SECTION II
UNCERTAINTY MODEL

2.1 GENERAL

Terms such as bias[3], precision error, uncertainty, standard deviation, NBS, traceability, calibration, and degrees of freedom and the statistical concepts and mathematical procedures to be employed were introduced in Section I. This section will describe the mathematical model with words, illustrations, and an example.

It is intended that the examples given will closely fit typical applications. However, the model is general, and if specific calibration hierarchies are more or less extensive than the examples, simply add or omit levels and apply the model as shown; if specific measurement systems are different from the examples, substitute the bias limit and precision index terms for the system components and apply the model.

To review briefly, there are two types of measurement error: precision and bias. Precision error is the variation of repeated measurements of the same quantity. The sample standard deviation (S) is used as an index of the precision. Bias is the difference between the true value and the average of many repeated measurements. A limit (B) for the bias is estimated based on judgment, experience, and testing. The formula for combining these into uncertainty (U) is

$$U = \pm(B + t_{95}S) \qquad \text{(II-1)}$$

$$U^- = B^- - t_{95}S \text{ and } U^+ = B^+ + t_{95}S$$

when nonsymmetrical biases are present.

Note that throughout this Handbook lower case notation always indicates elemental errors, i.e., s and b for elemental precision and bias, and upper case notation indicates the root-sum-square (RSS) combination of several errors, e.g.,

$$S - \pm\sqrt{s_1^2 + S_2^2 + s_3^2}$$

$$B = \pm\sqrt{b_1^2 + B_2^2 + b_3^2}$$

where

$$S_2 = \pm\sqrt{\Sigma_i s_i}$$

$$B_2 = \pm\sqrt{\Sigma_i b_i}$$

[3]For a definition of terms used in this Handbook, see the Glossary in Section IX.

The remainder of this section is devoted to illustration of a typical measurement uncertainty analysis and the propagation of errors to performance parameters.

2.2 MEASUREMENT ERROR SOURCES

For purposes of illustration, the elemental error sources for the force measurement system will be treated in this section. These error sources fall into three categories:

1. Calibration Hierarchy Errors (2.2.1)
2. Data Acquisition Errors (2.2.2)
3. Data Reduction Errors (2.2.3)

Elemental error sources for other measurements will be enumerated in the section dealing with each measurement.

2.2.1 Calibration Hierarchy Errors

To demonstrate traceability of measurements to the NBS, whose standards are by definition the "truth," it is necessary to establish calibration hierarchies. Each level in the hierarchy, including NBS, constitutes an error source which contributes to the error in the final measurement. Calibration of all measurement instruments at the NBS is possible; however, such calibrations would be inconvenient, time consuming, and very expensive. The purpose here is to illustrate a typical hierarchy and to enumerate the error sources within. Figure II-1 is a typical force transducer calibration hierarchy. Associated with each comparison in the calibration hierarchy is a pair of elemental errors. These errors are the unknown bias and the precision index in each process. Note that these elemental errors are independent, e.g., b_{21} is not a function of b_{11}. The error sources are listed in Table V.

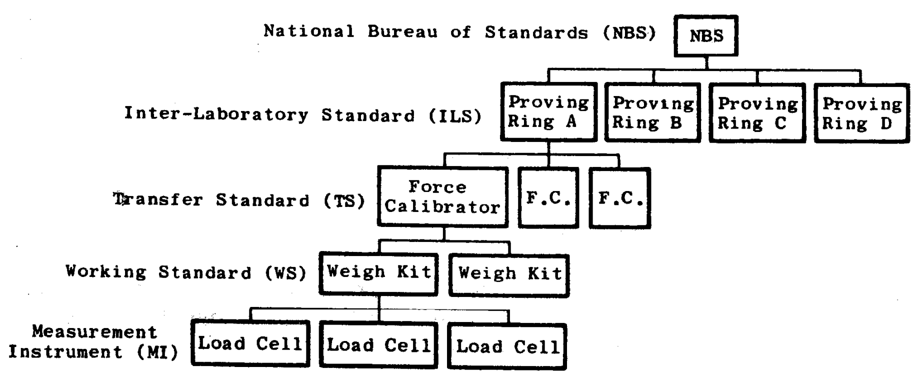


Fig. II-1 Force Measurement Calibration Hierarchy

Table V Calibration Hierarchy Error Sources

Calibration	Bias Limit	Precision Index	Degrees of Freedom
NBS – ILS	b_{11}	s_{11}	df_{11}
ILS – TS	b_{21}	s_{21}	df_{21}
TS – WS	b_{31}	s_{31}	df_{31}
WS–MI	b_{41}	s_{41}	df_{41}

2.2.2 Data Acquisition Errors

Data are acquired by measuring the electrical output resulting from force applied to a strain-gage-type force measurement instrument. Figure II-2 illustrates some of the error sources associated with data acquisition. Other error sources such as electrical simulation, thrust bed mechanics, and environmental effects are also present. The best method to determine the effects of all of these error sources is to perform end-to-end calibrations and compare known applied forces with measured values. However, it is not always possible or even desirable to do this, and if this is the case, it is necessary to evaluate each of the elemental errors and combine them to determine the overall error.

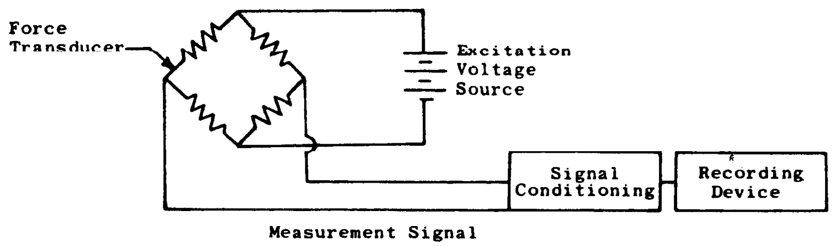


Fig. II-2 Data Acquisition System

All the data acquisition error sources are listed in Table VI. Symbols for the elemental bias and precision errors and for the degrees of freedom are shown.

2.2.3 Data Reduction Errors

Computers operate on raw data to produce output in engineering units. The errors in this process stem from calibration curve fits (Fig. II-3) and computer resolution.

Symbols for the data reduction error sources are listed in Table VII. These errors are often negligible in each process.

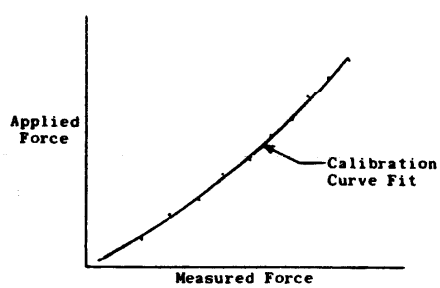


Fig. II-3 Calibration Curve

Table VI Data Acquisition Error Sources

Error Source	Bias Limit	Precision Index	Degrees of Freedom
Excitation Voltage	b_{12}	s_{12}	df_{12}
Electrical Simulation	b_{22}	s_{22}	df_{22}
Signal Conditioning	b_{32}	s_{32}	df_{32}
Recording Device	b_{42}	s_{42}	df_{42}
Force Transducer	b_{52}	s_{52}	df_{52}
Thrust Bed Mechanics	b_{62}	s_{62}	df_{62}
Environmental Effects	b_{72}	s_{72}	df_{72}

Table VII Data Reduction Error Sources

Error Source	Bias Limit	Precision Index	Degrees of Freedom
Calibration Curve Fit	b_{13}	s_{13}	df_{13}
Computer Resolution	b_{23}	s_{23}	df_{23}

2.3 MEASUREMENT UNCERTAINTY MODEL

The measurement process, defined for an entire engine test facility, is the totality of all the individual subprocesses and steps in the measurement system for a given engine type. That is, the process is the total of all the calibrations, all data acquisitions and all data reductions. Therefore, the precision error in each step of each subprocess is reflected as a precision error in the total process. The bias error in each subprocess is a bias error in the total process. (Another definition for the measurement process is discussed in Section VIII).

The precision index (S) at any stage in the total process is the root-sum-square of the elemental precision indices for that stage with the elemental precision indices for all of the preceding steps.

$$S = \pm\sqrt{\sum_j \sum_i s_{ij}^2} \qquad \text{(II-2)}$$

where j defines the subprocesses calibration, data acquisition, and data recording and i defines the steps within the subprocess.

For example: The precision index for the calibration process is the root-sum-square of the elemental precision indices of Table V.

$$S_{cal} = \pm\sqrt{s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2} \qquad \text{(II-3)}$$

The precision index for the data acquisition process is the root-sum-square of the precision indices of Table VI.

$$S_{\substack{\text{Data}\\ \text{Acquisition}}} = \pm\sqrt{s_{12}^2 + s_{22}^2 + s_{32}^2 + s_{42}^2 + s_{52}^2 + s_{62}^2 + s_{72}^2} \qquad \text{(II-4)}$$

The precision index for the data reduction process is the root-sum-square of the precision indices of Table VII.

$$S_{\substack{\text{Data}\\ \text{Reduction}}} = \pm\sqrt{s_{13}^2 + s_{23}^2} \qquad \text{(II-5)}$$

The force measurement precision index is the root-sum-square of all the elemental precision indices in the force measurement system.

$$S_{Force} = \sqrt{s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2 + s_{12}^2 + s_{22}^2 + s_{32}^2 + s_{42}^2 + s_{52}^2 + s_{62}^2 + s_{72}^2 + s_{13}^2 + s_{23}^2} \qquad \text{(II-6)}$$

The bias limit for any stage in the process is the root-sum-square of the elemental errors in the preceding steps of the process.

For example: The bias limit for the calibration hierarchy is

$$B_{Cal} = \pm\sqrt{b_{11}^2 + b_{21}^2 + b_{31}^2 + b_{41}^2} \quad \text{(II-7)}$$

The bias limit for the data acquisition process is

$$B_{\substack{Data\\Acquisition}} = \pm\sqrt{b_{12}^2 + b_{22}^2 + b_{32}^2 + b_{42}^2 + b_{52}^2 + b_{62}^2 + b_{72}^2} \quad \text{(II-8)}$$

The bias limit for the data reduction process is

$$B_{\substack{Data\\Reduction}} = \pm\sqrt{b_{13}^2 + b_{23}^2} \quad \text{(II-9)}$$

The bias limit for the force measurement process is

$$B_{Force} = \sqrt{b_{11}^2 + b_{21}^2 + b_{31}^2 + b_{41}^2 + b_{12}^2 + b_{22}^2 + b_{32}^2 + b_{42}^2 + b_{52}^2 + b_{62}^2 + b_{72}^2 + b_{13}^2 + b_{23}^2} \quad \text{(II-10)}$$

Biases associated with force measurement are equally likely in either the plus or minus directions, i.e., there are no nonsymmetrical bias limit estimates.

The degrees of freedom (df) associated with the precision index at any step in the process are calculated using the Welch-Satterthwaite formula. It is a function of the degrees of freedom and magnitude of each elemental precision index.

For example: The degrees of freedom for the calibration precision index (S_{Cal}) is

$$df = \frac{\left[s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2\right]^2}{\left[\frac{s_{11}^4}{df_{11}} + \frac{s_{21}^4}{df_{21}} + \frac{s_{31}^4}{df_{31}} + \frac{s_{41}^4}{df_{41}}\right]} \quad \text{(II-11)}$$

The degrees of freedom for the force measurement precision index is

$$df = \frac{\left[s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2 + s_{12}^2 + s_{22}^2 + s_{32}^2 + s_{42}^2 + s_{52}^2 + s_{62}^2 + s_{72}^2 + s_{13}^2 + s_{23}^2\right]^2}{\left[\frac{s_{11}^4}{df_{11}} + \frac{s_{21}^4}{df_{21}} + \frac{s_{31}^4}{df_{31}} + \frac{s_{41}^4}{df_{41}} + \frac{s_{12}^4}{df_{12}} + \frac{s_{22}^4}{df_{22}} + \frac{s_{32}^4}{df_{32}} + \frac{s_{42}^4}{df_{42}} + \frac{s_{52}^4}{df_{52}} + \frac{s_{62}^4}{df_{62}} + \frac{s_{72}^4}{df_{72}} + \frac{s_{13}^4}{df_{13}} + \frac{s_{23}^4}{df_{23}}\right]} \quad \text{(II-12)}$$

The uncertainty parameter (U) at any stage is the sum of the bias limit (B) for that stage and the precision limit ($t_{95}S$). The precision limit ($t_{95}S$) for any stage is the precision index (S) for that stage times the 95th percentile of the student's "t" distribution (when the degrees of freedom are greater than 30, 2.0 is used for the "t" value). The uncertainty parameter (U) defines the limits of the measurement error that might reasonably be expected in a well-defined measurement process:

$$U = \pm(B + t_{95}S) \quad \text{(II-13)}$$

Figure II-4 illustrates the uncertainty parameter (U).

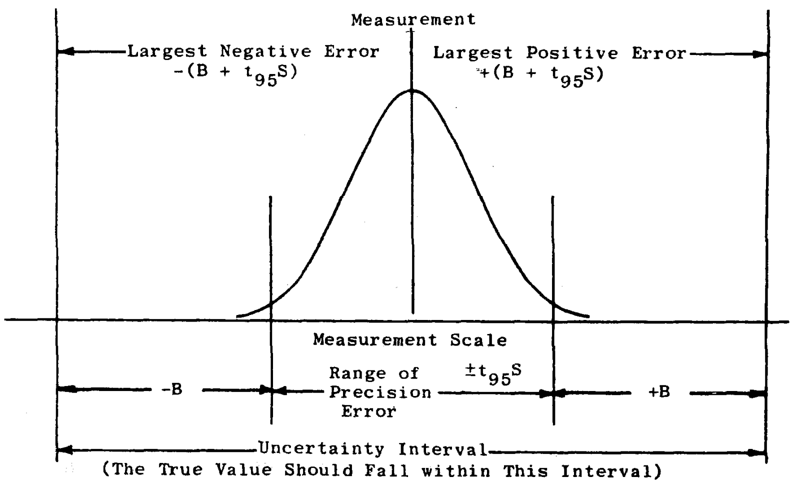


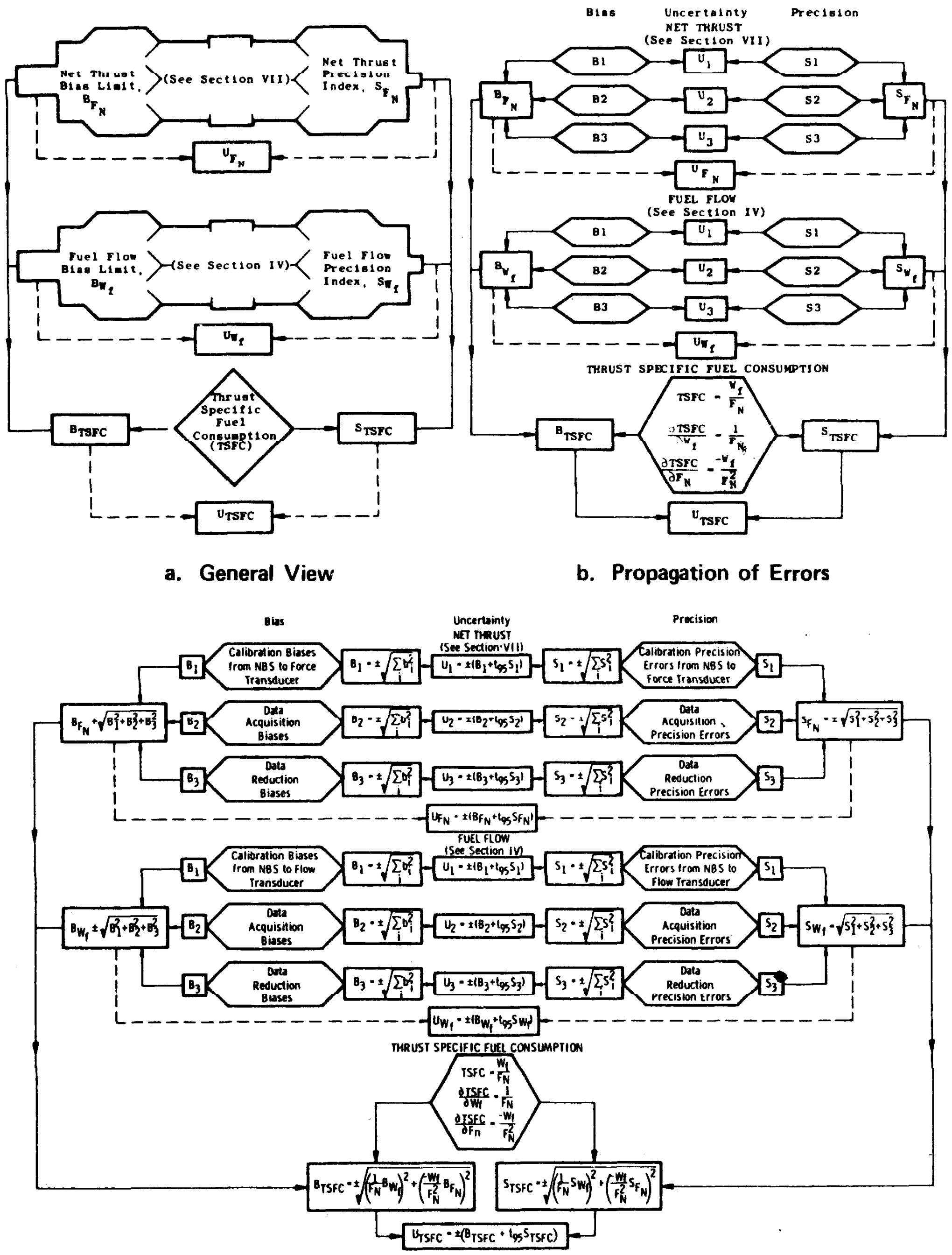
Fig. II-4 Uncertainty Parameter $U = \pm(B + t_{95}S)$

2.4 EXAMPLE OF THE MODEL

Figure II-5a is a block diagram showing the overall model for determining the uncertainty in gas turbine engine thrust specific fuel consumption. The blocks identify the two major parameters: net thrust and fuel flow. The dotted lines indicate the calculation of uncertainty for each parameter; solid lines indicate the propagation of bias limits and precision indices to the bias limit and precision index for thrust specific fuel consumption using Taylor's series methods. Detailed treatment of fuel flow measurement uncertainty and net thrust determination uncertainty is contained in Sections IV and VII, respectively.

In Fig. II-5b, it is seen that each parameter block contains three general types of errors: calibration errors, data acquisition errors, and data reduction errors. These are identified by S_1, S_2, and S_3, respectively, for precision indices and B_1, B_2, and B_3, respectively, for bias limits. The lines within each parameter indicate the calculation of uncertainty (U_1, U_2, and U_3) for each type of measurement. Other lines indicate the calculation of bias limits and precision indices for the individual parameters.

The final figure of the series (Fig. II-5c) illustrates the fact that each measurement is made up of several elemental sources of error. Examples of these are tabulated in Tables V, VI, and VII. In the figure, blocks indicate the formulas for the calculation of bias limits and precision indices at each level in the measurement process. The lines point out the procedures for determining bias limit (B), precision index (S), and uncertainty (U) for each process in the measurement chain and also for the calculated parameter, thrust specific fuel consumption.



a. General View

b. Propagation of Errors

c. Elemental Errors

Fig. II-5 Overall Uncertainty Model

As an example, the uncertainty in the thrust specific fuel consumption of a gas turbine engine will be calculated. Test conditions are that the engine is being tested on an outdoor sea-level test stand. Assume a nominal sea-level thrust specific fuel consumption of 1.0 lbm/lbf-hr and 10,000 lbf and 10,000 lbm/hr for net thrust and fuel flow, respectively. Detailed treatment of the errors in net thrust determination may be found in Section 3.3 for sea-level testing which applies to this example. Section VII details the treatment of errors in net thrust determination at altitude conditions. For simplicity, values for the bias and precision index for fuel flow have been assumed.

2.4.1 Net Thrust Measurement

It is assumed that:

1. The net thrust bias limit is

$$B_{F_N} = \pm 18.1 \text{ lbf}$$

2. The net thrust precision index is

$$S_{F_N} = \pm 37.8 \text{ lbf}$$

3. The net thrust nominal level = 10,000 lbf

Then,

4. Net thrust uncertainty is

$$U_{F_N} = \pm(B_{F_N} + t_{95}S_{F_N}), \quad t_{95} = 2.00 \text{ because df} > 30 \text{ for } S_{F_N}$$

$$U_{F_N} = \pm(18.1 + 2.00 \times 37.8)$$

$$= \pm 93.7 \text{ lbf}$$

2.4.2 Fuel Flow Measurement

It is assumed that:

1. The fuel flow bias limit is

$$B_{W_f} = \pm 50 \text{ lb/hr}$$

2. The fuel flow precision index is

$$S_{W_f} = \pm 50 \text{ lb/hr}, \; df_{W_f} = 60$$

3. The fuel flow nominal level = 10,000 lb/hr

Then,

4. Fuel flow uncertainty is

$$U_{W_f} = \pm\left(B_{W_f} + t_{95}S_{W_f}\right), t_{95} = 2.00 \text{ because df} > 30 \text{ for } S_{W_f}$$

$$U_{W_f} = \pm(50 + 2.00 \times 50)$$

$$= \pm 150 \text{ lb/hr}$$

2.4.3 Thrust Specific Fuel Consumption

The TSFC bias limit is

$$B_{TSFC} = \pm\sqrt{\left(\frac{1}{F_N} B_{W_f}\right)^2 + \left(\frac{-W_f}{F_N^2} B_{F_N}\right)^2}$$

$$B_{TSFC} = \pm\sqrt{\left(\frac{1}{10,000} \times 50\right)^2 + \left(\frac{-10,000}{10,000^2} \times 18.1\right)^2}$$

$$= \pm 0.0053 \text{ lbm/lbf-hr}$$

The TSFC precision index is

$$S_{TSFC} = \pm\sqrt{\left(\frac{1}{F_N} S_{wf}\right)^2 + \left(\frac{-W_f}{{F_N}^2} S_{F_N}\right)^2}$$

$$S_{TSFC} = \pm\sqrt{\left(\frac{1}{10.000} \times 50\right)^2 + \left(\frac{-10,000}{10,000^2} \times 37.8\right)^2}$$

$$= \pm 0.0063 \text{ lbm/lbf-hr}$$

The TSFC degree of freedom is

$$df_{TSFC} = \frac{\left[\left(\frac{\partial TSFC}{\partial W_f} S_{wf}\right)^2 + \left(\frac{\partial TSFC}{\partial F_N} S_{F_N}\right)^2\right]^2}{\frac{\left(\frac{\partial TSFC}{\partial W_f} S_{wf}\right)^4}{df_{wf}} + \frac{\left(\frac{\partial TSFC}{\partial F_N} S_{F_N}\right)^4}{df_{F_N}}}$$

$$df_{TSFC} = \frac{\left[\left(\frac{1}{F_N} S_{W_f}\right)^2 + \left(\frac{-W_f}{F_N^2} S_{F_N}\right)^2\right]^2}{\frac{\left(\frac{1}{F_N} S_{W_f}\right)^4}{df_{W_f}} + \frac{\left(\frac{-W_f}{F_N^2} S_{F_N}\right)^4}{df_{F_N}}}$$

When the degrees of freedom for each source of error are greater than thirty, the resultant degrees of freedom will also be greater than thirty. For illustrative purposes, the calculation of degrees of freedom are:

$$df_{TSFC} = \frac{\left[\left(\frac{1}{10{,}000} \times 50\right)^2 + \left(\frac{-10{,}000}{10{,}000^2} \times 37.8\right)^2\right]^2}{\frac{\left(\frac{1}{10{,}000} \times 50\right)^4}{60} + \frac{\left(\frac{-10{,}000}{10{,}000^2} \times 37.8\right)^4}{57}}$$

$$= 110$$

The result is greater than thirty, as expected. Therefore, $t_{95} = 2.00$.

The TSFC uncertainty is

$$U_{TSFC} = \pm(B_{TSFC} + t_{95}S_{TSFC})$$

$$U_{TSFC} = \pm(0.0053 + 2.00 \times 0.0063)$$

$$= \pm 0.018 \text{ lbm/lbf-hr}$$

2.5 SUMMARY

The statistical concepts and mathematical procedures used to develop the models are set forth in Section I.

In Section II, the uncertainty model is presented in mathematical, graphical, and block diagram form with a numerical example of how the model is to be used. These methods are summarized in Fig. II-6, a logic decision diagram. However, Sections I and II by no means provide full treatment of the problem of determining uncertainty in the gas turbine engine performance parameter, thrust specific fuel consumption. Some things which have not been treated in this section are:

1. Multiple measurements (see Appendix III)
2. Evaluation of elemental errors (see Sections III through VII)
3. Signed bias (see Section I).