B = \sum_{i=1}^{K} B_i = KB_i \quad \text{(VIII-3)}$$

The bias limits are added rather than root-sum-squared because the bias errors for similar engines are not usually independent. (If the bias errors are independent, the bias limits may be root-sum-squared.) Therefore, these errors do not tend to cancel, and the limit is best estimated as the sum of the limits. This is consistent with the philosophy that independent bias limits are combined by root-sum-squaring. Finally, the uncertainty for the installation can be calculated:

$$U = \pm\left[KB_i + (t_{95}) S_i \sqrt{K}\right] \quad \text{(VIII-4)}$$

where t_{95} is the student's "t" value for 95 percent (two-tailed) confidence and df_i times K are the degrees of freedom.

8.2.2 Example of a Four-Engine Installation

Suppose an aircraft installation consists of four, 20,000-lb-thrust engines, each with the following reported measurement uncertainty:

Bias limit	±36 lb	df	27.8
Precision limit	±75 lb	Uncertainty	±190 lb

The precision index for the installation is calculated from Eq. (VIII-1):

$$S_{F_T} = \pm(75)\sqrt{4} = \pm(75)(2) = \pm 150.0$$

The degrees of freedom (Eq. (VIII-2)) for the precision index are 27.8 x 4 = 111.2. The bias limit (Eq. (VIII-3)) for the installation is 4 times (±36) = ±144. The uncertainty limit (Eq. (VIII-4)) for the cluster is

$$U = \pm(144 + t_{95} \times 150) = \pm[144 + 2(150)] = \pm 444 \text{ lb}$$

Since the degrees of freedom exceed 30, $t_{95} = 2.0$ is used.

8.3 MEASUREMENT PROCESSES

The measurement uncertainty estimate is a function of a specific measurement process. In this section, two different but related processes are discussed. The models illustrate extremes: many tests over a long period of time versus two tests over a short time interval. In the paragraphs that follow, the general model for many engines and many test stands is contrasted with the model for back-to-back development testing of a single engine on a single stand.

Note that in the following examples the engine hardware, instrumentation, and test stand might be identical but that the uncertainties are different because the measurement of interest is different.

8.3.1 Many Stand, Many Engine Model

The general process, which was defined in Section 1.7 and discussed throughout this Handbook, pertains to the measurement process defined for many sets of measurement instruments, many test stands, many calibrations, and many months of operation. An example is the measurement of TSFC at an engine production facility. The problem is to determine the absolute level of performance.

The uncertainty for this measurement process is $U = \pm(B + t_{95} S)$, where B is the root-sum-square of all elemental bias limits and S is the root-sum-square of all elemental precision indices for the process. For example, Table XX lists the elemental errors for a measurement system with six error sources. The root-sum-square of the bias limits is ±0.9. The root-sum-square of the precision indices is also ±0.9. The uncertainty of the general process is ±2.7.

Table XX A Measurement System with Six Error Sources

Source	Bias Limit	Precision Index	Uncertainty
NBS, Interlab Standard	±0.1	±0.1	±0.3
Interlab, Transfer Standard	±0.1	±0.1	±0.3
Transfer, Working Standard	±0.5	±0.5	±1.5
Working, End Instrument	±0.5	±0.5	±1.5
Calibration Hierarchy Error	±0.72	±0.72	±2.16
Data Acquisition	±0.5	±0.5	±1.5
Data Reduction	±0.2	±0.2	±0.6
Combined Error	±0.9	±0.9	±2.7

8.3.2 Single Stand, Single Engine Model

During a gas turbine development program, many tests are devoted to evaluating new component designs. The objective is to obtain the most accurate determination of the incremental change in performance between the baseline and the new configuration.

The engine (or rig) is installed on a test stand, and a baseline calibration is performed. Then, an engine design change is made without modification to the stand or instrumentation. Typically, these changes may be made without removing the engine from the test stand. For example, the inlet guide vanes might be replaced or compressor stator vane angles adjusted. Then, the engine is tested again, and performance is measured for comparison with the baseline calibration. The measurement is the difference between corresponding performance values for the two tests. The difference should be due to only three causes: instrumentation precision error, engine repeatability (which has been assumed to be negligible), and the design change. Since the repetitive tests are confined to a single set of instruments and a single engine, the measurement uncertainty is reduced; bias errors are not considered because they will be the same for each test and will not affect the comparison. The only errors which need to be considered are the run-to-run precision errors of the measurement system for each of the runs. For the data in Table XX, the precision index of the data acquisition is 0.5 and reduction processes is 0.2 for each run. The uncertainty must be calculated for the difference in the two runs. It is the root-sum-square of the run-to-run precision error of the data acquisition and reduction precision error for each run:

$$U = t_{95}\sqrt{S_{\text{run one}}^2 + S_{\text{run two}}^2} = t_{95}\sqrt{S_{\text{data acq}}^2 + S_{\text{data red}}^2 + S_{\text{data acq}}^2 + S_{\text{data red}}^2}$$

$$= t_{95}\sqrt{2(0.5^2 + 0.2^2)} = 2\sqrt{2}\,\sqrt{0.29} = 1.523$$

where $t_{95} = 2.0$.

8.4 CONFIDENCE INTERVAL WHEN BIASES ARE NEGLIGIBLE OR CAN BE IGNORED

When the bias is very small (negligible) or can be ignored, i.e., as in a back-to-back development test, the uncertainty parameter (U) becomes a statistical confidence interval.

The interpretation and the use of the uncertainty parameter are not changed. However, it is now defined with known exact characteristics. An uncertainty limit of six pounds force means that it would be "reasonable to expect" that the true value would be within six pounds force of the measured value. For 95 percent confidence intervals, 95 percent of the intervals will contain the true value. The qualitative concept "reasonable to expect" is quantified by the confidence concept. It is not necessary to distinguish between uncertainty intervals with and without bias errors. The same simple interpretation of uncertainty applies.

As an example of a confidence interval, take the following measurement data for specific fuel consumption for a 10,000-lbf gas turbine engine:

Specific Fuel Consumption	= 0.88
Bias Limit	= 0.0
Precision Index (S)	= 0.02
Degrees of Freedom	= 25.2

The point estimate or unbiased estimate of specific fuel consumption is 0.88. The uncertainty (U) is a 95 percent confidence interval estimate:

$$U = \pm t_{95} \times 0.02 = \pm 0.041$$

That is the 95-percent confidence interval is 0.88 ± 0.041 or 0.839 to 0.921. In repeated sampling, such an interval will contain the true value with 95-percent frequency.

8.5 COMPRESSOR EFFICIENCY ERROR ANALYSIS

8.5.1 General

Consider a compressor efficiency uncertainty analysis for the many stand, many compressor model and the single stand, single compressor model. For each analysis, Eq. (VIII-5) is used for the calculation of adiabatic efficiency (η) of a compressor as a function of the pressure and temperature ratios across the compressor:

$$\eta = \left[\left(\frac{P_1}{P_o}\right)^{\frac{K-1}{K}} - 1.0\right] \Big/ \left[\left(\frac{T_1}{T_o}\right) - 1.0\right] \qquad \text{(VIII-5)}$$

where

P_o = inlet pressure

P_1 = exit pressure

T_o = inlet temperature

T_1 = exit temperature

K = ratio of specific heats

The nominal values and elemental bias limits and precision indices are listed in Table XXI. These data imply a pressure ratio of 6.5, inlet and exhaust temperatures of 530 and 960°R, and efficiency of 85 percent.

Table XXI Tabulation of the Elemental Errors

Source	Bias Limit	Precision Index
T_o Nominal, 530°R		
Thermocouples at 530°R	0.0°F to +0.10°F	0.20°F
Reference	-0.10°F to +0.10°F	0.10°F
Signal Conditioning	-0.10°F to +0.10°F	0.50°F
Data Reduction	Negligible	Negligible
Combining	-0.14°F to +0.17°F	0.55°F
T_1 Nominal, 960°R		
Thermocouples	0.0°F to +1.00°F	0.50°F
Reference	-0.10°F to +0.10°F	0.10°F
Signal Conditioning	-0.10°F to +0.10°F	0.50°F
Data Reduction	Negligible	Negligible
Combining	-0.14°F to +1.01°F	0.714°F
P_o Nominal 14.696 psia		
Transducers	±0.1% ~0.015 psia	0.15% ~0.022 psia
Recovery Factor	Negligible	Negligible
Signal Conditioning	±0.1% ~0.015 psia	0.10% ~0.015 psia
Data Reduction	Negligible	Negligible
Combining	±0.14% ~0.021 psia	0.18% ~0.027 psia
P_1 Nominal 95.524 psia		
Transducers	±0.10% ~0.10 psia	0.15% ~0.14 psia
Recovery Factor	±0.10% ~0.10 psia	Negligible
Signal Conditioning	±0.10% ~0.10 psia	0.10% ~0.10 psia
Data Reduction	Negligible	Negligible
Combining	±0.17% ~0.173 psia	0.18% ~0.17 psia

Degree of freedom is assumed >30.
K is ratio of specific heats, Nominal, 1.39, errors negligible.

8.5.2 The General Process

Many compressors are tested in many rigs with many instrumentation setups over a period of years.

The measurement uncertainty for this process is the uncertainty associated with the absolute level of performance for a compressor. The interval includes all the bias errors and precision measurement errors associated with compressor efficiency calculation. All of the error estimates listed in Table XXI contribute to the uncertainty.

For each of the four values T_o, T_1, P_o, and P_1, the elemental bias limits are root-sum-squared and the elemental precision indices are root-sum-squared. Table XXII presents a summary of the combined bias limits and precision indices from Table XXI.

Table XXII Summary of Errors

Source	Bias Limits	Precision Index
T_o	$B_{T_o}^- = -0.14°F$ $B_{T_o}^+ = 0.17°F$	$S_{T_o} = 0.55°F$
T_1	$B_{T_1}^- = -0.14°F$ $B_{T_o}^+ = 1.01°F$	$S_{T_1} = 0.714°F$
P_o	$B_{P_o} = \pm 0.14\%$ or 0.021 psia	$S_{P_o} = 0.18\%$ or 0.027 psia
P_1	$B_{P_1} = \pm 0.17\%$ or 0.173 psia	$S_{P_1} = 0.18\%$ or 0.17 psia

The errors in T_o, T_1, P_o, and P_1 are propagated to efficiency using the Taylor's series method described in Appendix B. The appropriate calculations for the propagation are

$$S_\eta^2 = \left(\frac{\partial\eta}{\partial T_o} S_{T_o}\right)^2 + \left(\frac{\partial\eta}{\partial T_1} S_{T_1}\right)^2 + \left(\frac{\partial\eta}{\partial P_o} S_{P_o}\right)^2 + \left(\frac{\partial\eta}{\partial P_1} S_{P_1}\right)^2 \qquad \text{(VIII-6)}$$

$$B_\eta^2 = \left(\frac{\partial\eta}{\partial T_o} B_{T_o}\right)^2 + \left(\frac{\partial\eta}{\partial T_1} B_{T_1}\right)^2 + \left(\frac{\partial\eta}{\partial P_o} B_{P_o}\right)^2 + \left(\frac{\partial\eta}{\partial P_1} B_{P_1}\right)^2 \qquad \text{(VIII-7)}$$

where

$$\frac{\partial\eta}{\partial T_o} = \frac{T_1\left[\left(P_1/P_o\right)^{\frac{K-1}{K}} - 1\right]}{(T_1 - T_o)^2}$$

the rate of change of η with respect to T_o

$$\frac{\partial\eta}{\partial T_1} = \frac{-T_o\left[\left(P_1/P_o\right)^{\frac{K-1}{K}} - 1\right]}{(T_1 - T_o)^2}$$

the rate of change of η with respect to T_1

$$\frac{\partial\eta}{\partial P_o} = \frac{-\left(\frac{K-1}{K}\right)\left(P_1/P_o\right)^{\frac{-1}{K}}\left(P_1/P_o^2\right)}{(T_1/T_o) - 1}$$

the rate of change of η with respect to P_o

$$\frac{\partial\eta}{\partial P_1} = \frac{\left(\frac{K-1}{K}\right)\left(\frac{1}{P_o}\right)\left(P_1/P_o\right)^{\frac{-1}{K}}}{(T_1/T_o) - 1}$$

the rate of change of η with respect to P_1

By substituting into Eqs. (VIII-6) and (VIII-7), the precision index and the upper and lower bias limits are calculated.

$$S_\eta^2 = [(0.0036)(0.55)]^2 + [(-0.00198)(0.714)]^2 + [(-0.0398)(0.027)]^2 + [(0.0061)(0.17)]^2$$

$$= 0.00000815$$

$$S_\eta = 0.002854$$

$$(B_\eta^-)^2 = [(0.0036)(-0.14)]^2 + [(-0.0020)(-0.14)]^2 + [(-0.0398)(0.021)]^2 + [(0.0061)(0.173)]^2$$

$$= 0.00000214$$

$$B_\eta^- = 0.00146$$

$$(B_\eta^+)^2 = [(0.0036)(0.17)]^2 + [(-0.0020)(1.01)]^2 + [(0.0398)(0.021)]^2 + [(0.0061)(0.173)]^2$$

$$= 0.00000626$$

$$B_\eta^+ = 0.0025$$

B_η^- is assigned a negative value since the lower limits in Table XXI are all negative.

The propagated values to efficiency are S = 0.0029 for the precision index, and B^- = -0.0015 and B^+ = 0.0025 for the bias limits. The error in specific heat parameters (K) is considered negligible.

The uncertainty for the general model is the interval contained between U^- and U^+ where:

$$U^- = +B_\eta^- - t_{95}S_\eta \qquad \text{(VIII-8)}$$

$$U^+ = +B_\eta^+ + t_{95}S_\eta \qquad \text{(VIII-9)}$$

In this case, t_{95} = 2.0 since all the degrees of freedom are greater than 30.

$$U^- = -0.0015 - 2(0.0029) = -0.0073$$

$$U^+ = +0.0025 + 2(0.0029) = 0.0083$$

For the nominal values of Table XXI, the uncertainty interval would be 85% - 0.0073 to 85% + 0.0083 or 84.27% to 85.83%.

8.5.3 Single Stand, Single Compressor Process

For a back-to-back development process, two tests are performed on a single stand with a single compressor. No changes are allowed to the stand system or data recording equipment. The process would involve testing the compressor to establish a baseline efficiency value, changing the compressor configuration and retesting to determine a new efficiency value and to determine the delta from the baseline efficiency value.

The uncertainty interval for these tests is a function of the precision error only of the measurement system. The original table of elemental errors (Table XXI) is now reduced to Table XXIII.

Table XXIII Elemental Errors

Parameter	T_o, °F	T_1, °F	P_o, psia	P_1, psia
Signal Conditioning	0.5	0.5	0.015	0.1
Precision Index Reference	0.1	0.1	---	---
Root-Sum-Square	0.51	0.51	0.015	0.1

The root-sum-square errors are propagated to efficiency using Taylor's series methods described above and in Appendix B. The measurement of concern is the delta between two tests. Therefore, the uncertainty must be propagated to the delta value:

$$\Delta\eta = \eta_2 - \eta_1$$

where η_2 is the efficiency for the second run and η_1 is the efficiency for the first run. The propagation for the precision term $S_{\Delta\eta}$ is:

$$S_{\Delta\eta} = \pm\sqrt{S_{\eta_2}^{\ 2} + (-1)^2 S_{\eta_1}^{\ 2}} = \pm\sqrt{2 S_\eta^{\ 2}} = \pm S_\eta\sqrt{2} \qquad \text{(VIII-10)}$$

The uncertainty resulting from the values of Table XXIII is 0.0065.

8.6 HOW TO INTERPRET UNCERTAINTY

Uncertainty is a function of the measurement process as discussed in Section 8.3. A different definition of the process would significantly change the uncertainty. Table XXIV lists the uncertainty values for the many stand, many engine model and for the single stand, single engine model. These values are significantly different, yet both are based on the elemental errors listed in Table XXI.

Table XXIV Uncertainty Values for Two Processes

Measurement Process	Uncertainty Interval
General Measurement Process (for Intercompany Comparisons)	+ 3.7
Back-to-Back Testing Measurement Process	+ 2.0

Uncertainty, then, is a function of the measurement process. It provides an estimate of the largest error that may reasonably be expected for that measurement process (Fig. VIII-1).

Errors larger than the uncertainty should rarely occur. On repeated runs within a given measurement process, the parameter values should be within the uncertainty interval. These differences might look like Fig. VIII-2. Run-to-run differences between corresponding values of Parameter A should be less than the uncertainty for A.

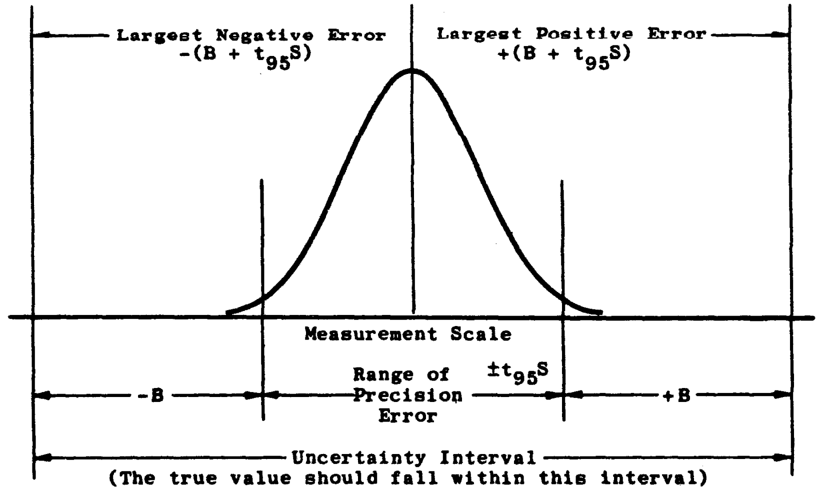

Fig. VIII-1 Measurement Uncertainty

If the difference to be detected in an experiment is of the same size or smaller than the projected uncertainty, corrective action should be taken to reduce the uncertainty. Therefore, measurement uncertainty analysis should always be done before the test or experiment. The corrective action to reduce the uncertainty may involve (1) improvements or additions to the instrumentation, (2) selection of a different function to obtain the parameter of interest, and/or (3) repeated testing. Cost and time will dictate the choice. If corrective action cannot be taken, the test should be cancelled as there is a high risk that the real differences will be lost in the uncertainty interval (undetected). If the measurement uncertainty analysis is made after the test, the opportunity for corrective action is lost, and the test may be wasted.

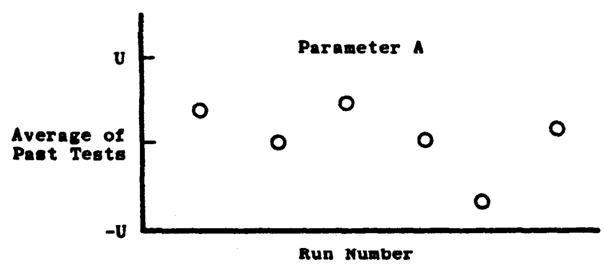

Fig. VIII-2 Run-to-Run Differences

8.7 DYNAMIC MEASUREMENT UNCERTAINTY

The same basic measurement uncertainty model may be applied to time-varying data. However, there are added complex problems to solve, and the services of a statistician are recommended. Some of these problems are

1. Time Lags
Different instrumentation will have different time lags which cause serious problems in determining time-variant parameters. For example, thrust specific fuel consumption (TSFC) is based on fuel flow and on thrust determinations. During a transient, if the measured value for one lags behind the other, the ratio, TSFC, will be in error.

2. Uncertainty Varies with Parameter Level
The uncertainty of the measurements will probably change as the level of the parameter changes. This will be hard to predict because some instruments have errors which are constant, independent of level (error is a constant percentage of full scale, for example); other instrument precision errors and biases vary as the level of the measured parameter varies (error as a percentage of point). Therefore, the uncertainty will be a combination of these two types of error and will neither remain a constant percentage nor change as a constant percentage of point.

3. Shifting Flow Profiles
Flow profiles are usually considered fixed at steady-state points. During transients, the profile will often shift or change. The uncertainty is increased by this added variation.

4. Autocorrelation between Measurements
Time-variant measurements on gas turbines will usually be highly related in time (autocorrelated). The degree of autocorrelation will have a significant effect on the uncertainty of the performance parameter.

5. Number of Probes, Location of Probes, Sampling Rate
If the parameter to be measured involves extreme values like inlet distortion and burner temperature pattern factor, the uncertainty will be highly dependent on the number and the location of the probes. If the parameter involves frequency, the time rate of sampling will be significant and the uncertainty will vary as a function of both sampling rate and frequency.

6. Outliers
Outliers in time-variant data are much more difficult to detect and flag because of the variation in the level of the parameter. This could result in more outliers being included as good data because they appear to be variations in the parameter. Therefore, for time-variant data, an outlier detection technique should be used very carefully.