SECTION VII
NET THRUST AND NET THRUST SPECIFIC FUEL CONSUMPTION

7.1 GENERAL

This section details an error analysis for net thrust and net thrust specific fuel consumption at an altitude test facility. In order to calculate net thrust, engine gross (jet) thrust must first be determined. Two independent techniques for the determination of gross thrust can be used: (1) external forces, or scale force method and (2) internal forces, or momentum balance method. Both techniques are utilized for most gas turbine engine performance programs when the determination of net thrust and thrust specific fuel consumption is a primary requirement. Performance determined by each method can be compared for agreement in order to improve the confidence in thrust data. Error analysis is presented only for the external forces or scale force method. The relationship between gross thrust and net thrust will be shown in Section 7.3.

The measurements associated with the determination of net thrust and net thrust specific fuel consumption include pressure, temperature, force, fuel flow, and airflow measurements. Error analysis of measurement systems have been presented in prior sections as follows: temperature and pressure measurements (Section V), force measurement (Section III), fuel flow measurement (Section IV), and airflow measurement (Section VI).

7.2 GROSS THRUST MEASUREMENT TECHNIQUES

7.2.1 Scale Force Method

The engine assembly and engine support mount are installed on a thrust stand which is flexure mounted on a model support cart. The engine inlet duct system contains a zero-leakage, labyrinth-type air seal. Resultant axial forces are measured by a strain-gage load cell. This installation permits the defining of a control volume (Fig. VII-1) which allows the calculation of gross (jet) thrust (F_G) from easily measurable parameters.[8]

The freebody diagram associated with the scale force method of thrust determination is shown in Fig. VII-1.

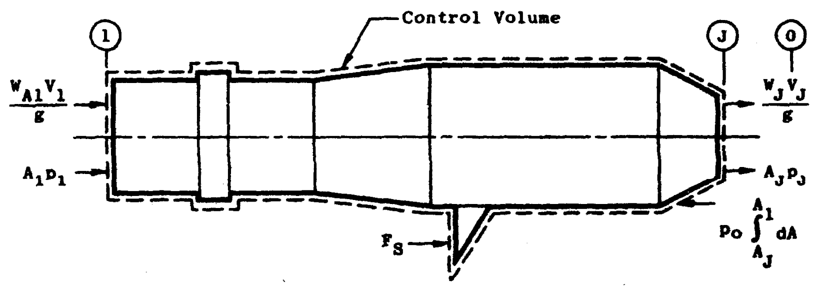
The derivation of gross thrust (F_G) from Fig. VII-1 is

$$\Sigma F_X = 0 = \frac{W_{A1} V_1}{g} + A_1 p_1 + F_S - p_o \int_{A_J}^{A_1} dA - \frac{W_J V_J}{g} - A_J p_J$$

Rearranging and combining terms give

$$\frac{W_J V_J}{g} + A_J(p_J - p_o) = \frac{W_{A1} V_1}{g} + A_1(p_1 - p_o) + F_S$$

[8]For a definition of terms used in this Handbook, see Glossary in Section IX.



W_{A1} - engine inlet airflow rate, lbm/sec

V_1 - engine inlet airflow velocity, ft/sec

g - dimensional constant, 32.174 lbm-ft/lbf-sec^2

A_1 - engine inlet duct cross-sectional area (OD), in.^{2}

p_1 - engine inlet duct static pressure, lbf/in.^{2}

F_S - force measuring transducer output (scale force), lbf

W_J - engine exhaust nozzle exit gas flow rate, lbm/sec

V_J - engine exhaust nozzle exit gas flow velocity, ft/sec

A_J - engine exhaust nozzle exit area, in.^{2}

p_J - engine exhaust nozzle exit static pressure, lbf/in.^{2}

p_o - free-stream (ambient) static pressure, lbf/in.^{2}

Fig. VII-1 Freebody Diagram for External Forces (Scale Force) Method of Determining Engine Gross (Jet) Thrust

Engine gross (jet) thrust in pounds force is, by definition,

$$F_G = \frac{W_J V_J}{g} + A_J(p_J - p_o)$$

therefore,

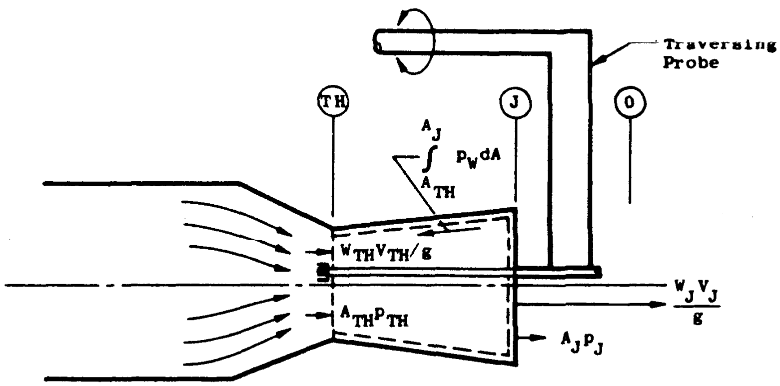
$$F_G = \frac{W_{A1} V_1}{g} + A_1(p_1 - p_o) + F_S \qquad \text{(VII-1)}$$

7.2.2 Momentum Balance Method

The momentum balance method of thrust determination utilizes the nozzle throat total pressure and temperature profiles obtained with a traversing probe and a mathematical flow field integration to determine the nozzle stream thrust.

Theoretical calculations utilized in the momentum balance method assume the fluid to be inviscid, thermally perfect, and non-heat-conducting. The method of calculation consists of direct numerical integration of the equations for continuity, momentum, and energy to define the gas state at the nozzle throat. The gas properties at the nozzle throat are then used to determine the exit momentum and pressure-area forces required to obtain the nozzle stream thrust.

The freebody diagram associated with the momentum balance method of thrust determination is shown in Fig. VII-2.



W_{TH} - gas flow rate at nozzle throat, lbm/sec

V_{TH} - gas flow velocity at nozzle throat, ft/sec

g - dimensional constant, 32.174 lbm-ft/lbf-sec^2

A_{TH} - nozzle throat area, in.^{2}

p_{TH} - nozzle throat static pressure, lbf/in.^{2}

W_J - gas flow rate at nozzle exit, lbm/sec

V_J - gas flow velocity at nozzle exit, ft/sec

A_J - nozzle exit area, in.^{2}

p_J - nozzle exit static pressure, lbf/in.^{2}

p_W - nozzle wall static pressure, lbf/in.^{2}

p_o - free-stream (ambient) static pressure, lbf/in.^{2}

Fig. VII-2 Freebody Diagram for Internal Forces (Momentum Balance) Method of Determining Engine Gross (Jet) Thrust

The derivation of gross thrust from Fig. VII-2 is

$$\Sigma F_X = 0 = \int_o^{A_{TH}} \frac{W_{TH}V_{TH}/g}{A}\, dA + \int_o^{A_{TH}} p_{TH}\, dA + \int_{A_{TH}}^{A_J} p_W\, dA - \int_o^{A_J} \frac{W_J V_J/g}{A}\, dA - \int_o^{A_J} p_J\, dA$$

Engine gross (jet) thrust in pounds force is, by definition,

$$F_G = \int_o^{A_J} \frac{W_J V_J/g}{A} dA + \int_o^{A_J} p_J dA - A_J p_o$$

therefore,

$$F_G = \int_o^{A_{TH}} \frac{W_{TH} V_{TH}/g}{A} dA + \int_o^{A_{TH}} p_{TH} dA + \int_o^{A_J} p_W dA - A_J p_o \qquad \text{(VII-2)}$$

7.3 PROPAGATION OF ERRORS TO NET THRUST

When F_G has been obtained by either the scale force or the momentum balance method, or both, identical equations are used to obtain net thrust. The external force (scale force) method for measuring net thrust is used as the example, and the derivation of this method is shown in Section 7.2.1. The Taylor's series method (Appendix B) of propagating error to net thrust is used.

The relationship of net thrust (F_N) to gross thrust (F_G) is

$$F_N = F_G - F_R \qquad \text{(VII-3)}$$

where

$$F_R = \frac{W_{A1} V_o}{g} \qquad \text{(VII-4)}$$

and V_o is the aircraft free-stream velocity in ft/sec, W_{A1} is the engine inlet airflow rate in lbm/sec, and g is a dimensional constant. Combining Eqs. (VII-1), (VII-3), and (VII-4) then results in the following equation for engine net thrust:

$$F_N = \frac{W_{A1}}{g}(V_1 - V_o) + A_1(p_1 - p_o) + F_S \qquad \text{(VII-5)}$$

where

$$V_1 = \sqrt{\frac{2KgRT_1}{K-1}\left[1 - \left(\frac{p_1}{P_1}\right)^{\frac{K-1}{K}}\right]} \qquad \text{(VII-6)}$$

$$V_o = \sqrt{\frac{2KgRT_1}{K-1}\left[1 - \left(\frac{p_o}{P_1}\right)^{\frac{K-1}{K}}\right]} \qquad \text{(VII-7)}$$

P_1 = engine inlet duct total pressure, lbf/in.^{2}

K = ratio of specific heats at T_1

R = gas constant for air at T_1, ft-lbf/lbm-°R

All other parameters are as designated in Fig. VII-1.

Net thrust in terms of pressure, temperature, area, airflow, a gas constant, a ratio of specific heats, a dimensional constant, and force measurement now becomes

$$F_N = \frac{W_{A1}}{g}\left(\frac{2KgRT_1}{K-1}\right)^{\frac{1}{2}}\left[\sqrt{1-\left(\frac{p_1}{P_1}\right)^{\frac{K-1}{K}}} - \sqrt{1-\left(\frac{p_o}{P_1}\right)^{\frac{K-1}{K}}}\right] + A_1(p_1 - p_o) + F_S \tag{VII-8}$$

The propagation formulas for the bias limit and precision index are derived from Eq. (VII-8) for F_N.

The bias limit propagation formula is the weighted root-sum-square of the bias limits for W_{A1}, g, R, K, T_1, P_1, p_1, p_o, A_1, and F_S:

$$B_{F_N}^2 = \pm\left\{\left(\frac{\partial F_N}{\partial W_{A1}} B_{W_{A1}}\right)^2 + \left(\frac{\partial F_N}{\partial g} B_g\right)^2 + \left(\frac{\partial F_N}{\partial K} B_K\right)^2 + \left(\frac{\partial F_N}{\partial T_1} B_{T_1}\right)^2 + \left(\frac{\partial F_N}{\partial R} B_R\right)^2 + \left(\frac{\partial F_N}{\partial P_1} B_{P_1}\right)^2 + \left(\frac{\partial F_N}{\partial p_1} B_{p_1}\right)^2 + \left(\frac{\partial F_N}{\partial p_o} B_{p_o}\right)^2 + \left(\frac{\partial F_N}{\partial A_1} B_{A_1}\right)^2 + \left(\frac{\partial F_N}{\partial F_S} B_{F_S}\right)^2\right\} \tag{VII-9}$$

In the same way, the precision index propagation formula is the weighted root-sum-square of the precision indices of W_{A1}, g, K, R, T_1, P_1, p_1, p_o, A_1, and F_S:

$$S_{F_N}^2 = \pm\left\{\left(\frac{\partial F_N}{\partial W_{A1}} S_{W_{A1}}\right)^2 + \left(\frac{\partial F_N}{\partial g} S_g\right)^2 + \left(\frac{\partial F_N}{\partial K} S_K\right)^2 + \left(\frac{\partial F_N}{\partial T_1} S_{T_1}\right)^2 + \left(\frac{\partial F_N}{\partial R} S_R\right)^2 + \left(\frac{\partial F_N}{\partial P_1} S_{P_1}\right)^2 + \left(\frac{\partial F_N}{\partial p_1} S_{p_1}\right)^2 + \left(\frac{\partial F_N}{\partial p_o} S_{p_o}\right)^2 + \left(\frac{\partial F_N}{\partial A_1} S_{A_1}\right)^2 + \left(\frac{\partial F_N}{\partial F_S} S_{F_S}\right)^2\right\} \tag{VII-10}$$

Errors associated with values of g, K, and R are generally assumed negligible.

The uncertainty for net thrust (F_N) is calculated using the uncertainty formula:

$$U = \pm(B + t_{95}S) \tag{VII-11}$$

The following list of partial derivatives is given as an aid to the analyst:

$$\frac{\partial F_N}{\partial W_{A1}} = \frac{1}{g}\left[\frac{2KgRT_1}{K-1}\right]^{\frac{1}{2}}\left[B_1^{\frac{1}{2}} - B_2^{\frac{1}{2}}\right]$$

$$\frac{\partial F_N}{\partial T_1} = \frac{W_{A1}}{g}\left[\frac{KgR}{(K-1)T_1}\right]^{\frac{1}{2}}\left[B_1^{\frac{1}{2}} - B_2^{\frac{1}{2}}\right]$$

$$\frac{\partial F_N}{\partial A_1} = p_1 - p_0$$

$$\frac{\partial F_N}{\partial F_S} = 1.0$$

$$\frac{\partial F_N}{\partial p_0} = W_{A1}\left(\frac{1}{p_0}\right)^{\frac{1}{K}}\left(\frac{1}{P_1}\right)^{\frac{K-1}{K}}\left(\frac{(K-1)RT_1}{2gKB_2}\right)^{\frac{1}{2}} - A_1$$

$$\frac{\partial F_N}{\partial p_1} = W_{A1}\left(\frac{1}{p_1}\right)^{\frac{1}{K}}\left(\frac{1}{P_1}\right)^{\frac{K-1}{K}}\left(\frac{(K-1)RT_1}{2gKB_1}\right)^{\frac{1}{2}} + A_1$$

$$\frac{\partial F_N}{\partial P_1} = W_{A1}\left(\frac{(K-1)RT_1}{2gK}\right)^{\frac{1}{2}}\left(P_1\right)^{\frac{1-2K}{K}}\left[\left((p_1)^{\frac{K-1}{K}} B_1^{-\frac{1}{2}}\right) - \left((p_0)^{\frac{K-1}{K}} B_2^{-\frac{1}{2}}\right)\right]$$

where

$$B_1 = 1 - \left(\frac{p_1}{P_1}\right)^{\frac{K-1}{K}} \quad \text{and} \quad B_2 = 1 - \left(\frac{p_0}{P_1}\right)^{\frac{K-1}{K}}$$

By using Eqs. (VII-9) and (VII-10), the partial derivative equations above, and the example values listed in Table XVIII, the propagation of error to net thrust can be determined.

In this example, values for the partial derivative terms above were approximated through the basic net thrust equation. First, the thrust level is determined from the measured values using performance Eq. (VII-8). Then each measured value in Eq. (VII-8) is changed, independently, by the amount of its precision index, and the resulting change in F_N is obtained. This process, repeated for each measured value, provides good approximate numerical values for each term in Eq. (VII-10). For example to obtain the approximate value for the term $[(\partial F_N/\partial W_{A1})S_{W_{A1}}]$ of Eq. (VII-10), determine F_N (Eq. (VII-8)) with the measured values. Then, determine the change in $F_N(\Delta F_N)$ by changing airflow (W_{A1}) by the amount of its precision index. The change in F_N resulting from the change in W_{A1} is approximately equal to the term $[(\partial F_N/\partial W_{A1})S_{W_{A1}}]$. The identical process is repeated for bias limits to obtain numerical values for the terms in Eq. (VII-9).

Table XVIII
Typical Measurement and Uncertainty Values Used in Net Thrust for Supersonic Afterburning Turbofan Engine

Flight Condition: 30,000-ft Altitude, Mach Number 0.9, Military Power

Component	Nominal Value	Bias Limit	Precision Index	Uncertainty, U	Degrees of Freedom**
F_S, Scale Force, lbf	4,388	7.90	3.95	15.80	105
g, Dimensional Constant, $\frac{lbm - ft}{lbf - sec^2}$	32.174	*	*	*	*
A_1, Inlet Duct Area (OD), in.^{2}	984	0.050	0.050	0.160	12
p_1, Inlet Duct Static Pressure, psia	6.50	0.0065	0.0065	0.0202	15
p_o, Free-Stream Static Pressure, psia	4.31	0.0099	0.0099	0.0310	15
R, Gas Constant for Air at T_1, $\frac{ft - lbf}{lbm - °R}$	53.329	*	*	*	*
T_1, Inlet Duct Total Temperature, °R	477	±2.48	±0.19	±2.86	100
K, Ratio of Specific Heats at T_1	1.4034	*	*	*	*
P_1, Inlet Duct Total Pressure, psia	7.43	±0.0074	±0.0111	±0.0312	15
W_f, Total Fuel Flow, lbm/hr	4662	±6.06	±5.13	±16.32	35

*Bias, precision, and uncertainty considered as negligible in thrust uncertainty.
**Degrees of freedom determined by the Welch-Satterthwaite formula (Eq. I-11).

By using Eqs. (VII-6), (VII-7), and (VII-8) and the measurement values and their uncertainty components listed in Table XVIII, the propagation of error to V_o, V_1, F_R, F_N, and TSFC are shown in Table XIX. Wa_1 of Table XIX was obtained from Section VI.

Table XIX
Derived Measurement Uncertainty Values

Parameter	Nominal	Bias Limit	Precision Index	Uncertainty	Degrees of Freedom
V_o, Free-Stream Velocity, ft/sec	908.4	±3.049	±2.119	±7.414	26
V_1, Inlet Duct Velocity, ft/sec	463.4	±2.684	±3.057	±8.981	26
Wa_1, Inlet Duct Airflow, lbm/sec	115.5	±0.531	±0.139	±0.825	16
F_R, Ram Drag, lbf	3261.	±18.563	±8.559	±35.681	38
F_N, Net Thrust, lbf	4945	±12.498	±7.471	±27.440	59
TSFC, Net Thrust Specific Fuel Consumption, lbm/lbf-hr	0.943	±0.0027	±0.0018	±0.0063	94

7.4 PROPAGATION OF ERROR TO NET THRUST SPECIFIC FUEL CONSUMPTION

$$\text{TSFC} = \frac{W_F}{F_N}, \frac{\text{lbm/hr}}{\text{lbf}} \qquad \text{(VII-12)}$$

where W_F is total engine fuel flow (lbm/hr), and F_N is net thrust (lbf).

The error analysis for fuel flow is shown in Section IV, and the error analysis for net thrust is shown in Section 7.3.

The bias limit propagation formula is the root-sum-square of the bias limits for W_F and F_N weighted by the partial derivatives (Appendix B):

$$B_{TSFC}^2 = \pm\left[\left(\frac{\partial TSFC}{\partial W_F} B_{W_F}\right)^2 + \left(\frac{\partial TSFC}{\partial F_N} B_{F_N}\right)^2\right] \qquad \text{(VII-13)}$$

In the same way, the precision index is calculated as the root-sum-square of the precision indices for W_F and F_N:

$$S_{TSFC}^2 = \pm\left[\left(\frac{\partial TSFC}{\partial W_F} S_{W_F}\right)^2 + \left(\frac{\partial TSFC}{\partial F_N} S_{F_N}\right)^2\right] \qquad \text{(VII-14)}$$

The partial derivatives for the terms in Eqs. (VII-13) and (VII-14) are

$$\frac{\partial TSFC}{\partial W_F} = \frac{1}{F_N} \quad \text{and} \quad \frac{\partial TSFC}{\partial F_N} = -\frac{W_F}{F_N^2}$$

Equations (VII-13) and (VII-14) are now evaluated using the above partials and the measurement values and uncertaintv components from Tables XVIII and XIX for W_F and F_N, respectively.

$$B_{TSFC} = \pm\sqrt{[(0.0002022)(6.06)]^2 + [(0.0001906)(12.498)]^2}$$
$$= \pm 0.0027 \text{ lbm/hr/lbf}$$

$$S_{TSFC} = \pm\sqrt{[(0.0002022)(5.13)]^2 + [(0.0001906)(7.471)]^2}$$
$$= \pm 0.0018 \text{ lbm/hr/lbf}$$

The uncertainty for TSFC is calculated using the uncertainty formula:

$$U = \pm(B + t_{95}S)$$

Here t_{95} is = 2.0 since the degrees of freedom are greater than 30.

$$U = \pm[0.0027 + 2(0.0018)]$$
$$= \pm 0.0063 \text{ lbm/hr/lbf}$$