SECTION IV
FUEL FLOW MEASUREMENT

4.1 GENERAL

Fuel flow measurements are difficult because there is no standard for measuring volume per unit of time; therefore, the flow calibration[5] must be referenced to standards for weight or volume. Furthermore, there is no universal method for calibrating a flowmeter.

Turbine meters are the most widely used instruments for measuring fuel flows. They generate an alternating voltage with frequency proportional to the volumetric flow rate. The frequency of the output is converted to an analog voltage and then to digital counts. Another method for recording turbine meter signals is to count the voltage excursions (pulses) over some preset period of time to determine the signal frequency. Figure IV-1 illustrates a typical turbine meter signal.

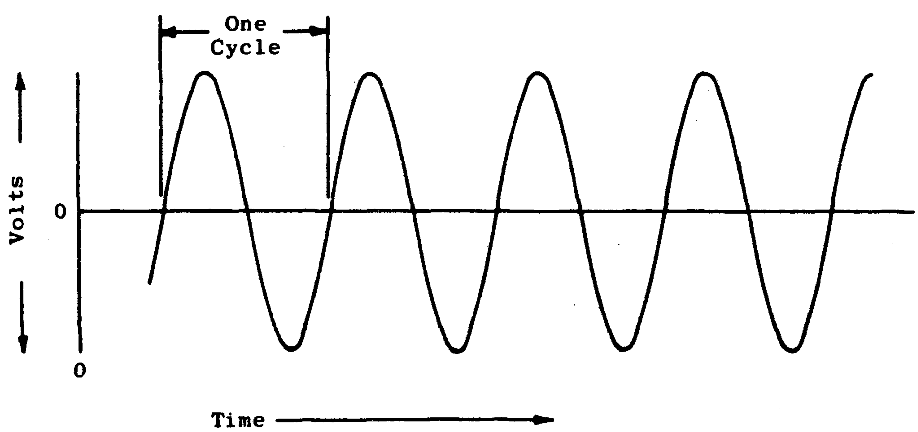

Fig. IV-1 Turbine Meter Signal

Turbine meters may be calibrated by three methods:

1. Volumetric: flowing a measured volume of fluid through the meter to establish a pulses-per-gallon factor,
2. Gravimetric: flowing a measured mass of fluid through the meter, determining the density, and converting the mass to volume to establish a pulses-per-gallon factor, and
3. Comparative calibration: comparing the meter against a master meter.

[5]For a definition of terms used in this Handbook, see the Glossary in Section IX.

Calibration factors are determined over a range of turbine meter frequencies to develop a calibration curve. Figure IV-2 is an illustration of a typical turbine meter calibration curve.

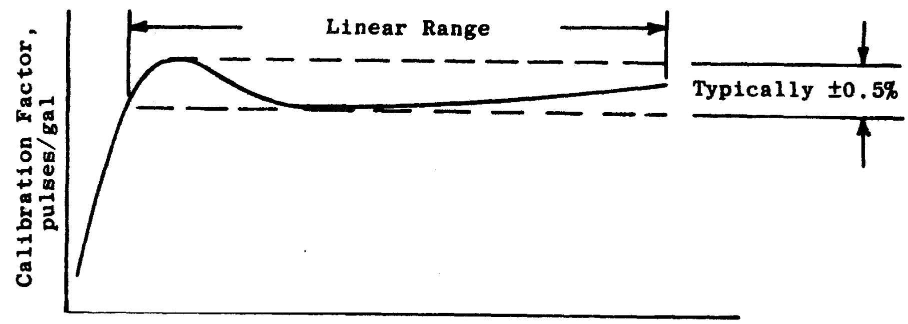

Fig. IV-2 Turbine Meter Calibration Curve

The range of calibration factors, over the linear range of a turbine meter, is typically ±0.5 percent. However, meter range may vary somewhat and is influenced by meter size and design, plus fluid viscosity. A complete analysis of turbine meter performance is given by the "Turbine Flowmeter Performance Model." (AD825354) prepared by the Greyrad Corporation.

Multiple measurement of fuel flow is recommended for several reasons, the chief ones being reliability and accuracy. Multiple measurements are readily accomplished by simply installing two or more turbine meters in series. The meters should have independent calibrations as far back in the calibration hierarchy as possible.

4.2 FUEL FLOW MEASUREMENT ERROR SOURCES

Errors in fuel flow measurement fall into three major categories: (1) calibration, (2) data acquisition, and (3) data reduction. The calibration elemental bias and precision error sources will vary according to the method of calibration used, while the data acquisition and reduction errors will not. The next section will contain three parts, one for each method of calibration. All three may not apply to a particular test facility. The ones that do not should be ignored. The remainder of the section will be devoted to discussions of the elemental errors in the data acquisition and data reduction processes.

4.2.1 Calibration Errors

Turbine meters are calibrated by three methods or a combination thereof. The first is volumetric calibration. It is accomplished by flowing a measured volume of fluid through the meter and recording the total number of turbine meter cycles (pulses) generated. The calibration factor (K factor) is then calculated

$$K = \frac{\text{total pulses}}{\text{total gallons}} = \frac{\text{pulses}}{\text{gallon}} \qquad \text{(IV-1)}$$

The second method, gravimetric calibration, is accomplished by flowing a measured mass of fluid through the meter and again recording the total number of turbine meter pulses. Measurement of fluid temperature and pressure will allow determination of fluid density. With these data, a meter K factor may be calculated:

$$K = \frac{\text{total pulses}}{\text{total pounds}} \times \frac{\text{pounds}}{\text{gallon}}$$

$$= \frac{\text{pulses}}{\text{gallon}} \qquad \text{(IV-2)}$$

where density = pounds per gallon.

The third method is comparative calibration. The meter being calibrated is compared against a working standard turbine meter. In some applications turbine meters are so repeatable that the greater part of any precision error incurred results from nonrepeatability of the calibrating device. In this case, better calibration results can be obtained by simply flowing the calibration fluid through a standard turbine meter in series with the meter being calibrated. Data recorded are frequency of each meter and the ratio (R) of total pulses from the meter being calibrated to total pulses from the standard meter. The calibration factor (K_{Cal}) for the meter being calibrated is

$$K_{Cal} \frac{\text{pulses}}{\text{gallon}} = (R)K_{standard} \qquad \text{(IV-3)}$$

where $K_{standard}$ = the $\frac{\text{pulses}}{\text{gallon}}$ factor for the standard meter at the set frequency.

4.2.1.1 Volumetric Calibration

At the apex of the calibration hierarchy, Fig. IV-3, for volumetric calibrations is the National Bureau of Standards Dynamic Weigh Calibrator. The Interlab Standard Turbine Meter is calibrated against this calibrator and in turn is used to calibrate the working standard volumetric calibrator in the company laboratory.

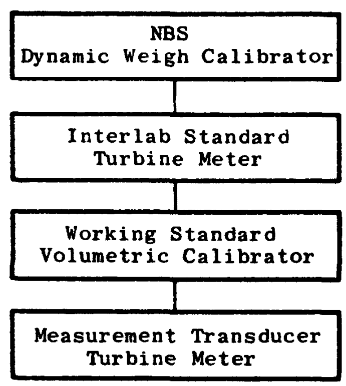

Fig. IV-3 Turbine Meter Volumetric Calibration Hierarachy

4.2.1.1.1 Calibration of the Interlab Standard

Beginning with the interlab standard, calibration methods and elemental error evaluation techniques at each level in the hierarchy will be discussed.

Turbine meters are calibrated at the NBS as follows:

1. At each pulse frequency, specified by the owner, and controlled to within one-half percent, the total number of pulses generated are counted during the period required to flow a weighed quantity of liquid through the meter.

2. Liquid density is determined using measured values of liquid temperature and pressure.

3. The weighed quantity of fluid is converted to gallons using the density determined in step (2).

4. A pulses/gallon (K) factor is calculated from the data obtained in steps (1), (2), and (3).

5. Steps (1) through (4) are repeated five times successively on each of two different days making a total of ten (10) separate observations.

The K factor reported is the arithmetic mean $\bar{K}$ of the ten observations. Data from many similar calibrations yield a standard deviation (s) of about 0.03 percent for the NBS calibration procedure. The precision index ($s_{\bar{K}}$) for the mean value, is then

$$s_{\bar{K}} = \frac{s}{\sqrt{N_i}} = \frac{0.03\%}{\sqrt{10}} = 0.01\% \qquad \text{(IV-4)}$$

where N_i = the number of observations in the determination of $\bar{K}$. N_i usually is 10; the degrees of freedom is $N_i - 1$ or usually 9. Of course, this is the precision index at just one pulse frequency. The precision indices $s_{\bar{K}_i}$ for M frequencies must be pooled to produce the precision index (s_{NBS}) for the calibration process:

$$s_{NBS} = \pm\sqrt{\frac{\sum_{i=1}^{M}(N_i - 1)\, s_{\bar{K}_i}^{2}}{\sum_{i=1}^{M}(N_i - 1)}} \qquad \text{(IV-5)}$$

with degrees of freedom $df_{NBS} = \sum_{i=1}^{M}(N_i - 1)$. If the number of tests at each frequency is 10, then these calculations reduce to

$$s_{NBS} = \pm\sqrt{\frac{\sum_{i=1}^{M} s_{\bar{K}_i}^{2}}{M}} \qquad \text{(IV-6)}$$

and

$$df_{NBS} = M(n-1) = 9M \qquad \text{(IV-7)}$$

Bias limits in the NBS dynamic weight calibrator have been established by repeated comparison calibrations of reference turbine meters on both volumetric (stand pipe) calibrators and gravimetric (weigh) calibrators. These comparison calibrations were performed at several different locations and yielded a bias limit of ±0.1 percent. This then is the bias limit (b_{NBS}) reported by the NBS for turbine meter calibration.

Uncertainty in NBS turbine meter calibrations, is then

$$U = \pm(b_{NBS} + t_{95}s_{NBS}) \qquad \text{(IV-8)}$$

If, for example, the number of frequency settings is equal to ten, the degrees of freedom are

$$df_{NBS} = 9 \times 10 = 90$$

Since the degree of freedom is greater than thirty,

$$t_{95} = 2.00$$

and

$$U = \pm(0.1 + 2.00 \times 0.01) = 0.12\%$$

4.2.1.1.2 Uncertainty in the Working Standard

The working standard volumetric calibrator generally consists of a standpipe arrangement with liquid level sensors. These mark the top and bottom of a constant volume interval (Fig. IV-4). The interlab standard flowmeter is connected in series with the calibrator.

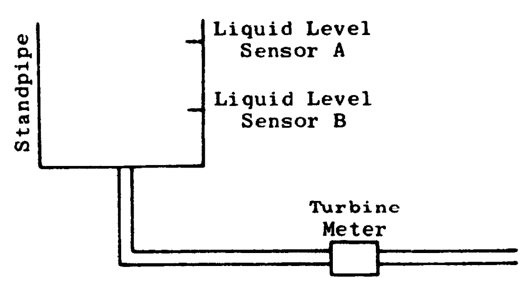

Fig. IV-4 Volumetric Calibrator

To calibrate, liquid is forced out of or into the calibrator at a constant flow rate. The number of pulses generated by the flowmeter are counted while the liquid between sensors A and B is being displaced. The volume (V_i) between A and B is then the quotient of the number of pulses counted (C_i) divided by the turbine meter K factor (K_{NBS}) determined at NBS:

$$V_i = \frac{C_i \text{ pulses}}{K_{NBS} \text{ pulses/gallon}} = \text{gallons} \qquad \text{(IV-9)}$$

Repeating the calibration process N times improves the estimate of the volume, and the average of the N calibrations is

$$\bar{V} = \frac{\sum_{i=1}^{N} V_i}{N} \qquad \text{(IV-10)}$$

The precision index estimate for this determination of volume is the precision index for the N determinations divided by the square root of N.

$$s_{\overline{V}} = \frac{s_V}{\sqrt{N}} = \sqrt{\frac{\sum_{i=1}^{N}(V_i - \overline{V})^2}{N\,(N-1)}} \qquad \text{(IV-11)}$$

This is called the standard error of the mean for the N determinations. The associated degrees of freedom is N - 1. Please note in this situation, the precision index of an average value ($\overline{V}$) is of interest rather than the precision index of the individual determinations (V_i). The estimating formula is corrected to provide that estimate by dividing by the square root of N, the number of determinations.

The bias limit ($b_{\overline{V}}$) for this process is the root-sum-square of the bias limit reported by the NBS (b_{NBS}) and the best estimate of any additional bias contributed by the calibration process.

Another way to evaluate the precision error of the calibrator is to determine the standpipe volume by some other means and then compare K factors determined by the calibrator with those produced by the NBS calibrator. The standpipe volume between A and B may be determined by physical measurement of the standpipe dimensions or by measuring the volume of liquid between A and B with fixed volume standards, e.g., 5-, 10-, 50-, 100-gal containers. Then calibrations of the interlab standard turbine meter can be performed with the calibrator by forcing liquid into or out of the standpipe through the turbine meter at a constant flow rate. The total number of pulses generated while flowing the volume of liquid between A and B divided by the measured volume yields the meter K factor:

$$K = \frac{\text{total pulses}}{\text{total gallons}} = \frac{\text{pulses}}{\text{gallon}} \qquad \text{(IV-12)}$$

By repeating this procedure several times and calculating a mean calibration factor ($\overline{K}$) for a constant flow rate as the average of the observed K factors,

$$\overline{K} = \frac{\sum_{i=1}^{N} K_i}{N} \qquad \text{(IV-13)}$$

where N = the number of observations used to determine $\overline{K}$

K_i = K calculated from the i th observation

and the precision index of $\overline{K}$ is estimated from the variation of K_i about $\overline{K}$; it is the precision index for K divided by the square root of the number of observations (N):

$$s_{\overline{K}} = \frac{s_K}{\sqrt{N}} = \sqrt{\frac{\sum_{i=1}^{N}(K_i - \overline{K})^2}{N\,(N-1)}} \qquad \text{(IV-14)}$$

The degrees of freedom associated with $s_{\bar{K}}$ is N - 1.

If an average K factor ($\bar{K}$) is determined at several different flow rates over the range of the meter and corresponding precision index, $S_{\bar{K}}$, is calculated at each flow rate; then pooling the precision indices provides an estimate of overall calibrator precision index:

$$s_{ws} = \pm \sqrt{\frac{\sum_{j=1}^{M} (N_j - 1) s_{K_j}^2}{\sum_{J=1}^{M} (N_j - 1)}} \qquad \text{(IV-15)}$$

The degrees of freedom associated with the pooled precision index (the measurement instrument calibration process) are the sum of the degrees of freedom for each flow rate:

$$df_{ws} = \sum_{j=1}^{M} (N_j - 1) \qquad \text{(IV-16)}$$

where M = the number of pulse frequencies for which a K is determined and N_j = the number of observations made at each frequency setting. The bias in this process can be estimated as follows:

1. Calculate an average calibration factor $\bar{\bar{K}}$ from K_j values calculated from the NBS calibration data
2. Calculate an average calibration factor $\bar{\bar{\bar{K}}}$ from all of the $\bar{K}$'s calculated from volumetric calibrator data
3. Make the correction $\bar{\bar{\bar{K}}}$ - $\bar{\bar{K}}$
4. Estimates of the limits of unknown bias (b_j) for the process should be based on data from interlaboratory or interfacility comparisons and engineering judgment. Then

$$b_{ws} = \pm \sqrt{(b_{NBS})^2 + (b_j)^2} \qquad \text{(IV-17)}$$

Finally, perform the calibration of the measurement turbine meter against the volumetric calibrator in essentially the same manner that the interlab standard turbine meter was calibrated. The precision index (s_j) for this calibration is calculated using Eqs. (IV-15) and (IV-16). The bias limit (b) for the measurement meter calibration process is simply the best estimate based on interlab or interfacility comparison history and engineering experience.

The precision index for the calibration hierarchy is

$$S_1 = \pm \sqrt{\sum_i s_i^2} \qquad \text{(IV-18)}$$

$$S_1 = \pm\sqrt{s_{NBS}^2 + s_{\bar{V}}^2 + s_j^2} \quad \text{(IV-19)}$$

or

$$S_1 = \pm\sqrt{s_{NBS}^2 + s_{ws}^2 + s_j^2} \quad \text{(IV-20)}$$

Degrees of freedom (df) for the calibration hierarchy are calculated by

$$df_1 = \frac{\left(s_{NBS}^2 + s_{\bar{V}}^2 + s_j^2\right)^2}{\frac{s_{NBS}^4}{df_{NBS}} + \frac{s_{\bar{V}}^4}{df_{\bar{V}}} + \frac{s_j^4}{df_j}} \quad \text{(IV-21)}$$

or

$$df_1 = \frac{\left(s_{NBS}^2 + s_{ws}^2 + s_j^2\right)^2}{\frac{S_{NBS}^4}{df_{NBS}} + \frac{S_{ws}^4}{df_{ws}} + \frac{S_j^4}{df_j}} \quad \text{(IV-22)}$$

Bias limits for the hierarchy are

$$B_1 = \sqrt{\sum_i b_i^2} \quad \text{(IV-23)}$$

$$B_1 = \sqrt{b_{NBS}^2 + b_{\bar{V}}^2 + b_j^2} \quad \text{(IV-24)}$$

or

$$B_1 = \sqrt{b_{NBS}^2 + b_{ws}^2 + b_j^2} \quad \text{(IV-25)}$$

Uncertainty in the calibration hierarchy is calculated by

$$U_1 = \pm(B_1 + t_{95}S_1) \quad \text{(IV-26)}$$

t_{95} is evaluated at df_1 degrees of freedom.

4.2.1.2 Gravimetric Calibrations

The gravimetric flow calibration system has a calibration hierarchy (Fig. IV-5) similar to that for volumetric calibrations. The only difference is that the working standard is a dynamic weigh calibrator rather than a volumetric calibrator.

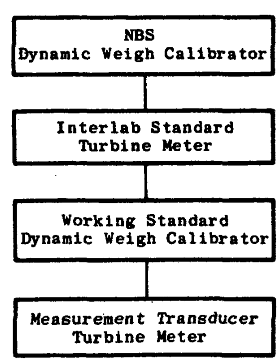

Fig. IV-5 Turbine Meter Gravimetric Calibration Hierarchy

Elemental errors in the gravimetric calibration hierarchy are evaluated in exactly the same way as those for the volumetric hierarchy except for the working standard.

Turbine meter gravimetric calibrations are accomplished by flowing a weighed quantity of liquid through the meter. Liquid temperature and pressure are measured to determine liquid density for conversion of the weighed quantity to gallons. The K factor is calculated by dividing the total number of pulses recorded by the weighed quantity in pounds and multiplying by the density in pounds per gallon:

$$K = \frac{\text{total pulses}}{\text{total pounds}} \times \frac{\text{pounds}}{\text{gallon}} = \frac{\text{pulses}}{\text{gallon}} \tag{IV-27}$$

Figure IV-6 illustrates the basic idea of gravimetric calibration.

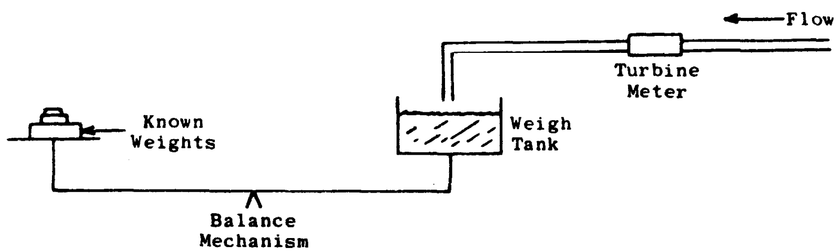

Fig. IV-6 Gravimetric Calibrator

By flowing liquid at a constant flow rate through the turbine meter and into the weigh tank until the weight of the liquid exactly balances the standard weights, the weighed quantity (W) is established in pounds. With the total number of pulses generated (C) and liquid density (ρ) in pounds per gallon, K_i is calculated as follows:

$$K_i = \frac{\text{pulses}}{\text{gallon}} = \frac{C}{W}\rho \tag{IV-28}$$

If N_j observations of K_i are made at one flow rate, average ($\overline{K}$) and precision index ($s_{\overline{K}_j}$) can be calculated:

$$\overline{K} = \frac{\sum_{i=1}^{N_j} K_i}{N_j} \tag{IV-29}$$

$$s_{\overline{K}_j} = \sqrt{\frac{\sum_{i=1}^{N_j} (K_i - \overline{K})^2}{N_j(N_j - 1)}} \tag{IV-30}$$

Then if $\bar{K}$ is established for M different flow rates, the precision index for the calibration process is the pooled precision index (S_{ws}) of the $s_{\bar{K}_j}$ indices:

$$S_{ws} = \pm \sqrt{\frac{\sum_{j=1}^{M} (N_j - 1) s_{\bar{K}_j}^{2}}{\sum_{j=1}^{M} (N_j - 1)}} \qquad \text{(IV-31)}$$

Degrees of freedom (df_{ws}) for this process are calculated by

$$df_{ws} = \sum_{j=1}^{M} (N_j - 1) \qquad \text{(IV-32)}$$

Bias in this calibration process is estimated by the method described in Section 4.2.1.1.2.

Calibration hierarchy precision index (S_1), degrees of freedom (df_1), bias (B_1), and uncertainty (U_1) are calculated by Eqs. (IV-18) through (IV-26).

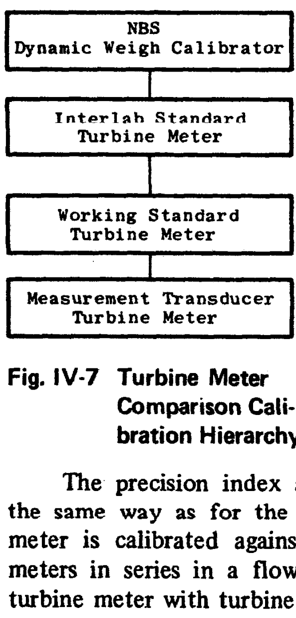

Fig. IV-7 Turbine Meter Comparison Calibration Hierarchy

4.2.1.3 Calibration by Comparison

The third method of calibration is comparison of the measurement meter with a working standard turbine meter. This method substitutes a turbine meter for the volumetric or gravimetric working standards in the calibration hierarchy (Fig. IV-7).

The comparison method has not been widely accepted. However, it does have considerable merit especially in hydrocarbon fuel applications. The NBS has recognized the use of turbine meters as transfer standards because they are very repeatable. The NBS has, in fact, used turbine meters as transfer standards in evaluating bias in the NBS dynamic weigh calibrator.

The precision index and bias limit estimates for the interlab standard are made in the same way as for the volumetric calibration system. The working standard turbine meter is calibrated against the interlab standard turbine meter by installing the two meters in series in a flow system. Figure IV-8 illustrates a typical setup for comparing turbine meter with turbine meter.

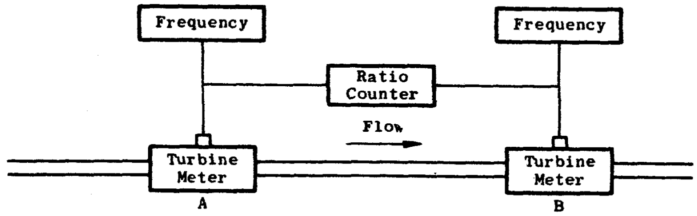

Fig. IV-8 Comparative Calibration

By considering, for example, meter A to be an interlab standard and meter B to be a working standard, a calibration can be performed as follows:

1. Adjust the flow rate by observing the frequency of meter A. From the NBS calibration curve, determine the interlab standard K factor for the particular frequency.

2. With the flow rate adjusted and constant, allow the ratio counter to count the total number of pulses generated by meter A over some predetermined time period. The counter will simultaneously count the total number of pulses generated by meter B over the same time period. The counter will then display digitally the ratio (R):

$$R = \frac{\text{total pulses} - A}{\text{total pulses} - B} \tag{IV-33}$$

then

$$K_B = \frac{K_A}{R} \tag{IV-34}$$

and

$$f_B = \frac{f_A}{R} \tag{IV-35}$$

Thus, calculate K_i for each frequency setting f_A. If N_i observations are made at each frequency setting, an average calibration factor ($\overline{K}$) can be calculated for meter B at f_B by

$$\overline{K} = \frac{\sum_{i=1}^{N_j} K_i}{N_j} \tag{IV-36}$$

The precision index for this average calibration factor is the precision index of the calibration values divided by the square root of the number of observations (N_j):

$$s_{\overline{K}_j} = \frac{s_{\overline{K}}}{\sqrt{N_j}} = \pm\sqrt{\frac{\sum_{i=1}^{N_j} (K_{B_i} - \overline{K})^2}{N_j(N_j - 1)}} \tag{IV-37}$$

If a K is determined at M different frequencies over the range of the meter, the precision index (s_{ws}) for the calibration process is the pooled value for all frequencies:

$$s_{ws} = \pm\sqrt{\frac{\sum_{j=1}^{M} (N_j - 1)\, s_{\overline{K}_j}^2}{\sum_{j=1}^{M} (N_j - 1)}} \tag{IV-38}$$

The degrees of freedom (df_{ws}) for the calibration process are

$$df_{ws} = \sum_{j}^{M} (N_j - 1) \qquad \text{(IV-39)}$$

Bias limits (b_{ws}) for the process are again based on interlab and interfacility comparison history and engineering judgment.

Precision index (S_1), degrees of freedom (df_1), bias limits (B_1), and uncertainty (U_1) for the calibration hierarchy are calculated using Eqs. (IV-18) through (IV-26).

Some important considerations which have not been mentioned heretofore are:

1. At each level in the calibration hierarchy, the flowmeter being calibrated should be accompanied by the plumbing upstream and downstream of the meter in its use condition. Tests have shown that inadequate control of the velocity profile is perhaps the strongest argument against the use of reference turbine meters as standards (see page 184 of "Turbine Flowmeter Performance Model," (AD825354) prepared by Greyrad Corporation).

2. If at all possible, turbine meters should be calibrated with the use liquid at run conditions of temperature and pressure.

These considerations will minimize the errors.

4.2.2 Data Acquisition Errors

The effect of data acquisition errors is determined by applying a known frequency (X) to the data acquisition equipment (Fig. IV-9).

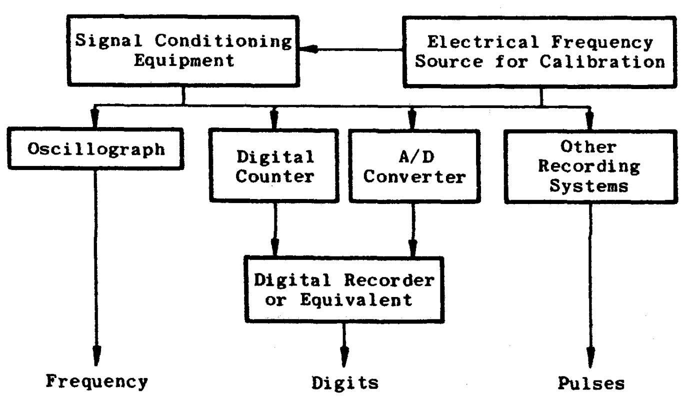

Fig. IV-9 Data Acquisition System Calibration

In the case of digital counters, the total number of input pulses is counted over some preset period of time and the digital indication is recorded. Then the recorded value can be compared with the input. If M recordings on N channels are made, then the average ($\overline{X}_j$) for one channel is

$$\overline{X}_j = \sum_{i=1}^{M} \frac{X_i}{M} \tag{IV-40}$$

where X_i = the recorded value for the ith recording. The grand average ($\overline{X}$) for all channels is

$$\overline{X} = \frac{\sum_{j=1}^{N} \overline{X}_j}{N} \tag{IV-41}$$

The precision index ($s_{\overline{X}}$) for digital counter channels is

$$s_{\overline{X}} = \pm\sqrt{\frac{\sum_{j=1}^{N} (\overline{X}_j - \overline{X})^2}{N - 1}} \tag{IV-42}$$

The degrees of freedom (df_j) are the number of recordings minus one, i.e., N-1.

Bias limits for the data acquisition process are left to the judgment of the most knowledgeable data recording engineers. Errors incurred by frequency-to-analog and analog-to-digital conversions are evaluated by comparing known input frequencies (f) with recorded frequencies. The known frequency (f) is applied to N channels, and M multiple scan recordings are made. A multiple scan average ($\overline{f}_{ij}$) is calculated for each channel for each recording. The average ($\overline{f}_j$) for each recording on the jth channel is

$$\overline{f}_j = \frac{\sum_{i=1}^{M} f_{ij}}{M} \tag{IV-43}$$

The grand average ($\overline{f}$) for all channels is

$$\overline{f} = \frac{\sum_{j=1}^{N} \overline{f}_j}{N} \tag{IV-44}$$

The precision index ($s_{\overline{f}}$) is then

$$s_{\overline{f}} = \pm\sqrt{\frac{\sum_{j=1}^{N} (\overline{f}_j - \overline{f})^2}{N - 1}} \tag{IV-45}$$

The degrees of freedom ($df_{\bar{f}}$) in this case are

$$df_{\bar{f}} = N - 1 \qquad \text{(IV-46)}$$

Bias limits (b_f) for frequency-to-analog and analog-to-digital conversion must be estimated and root-sum-squared with the bias limits (b_{f_o}) for the input frequency (f) as determined from the frequency standard calibration hierarchy, i.e.,

$$b_i = \pm\sqrt{b_f^2 + b_{f_o}^2}$$

4.2.2.1 Multiple Instruments

Because of the lack of flow rate standards, i.e., gallons per minute or pounds per second standards, multiple instrumentation for flow measurement is highly recommended. Correct application of multiple measurement statistics will never yield an estimate of precision error larger than that obtained with single measurements. The typical multiple instrumentation situation provides a reduction in precision error. The measurement provided by the average of several instruments is more precise than any individual instrument of that set. The reduction of precision error is indicated by the precision index (s_{avg}) of several instruments:

$$s_{avg} = \frac{s_{ind}}{\sqrt{K}} \qquad \text{(IV-47)}$$

where s_{ind} = the precision index of the individual instrument and K is the number of instruments. The formula for calculating s_{avg} is as follows when the simple average of multiple instruments is used:

$$s_{avg} = \sqrt{\frac{\Sigma s_i^2}{K}}$$

If individual instruments are weighted when combined, the formula for s_{avg} is more complex.

Analysis of flowmeter-to-flowmeter multiple measurements yields

1. Pooled within-run precision index (S_{wr}) and
2. Pooled run-to-run precision index (S_{rr}).

Appendix C gives derivations and formulation for calculating the above prècision indices.

Further analysis of multiple measurement data will provide an estimate of flowmeter calibration-to-calibration precision error (S_{cc}) which includes

1. Flowmeter nonrepeatability during calibration,
2. Installation effects between calibration facility and engine test stand, and

3. Calibration facility nonrepeatability, which includes temperature and pressure errors in defining density during calibration.

Derivations and formulation for evaluating S_{cc} are also in Appendix C.

4.2.3 Data Reduction Errors

Data reduction errors are those errors incurred in reducing measured units to units of flow and fall into the following three categories:

1. Density determination errors,
2. Precision errors resulting from test dynamics, and
3. Computer resolution errors.

4.2.3.1 Density Determination Errors

Errors in density determination are a result of errors in the measurement of fuel temperatures and pressures. At this point, these errors may be called s_{de} for the precision index and b_{de} for the bias limits.

The effect of fuel pressure precision errors on fuel density is calculated from

$$S_{de_1} = \pm \frac{1}{\sqrt{K}} \frac{\partial \rho}{\partial P} S_P \qquad \text{(IV-48)}$$

where S_P = Pressure measurement precision index

$\frac{1}{\sqrt{K}}$ = Factor to account for K multiple pressure measurements

$\frac{\partial \rho}{\partial P}$ = The partial derivative of the fuel density versus pressure relationship

The degrees of freedom associated with S_{de} are the same as that for the pressure measurement:

$$df_{de_1} = df_P \qquad \text{(IV-49)}$$

The effect of fuel pressure measurement bias on fuel density is

$$B_{de_1} = \frac{\partial \rho}{\partial P} B_P \qquad \text{(IV-50)}$$

The effect of fuel temperature precision error on fuel density (S_{de_2}) is calculated as follows:

$$S_{de_2} = \pm\frac{1}{\sqrt{K}}\frac{\partial\rho}{\partial T}S_T \qquad \text{(IV-51)}$$

where $\frac{1}{\sqrt{K}}$ = factor to account for K multiple temperature measurements

$\frac{\partial\rho}{\partial T}$ = the partial derivative of the fuel density versus temperature relationship.

The degrees of freedom associated with S_{de_2} are the same as the degrees of freedom for the temperature measurement precision index, i.e.,

$$df_{de_2} = df_T \qquad \text{(IV-52)}$$

The effect of fuel temperature bias on fuel density (B_{de_2}) is calculated from

$$B_{de_2} = \frac{\partial\rho}{\partial T}B_T \qquad \text{(IV-53)}$$

4.2.3.2 Computer Resolution

Computer resolution is the source of a small elemental error. Even the small computers used in experimental test applications have six digit resolution (most have eight or more). The resultant full-scale error would be plus or minus one in 10^6. Even though this error is probably negligible, some consideration should be given to it.

Two types of measurement resolution systems are in use, truncating systems and rounding systems. Consideration will be given the elemental bias and precision errors inherent in each of these.

In multiplying or dividing with six or eight digit numbers, the results may have more digits than the system resolution. The truncating system will eliminate digits on the right until the maximum allowable are left. This results in a bias of 1/2 digit and a uniform distribution of precision errors over the interval ±1/2 digit. The elemental precision index for this type of distribution is derived from the precision index of a uniform distribution (Appendix A):

$$s = \sqrt{\frac{(\text{lower limit - upper limit})^2}{12}} = \sqrt{\frac{1^2}{12}} = \pm 0.3 \qquad \text{(IV-54)}$$

The elemental precision error will be ±3/10 of a digit.

The rounding system will also reduce the number of digits to the resolution of the computer. In doing this it will increase the first digit on the right by one approximately half of the time. The decision to increase the digit is based on the size of the last digit eliminated (and others if necessary).

The bias error of 1/2 digit experienced with the truncating system is eliminated. The precision error, however, is not eliminated. It remains the same, ±3/10 of a digit. For a six-digit computer, the precision error is

$$S_{CR} = \pm 3/10 \times 10^{-6}$$

The conclusion is that the rounding system is superior to the truncating system, and if a choice exists, the rounding system should be used.

4.3 FUEL FLOW MEASUREMENT ERRORS

Fuel flow measurement errors when flowmeter calibrations are performed off site and multiple measurements are made, are then

$$S = \pm \frac{1}{\sqrt{K}} \sqrt{S_1^2 + s_{\overline{X}}^2 + S_{wr}^2 + S_{rr}^2 + S_{cc}^2 + s_{de}^2} \tag{IV-55}$$

where S = precision index for the fuel flow measurement and $\frac{1}{\sqrt{K}}$ = the factor to account for K multiple instruments which are averaged to provide the measurement.

$$df = \frac{\left(S_1^2 + s_{\overline{X}}^2 + S_{wr}^2 + S_{rr}^2 + S_{cc}^2 + s_{de}^2\right)^2}{\frac{S_1^4}{df_1} + \frac{s_{\overline{X}}^4}{df_{\overline{X}}} + \frac{S_{wr}^4}{df_{wr}} + \frac{S_{rr}^4}{df_{rr}} + \frac{S_{cc}^4}{df_{cc}} + \frac{s_{de}^4}{df_{de}}} \tag{IV-56}$$

where df = degrees of freedom associated with the precision index (S). The bias limit usually is not reduced when multiple instruments are used because they usually have equal biases. Bias errors would be reduced by averaging if the meters were calibrated independently at different facilities. The bias limit would be

$$B = \pm\sqrt{B_1^2 + b_{\overline{X}}^2 + b_{de}^2} \tag{IV-57}$$

where B = the bias limits for the measurement process. In Eqs. (IV-55) through (IV-57) if a frequency-to-digital conversion process was employed rather than counters, $s_{\overline{X}}$, $b_{\overline{X}}$, and $df_{\overline{X}}$ may be replaced with $s_{\overline{f}}$, $b_{\overline{f}}$, and $df_{\overline{f}}$. The uncertainty would be

$$U = \pm(B + t_{95}S) \tag{IV-58}$$

4.4 END-TO-END CALIBRATION

The best method for determining the errors incurred in the flow measurement process would be to flow a weighed quantity or measured volume of fuel through the turbine meters during an engine run. That is, an in-place, end-to-end calibration would be performed during an engine run. Figure IV-10 illustrates a typical arrangement of components for performing calibrations of this nature.

As an example, suppose a measured quantity (gravimetrically or volumetrically) of fuel was flowed through two flowmeters in series to the gas turbine engine during a run. During the period required to flow the measured quantity, record the following:

1. Total pulses generated by each meter,
2. Fuel temperature and pressure for density determination, and
3. The value for the measured quantity in gallons or pounds.

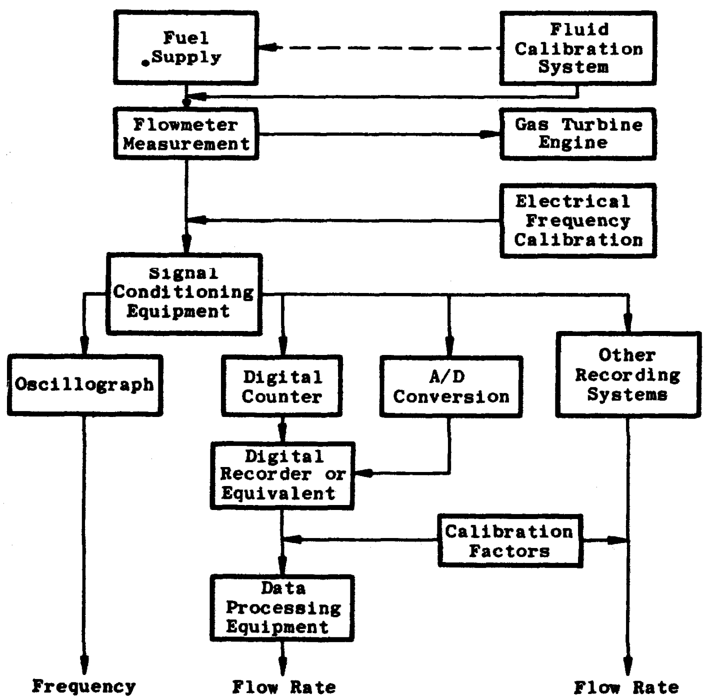

Fig. IV-10 End-to-End Calibration

From these data a meter factor can be calculated.

$$K_i = \frac{\text{total pulses}}{\text{total gallons}} \tag{IV-59}$$

for each meter. Then if N runs are made the precision index for data acquisition and reduction for each meter is

$$s_i = \pm\sqrt{\frac{\sum_{i=1}^{N}(K_i - \bar{K})^2}{N - 1}} \tag{IV-60}$$

This precision index will include the effects of

1. Meter nonlinearity
2. Electrical calibration
3. Counter or other frequency-to-digital conversion
4. Digital recording
5. Density determination
6. Computer resolution
7. Calibration to calibration precision error

The only precision errors not included in s_i are within-run-precision error (S_{wr}) and the precision error (s_{ws}) of the working standard (calibration system) calibration process. Fuel flow measurement precision index (S) is then

$$S = \pm \frac{1}{\sqrt{K}} \sqrt{s_i^2 + S_{wr}^2 + s_{ws}^2} \qquad \text{(IV-61)}$$

where $\frac{1}{\sqrt{K}}$ = the factor to account for averaging K multiple instruments. Derivations and formulation for calculating S_{wr} are in Appendix C. The working standard precision error can be determined using Eq. (IV-15) or (IV-30), depending on the type of working standard used.

Degrees of freedom for the flow measurement process are

$$df = \frac{\left(s_i^2 + S_{wr}^2 + s_{ws}^2\right)^2}{\frac{s_i^4}{df_i} + \frac{S_{wr}^4}{df_{wr}} + \frac{s_{ws}^4}{df_{ws}}} \qquad \text{(IV-62)}$$

where df defines individual degrees of freedom associated with each precision index.

Bias limits (B) in the flow measurement made by this method are the bias in the working standard plus any additional bias based on engineering judgment.

Uncertainty in the measurement is then

$$U = \pm(B + t_{95}S) \qquad \text{(IV-63)}$$

4.5 SUMMARY

In summary, the elemental errors to be evaluated are listed in Table XIII when off-site calibrations are performed. If end-to-end calibrations are performed while an engine test is in progress, the agony of evaluating all of the above errors is eliminated except those errors determined from multiple measurement statistics which must be evaluated in either case.

Table XIII Elemental Errors

Source	Bias Limit	Precision Index	Degrees of Freedom
Calibration	B_1	S_1	df_1
Data Acquisition	$b_{\bar{x}}$	$s_{\bar{x}}$, S_{wr}, S_{rr}, S_{cc}	$df_{\bar{x}}$, df_{wr}, df_{rr}, df_{cc}
Data Reduction	b_{de}	s_{de}	df_{de}