SECTION I
INTRODUCTION

1.1 OBJECTIVE

The objective of this Handbook is to present a standard method of treating measurement error[1] or uncertainty for gas turbine engine performance parameters, such as thrust, airflow, and thrust specific fuel consumption. The need for a standard is obvious to those who have reviewed the numerous methods currently used. The subject is complex and involves both engineering and statistics. Only one method is presented herein without alternative paths. A single standard method is required to make comparisons between engine manufacturers and between facilities. However, it must be recognized that no single method will give a rigorous, scientifically correct answer for all situations. Further, even for a single set of data, the task of finding and proving one method to be correct is usually impossible. The method selected is believed to be most universally applicable. It is identical with the measurement uncertainty model used in the rocket engine industry which has been well received ("ICRPG Handbook for Estimating the Uncertainty in Measurements made with Liquid Propellant Rocket Engine Systems," CPIA No. 180, AD855130, April 30, 1969).

There are numerous examples for illustration. An effort has been made to use simple prose with a minimum of jargon.

1.2 SCOPE

This Handbook presents a working outline detailing and illustrating the techniques for estimating measurement uncertainty. Section II describes the mathematical model for a typical performance parameter (thrust specific fuel consumption). Sections III, IV, V, and VI treat errors associated with the measurement of force, fuel flow, pressure and temperature, and airflow. Each section includes a discussion of the methods of calibration and lists of the elemental errors and examples of the statistical techniques. Section VII describes the calculations of the uncertainty in net thrust and thrust specific fuel consumption at altitude conditions. Section VIII describes and illustrates several special methods. Section IX is the Glossary. Appendixes with tables, derivations, and proofs are found at the end of the Handbook.

1.3 MEASUREMENT ERROR

All measurements have measurement errors. These errors are the differences between the measurements and the true value defined by the National Bureau of Standards (NBS). Uncertainty is the maximum error which might reasonably be expected and is a measure of accuracy, i.e., the closeness of the measurement to the true value. Measurement error has two components: a fixed error and a random error.

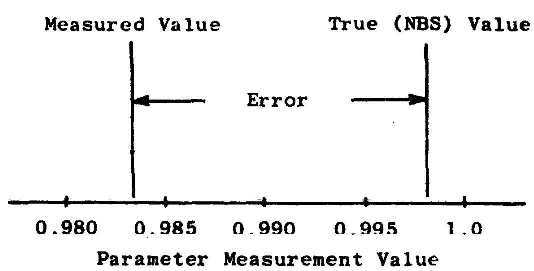

Fig. I-1 Measurement Error

[1]For a definition of terms used in this Handbook, see the Glossary in Section IX.

1.3.1 Precision (Random Error)

Random error is seen in repeated measurements. Measurements do not and are not expected to agree exactly. There are always numerous small effects which cause disagreements. The variation between repeated measurements is called precision error. The standard deviation (σ) is used as a measure of the precision error. A large standard deviation means large scatter in the measurements. The statistic (s) is calculated to estimate the standard deviation and is called the precision index

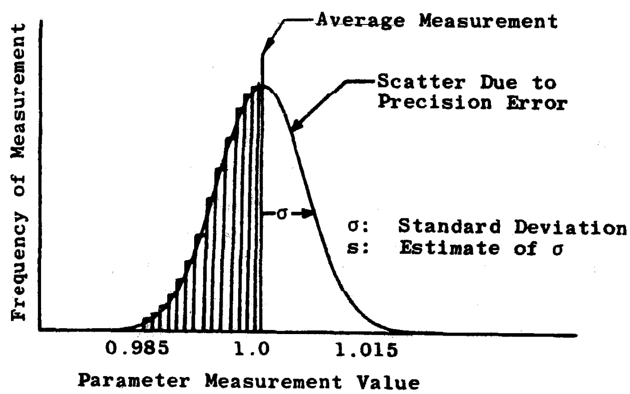

Fig. I-2 Precision Error

$$s = \sqrt{\frac{\sum_{i=1}^{N}(X_i - \overline{X})^2}{N - 1}}$$

where N is the number of measurements made and $\overline{X}$ is the average value of individual measurements X_i.

1.3.2 Bias (Fixed Error)

The second component, bias, is the constant or systematic error. In repeated measurements, each measurement has the same bias. The bias cannot be determined unless the measurements are compared with the true value of the quantity measured.

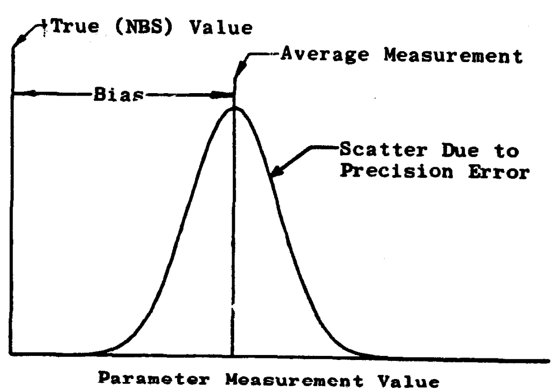

Fig. I-3 Bias Error

Bias is categorized into five classes: (1) large known biases, (2) small known biases, (3) large unknown biases, and small unknown biases which may have (4) unknown sign (±) or (5) known sign.

	Known Sign and Magnitude	Unknown Magnitude	
Large	(1) Calibrated Out	(3) Assumed to be Eliminated	
Small	(2) Negligible, Contributes to Bias Limit	(4) Unknown Sign	(5) Known Sign
		Contributes to Bias Limit	

Fig. I-4 Five Types of Bias Errors

1.3.2.1 Large Known Biases

The large known biases are eliminated by comparing the instrument with a standard instrument and obtaining a correction. This process is called calibration.

1.3.2.2 Small Known Biases

Small known biases may or may not be corrected depending on the difficulty of the correction and the magnitude of the bias.

1.3.2.3 Large Unknown Biases

Unknown biases are not correctable. That is, they may exist, but the magnitude of the bias is not known, and perhaps even the sign is not known.

Every effort must be made to eliminate all large unknown biases. The introduction of such errors converts the controlled measurement process into an uncontrolled worthless effort. Large unknown biases usually come from human errors in data processing, incorrect handling and installation of instrumentation, and unexpected environmental disturbances such as shock and bad flow profiles. In a well-controlled measurement process, the assumption is that there are no large unknown biases. To ensure that a controlled measurement process exists, all measurements should be monitored with statistical quality control charts. A list of references describing the use of statistical quality control charts is included at the end of this section. Drifts, trends, and movements leading to out-of-control situations should be identified and investigated. Histories of data from calibrations are required for effective control. It is assumed throughout this Handbook that these precautions are observed and that the measurement process is in control; if not, the methods contained herein are invalid.

1.3.2.4 Small Biases, Unknown Sign, and Unknown Magnitude

In most cases, the bias error is equally likely to be plus or minus about the measurement. That is, it is not known if the limit is positive or negative, and the estimate reflects this. The bias limit is estimated as an upper limit on the maximum fixed error. For example, ±5 pounds is a typical bias limit.

It is both difficult and frustrating to estimate the limit of an unknown bias. To determine the exact bias in a measurement, it would be necessary to compare the true value and the measurements. This is almost always impossible. An effort must be made to obtain special tests or data that will provide bias information. The following are examples of such data:

1. Interlab, interfacility, intercompany tests on measurement devices, test rigs, and full-scale engines.

2. Flight test data versus altitude test chamber data versus ground test data.

3. Special comparisons of standards with instruments in the actual test environment.

4. Ancillary or concomitant functions that provide the same performance parameter; i.e., in an altitude engine test, airflow may be measured with (1) an orifice and (2) a bellmouth, (3) estimated from compressor speed-flow rig data, (4) estimated from turbine flow parameter, and (5) jet nozzle calibrations.

5. When it is known that a bias results from a particular cause, special calibrations may be performed allowing the cause to perturbate through its complete range to determine the range of bias.

If there is no source of data for bias, the judgment of the most knowledgeable instrumentation expert on the measurement must be used. However, without data, the upper limit on the largest possible bias error must reflect the lack of knowledge.

1.3.2.5 Small Biases, Known Sign, and Unknown Magnitude

Sometimes the physics of the measurement system provide knowledge of the sign but not the magnitude of the bias. For example, thermocouples radiate and conduct energy to indicate lower temperatures. The bias limits which result are nonsymmetrical, i.e., not of the form $\pm b$. They are of the form $^{+b}_{-c}$ where both limits may be positive or negative or the limits may be of mixed sign as indicated. Table I below lists several nonsymmetrical bias limits for illustration.

Table I Nonsymmetrical Bias Limits

Bias Limits	Explanation
0, +10 deg	The bias will range from zero to plus 10 deg.
-5, +15 lb	The bias will range from minus 5 to plus 15 lb.
+3, +7 psia	The bias will range from plus 3 to plus 7 psia.
-8, -3 deg	The bias will range from minus 8 to minus 3 deg.

In summary, measurement systems are subject to two types of errors, bias and precision error (Fig. I-5). One sample standard deviation is used as the precision index. The bias limit is estimated as an upper limit on the maximum fixed error.

1.4 MEASUREMENT UNCERTAINTY

For simplicity of presentation a single number (some combination of bias and precision) is needed to express a reasonable limit for error. The single number must have a simple interpretation (the largest error reasonably expected) and be useful without complex explanation. It is impossible to define a single rigorous statistic because the bias is an upper limit based on judgment which has unknown characteristics. Any function of these two numbers must be a hybrid combination of an unknown quantity (bias) and a statistic (precision). However, the need for a single number to measure error is so great that the adoption of an arbitrary standard is warranted. The standard most widely used is

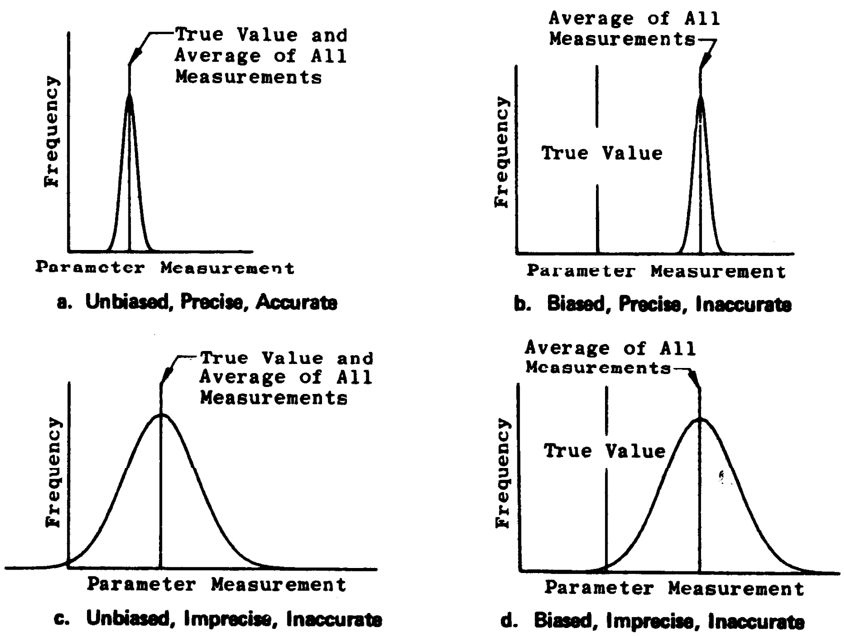

Fig. I-5 Measurement Error (Bias, Precision, and Accuracy)

the bias limit plus a multiple of the precision index. This method is recognized and recommended by the NBS[2] and has been widely used in industry.

Uncertainty (Fig. I-6) may be centered about the measurement and is defined herein as:

$$U = \pm(B + t_{95} S) \qquad \text{(I-1)}$$

where B is the bias limit, S is the precision index, and t_{95} is the 95th percentile point for the two-tailed Students "t" distribution (Table E-1, Appendix E). The t value is a function of the number of degrees of freedom (df) used in calculating S. For small samples, t will be large, and for larger samples t will be smaller, approaching 1.96 as a lower limit. The use of the t arbitrarily inflates the limit U to reduce the risk of underestimating S when a small sample is used to calculate S. Since 30 degrees of freedom yield a t of 2.04 and infinite degrees of freedom yield a t of 1.96, an arbitrary selection of t = 2 for values of df from 30 to infinity was made, i.e., $U = \pm(B + 2S)$, when df $\geqslant$ 30.

In a sample, the number of degrees of freedom is the size of the sample. When a statistic is calculated from the sample, the degrees of freedom associated with the statistic

[2]Eisenhart, C. "Expression of Uncertainties of Final Results, Precision Measurement and Calibration," NBS Handbook 91, Vol I, February 1969, pp. 69-72.

Ku, H. H. "Expressions of Imprecision, Systematic Error, and Uncertainty Associated with a Reported Value, Precision Measurement and Calibration," NBS Handbook 91, Vol I, February 1969, pp. 73-78.

are reduced by one for every estimated parameter used in calculating the statistic. For example, from a sample of size N, $\overline{X}$ is calculated:

$$\overline{X} = \sum_{i=1}^{N} X_i/N \tag{I-2}$$

which has N degrees of freedom and

$$S = \sqrt{\frac{\sum_{i=1}^{N}(X_i - \overline{X})^2}{N-1}} \tag{I-3}$$

which has N-1 degrees of freedom because $\overline{X}$ (based on the same sample of data) is used to calculate S. In calculating other statistics, more than one degree of freedom may be lost. For example, in calculating the standard error of a curve fit, the number of degrees of freedom which are lost is equal to the number of estimated coefficients for the curve.

It is recommended that the uncertainty parameter (U) be used for simplicity of presentation; however, the components (bias, precision, and degrees of freedom) should be available in an appendix or in supporting documentation. These three components may be required (1) to substantiate and explain the uncertainty value, (2) to provide a sound technical base for improved measurements, and (3) to propagate the uncertainty from measured parameters to performance parameters, and from performance parameters

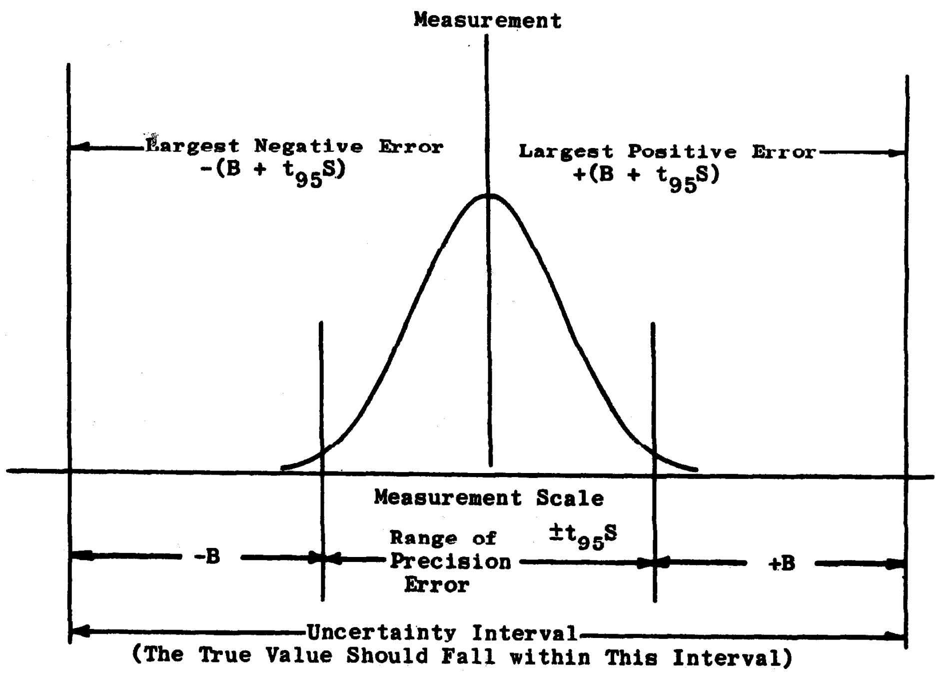

Fig. I-6 Measurement Uncertainty, Symmetrical Bias

to other more complex performance parameters (i.e. fuel flow to Thrust Specific Fuel Consumption (TSFC), TSFC to aircraft range, etc.). Although uncertainty is not a statistical confidence interval, it is an arbitrary substitute which is probably best interpreted as the largest error expected. Under any reasonable assumption for the distribution of bias, the coverage of U is greater than 95 percent, but this cannot be proved as the distribution of bias is both unknown and unknowable.

If there is a nonsymmetrical bias limit (Fig. I-7), the uncertainty U is no longer symmetrical about the measurement. The upper limit of the interval is defined by the upper limit of the bias interval (B^+). The lower limit is defined by the lower limit of the bias interval (B^-).

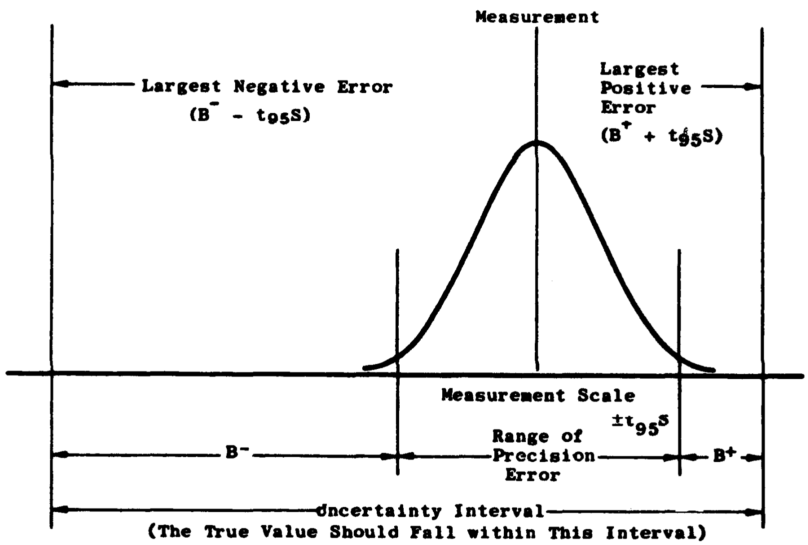

Fig. I-7 Measurement Uncertainty, Nonsymmetrical Bias

The uncertainty interval U is $U^- = B^- - t_{95}S$ to $U^+ = B^+ + t_{95}S$.

Table II shows the undertainty U for the nonsymmetrical bias limits of Table I. The S and t_{95} are assumed to be 1 unit and 2 units for each case.

Table II Uncertainty Intervals Defined by Nonsymmetrical Bias Limits

B^-	B^+	$t_{95}S$	U^- (Lower limit for U)	U^+ (Upper limit for U)
0 deg	+10 deg	2 deg	-2 deg	+12 deg
-5 lb	+15 lb	2 lb	-7 lb	+17 lb
+3 psia	+7 psia	2 psia	+1 psia	+9 psia
-8 deg	-3 deg	2 deg	-10 deg	-1 deg

The proper method for combining elemental measurement uncertainty values is to determine the root-sum-square values of the elemental bias limits and the elemental precision indices separately. Then, apply the uncertainty formula to the combined bias limits and precision indices. In some cases, the same value will be obtained if the uncertainties are root-sum-squared directly. However, this is not a general rule, and large errors in the combined uncertainty (10 to 25 percent) can result. Further, the root-sum-squared uncertainty value will be smaller (optimistic) than the proper uncertainty estimate, and the estimate is a significant underestimate of the true measurement error.

For example, in combining the following uncertainties the root-sum-square of the uncertainties was 18.38 units. The correct value was 23.21 units.

Bias Limit (B)	Precision Index (S)	Uncertainty
1	6	±13
11	1	±13

where Uncertainty = ±(B + 2S).

Now the bias limit for the combined parameter is the root-sum-square of 1 and 11:

$$B = \sqrt{1^2 + 11^2} = \sqrt{122} = 11.05$$

The precision index for the combined parameter is the root-sum-square of 6 and 1:

$$S = \sqrt{6^2 + 1^2} = \sqrt{37} = 6.08$$

The Uncertainty is thus:

$$U = \pm(B + 2S) = \pm[11.05 + 2(6.08)] = \pm 23.21$$

The root-sum-square of the original uncertainties is

$$\sqrt{(13)^2 + (13)^2} = \sqrt{169 + 169} = \sqrt{338} = 18.38$$

Now,

$$\frac{23.21 - 18.38}{18.38} \times 100 = 26.3\%$$

and over 25 percent error has been introduced just because of the wrong propagation of error formula.

1.5 PROPAGATION OF MEASUREMENT ERRORS

Rarely are performance parameters measured directly; usually more basic quantities such as temperature, force, pressure, and fuel flow are measured, and the performance parameter is calculated as a function of the measurements. Error in the measurements is propagated to the parameter through the function. The effect of the propagation may be approximated with the Taylor's series methods.

1.5.1 Engine Inlet Airflow

Engine inlet airflow is determined by the use of a choked venturi and measurements of upstream temperature and stagnation pressure (Fig. I-8).

The flow (W_a) is calculated from

$$W_a = F_A C^* A_{eff} P_1 / \sqrt{T_1} \quad \text{(I-4)}$$

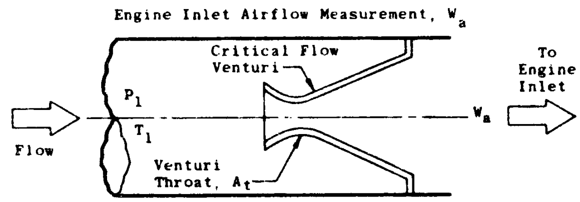

Fig. I-8 Flow through a Choked Venturi

where

F_A is the factor to account for thermal expansion of the venturi
A_{eff} is the effective venturi throat area
P_1 is the total (stagnation) pressure upstream
T_1 is the total temperature upstream
C^* is the factor to account for the properties of the air (critical flow constant)

The precision index for the flow (S_{W_a}) is calculated using the Taylor's series expansion (this method is derived in Appendix B):

$$S_{W_a} = \sqrt{\left(\frac{\partial W_a}{\partial F_A} S_{F_A}\right)^2 + \left(\frac{\partial W_a}{\partial C^*} S_{C^*}\right)^2 + \left(\frac{\partial W_a}{\partial A_{eff}} S_{A_{eff}}\right)^2 + \left(\frac{\partial W_a}{\partial P_1} S_{P_1}\right)^2 + \left(\frac{\partial W_a}{\partial T_1} S_{T_1}\right)^2} \quad \text{(I-5)}$$

where

$\frac{\partial W_a}{\partial F_A}$ denotes the partial derivative of W_a with respect to F_A.

Taking the necessary derivatives gives

$$S_{W_a} = \sqrt{\left(\frac{C^* A_{eff} P_1}{\sqrt{T_1}} S_{F_A}\right)^2 + \left(\frac{F_A A_{eff} P_1}{\sqrt{T_1}} S_{C^*}\right)^2 + \left(\frac{F_A C^* P_1}{\sqrt{T_1}} S_{A_{eff}}\right)^2 + \left(\frac{F_A C^* A_{eff}}{\sqrt{T_1}} S_{P_1}\right)^2 + \left(\frac{F_A C^* A_{eff} P_1}{-2\sqrt{T_1^3}} S_{T_1}\right)^2} \quad \text{(I-6)}$$

By inserting the nominal values and precision errors from Table III into Eq. (I-6), the precision index of 0.3658 lb/sec for engine airflow is obtained.

The bias limit in the flow calculation is propagated from the bias limits of the measured variables. Using the Taylor's series formula gives

$$B_f = \sqrt{\left(\frac{\partial f}{\partial x_1} B_{x_1}\right)^2 + \left(\frac{\partial f}{\partial x_2} B_{x_2}\right)^2 + \left(\frac{\partial f}{\partial x_3} B_{x_3}\right)^2 + \cdots + \left(\frac{\partial f}{\partial x_n} B_{x_n}\right)^2} \quad \text{(I-7)}$$

For this example, where $W_a = F_A C^* A_{eff} P_1 / \sqrt{T_1}$:

$$B_{W_a} = \sqrt{\left(\frac{\partial W_a}{\partial F_a} B_{F_a}\right)^2 + \left(\frac{\partial W_a}{\partial C^*} B_{C^*}\right)^2 + \left(\frac{\partial W_a}{\partial A_{eff}} B_{A_{eff}}\right)^2 + \left(\frac{\partial W_a}{\partial P_1} B_{P_1}\right)^2 + \left(\frac{\partial W_a}{\partial T_1} B_{T_1}\right)^2} \quad \text{(I-8)}$$

Taking the necessary derivatives gives

$$B_{W_a} = \sqrt{\left(\frac{C^* A_{eff} P_1}{\sqrt{T_1}} B_{F_A}\right)^2 + \left(\frac{F_a A_{eff} P_1}{\sqrt{T_1}} B_{C^*}\right)^2 \left(\frac{F_a C^* P_1}{\sqrt{T_1}} B_{A_{eff}}\right)^2 + \left(\frac{F_a C^* A_{eff}}{\sqrt{T_1}} B_{P_1}\right)^2 + \left(\frac{-F_a C^* A_{eff}}{-2\sqrt{T_1{}^3}} B_{T_1}\right)^2} \quad \text{(I-9)}$$

By inserting the nominal values and bias limits of the measured parameters from Table III into Eq. (I-9), a bias limit of 0.6987 lb/sec is obtained for a nominal engine airflow of 248.23 lb/sec.

Table III Flow Data

$W_a = F_A C^* A_{eff} P_1 / \sqrt{T_1}$

Parameter	Units	Nominal	Precision Index (One Standard Deviation)	Bias Limit
F_A	—	1.00	0.0	0.001
C^*	$\frac{\text{lbm } R^{1/2}}{\text{lb sec}}$	0.532	0.0	0.000532
A_{eff}	in.^{2}	296.	0.148	0.592
P_1	psia	36.8	0.05	0.05
T_1	°R	545.	0.3	0.3
..W_a	$\frac{\text{lbm}}{\text{sec}}$	248.23	0.3658	0.6987

To propagate nonsymmetrical bias limits, the bias limit portion of the analysis must be completed for both the upper and the lower limits. Then, the two results are combined as illustrated in Table II. There is a more detailed illustration of propagation of nonsymmetrical bias limits in Section VIII.

1.5.2 Thrust Specific Fuel Consumption (TSFC)

The goal of any analysis of measurement system errors is to determine the resulting errors in the reduced parameters, for example TSFC, which is calculated as the ratio of fuel flow (W_f) to net thrust (F_N); TSFC = W_f/F_N. Net thrust and TSFC uncertainty calculations are described in Section VII. The technique for relating the errors of measurement to the errors in the reduced parameters is based on a Taylor's Series expansion from the calculus. The Taylor's expression for errors in thrust specific fuel consumption is

$$\Delta TSFC = \frac{\partial TSFC}{\partial W_f} \Delta W_f + \frac{\partial TSFC}{\partial F_N} \Delta F_N = \frac{1}{F_N} \Delta W_f - \frac{W_f}{F_N^2} \Delta F_N \qquad \text{(I-10)}$$

Where $\partial TSFC/\partial W_f$ and $\partial TSFC/\partial F_N$ are the partial derivatives of thrust specific fuel consumption with respect to fuel flow and net thrust. The precison index is approximated by

$$S_{TSFC} = \sqrt{\left(\frac{\partial TSFC}{\partial W_f} S_{W_f}\right)^2 + \left(\frac{\partial TSFC}{\partial F} S_F\right)^2} = \sqrt{\left(\frac{1}{F} S_{W_f}\right)^2 + \left(\frac{-W_f}{F^2} S_F\right)^2} \qquad \text{(I-11)}$$

For example, the following hypothetical data were used to estimate thrust specific fuel consumption uncertainty:

Parameter	Nominal	Bias Limit	Precision Index	Degrees of Freedom	Uncertainty Limit
Thrust (F_N)	10,000	18.1 lbf	37.8 lbf	57	93.7 lbf
Fuel Flow (W_f)	10,000	50 lbm/hr	50 lbm/hr	60	150 lbm/hr

The nominal thrust specific fuel consumption is calculated from W_f/F_N:

$$\frac{W_f}{F_N} = \frac{10{,}000}{10{,}000} \frac{\text{lbm/hr}}{\text{lbf}} = 1.0 \text{ lbm/lbf-hr}$$

The precision index of thrust specific fuel consumption is

$$S_{TSFC} = \sqrt{\left(\frac{1}{F_N} S_{W_f}\right)^2 + \left(\frac{-W_f}{F_N^2} S_{F_N}\right)^2} = \sqrt{\left(\frac{50}{10{,}000}\right)^2 + \left(\frac{-10{,}000}{10{,}000^2} \times 37.8\right)^2}$$

$$= \pm 0.0063 \text{ lbm/lbf-hr}$$

The propagation formula is similar for bias

$$B_{TSFC} = \sqrt{\left(\frac{\partial TSFC}{\partial W_f} B_{W_f}\right)^2 + \left(\frac{\partial TSFC}{\partial F_N} B_{F_N}\right)^2} \qquad \text{(I-12)}$$

$$B_{TSFC} = \sqrt{\left(\frac{1}{F_N} B_{W_f}\right)^2 + \left(\frac{-W_f}{F_N^2} B_{F_N}\right)^2} \qquad \text{(I-13)}$$

$$B_{TSFC} = \sqrt{\left(\frac{50}{10{,}000}\right)^2 + \left(\frac{-10{,}000}{10{,}000^2} 18.1\right)^2}$$

$$= \pm 0.0053 \text{ lbm/lbf-hr}$$

The degrees of freedom for the TSFC precision index can be found using the Welch-Satterthwaite technique. In this situation, the partial derivative weighting factors, which are used in the calculation of the precision index, must also be used in the Welch-Satterthwaite formula. Note: The calculation is carried out to illustrate the use of the partial derivatives with the Welch-Satterthwaite. It is not necessary to calculate the degrees of freedom for TSFC since the degrees of freedom for thrust and fuel flow are 57 and 60, respectively. The expected minimum result would be 57. The t multiple is essentially 2.0 for degrees of freedom greater than thirty (Section 1.4). When the degrees of freedom for each component are greater than 30, the Welch-Satterthwaite procedure can be omitted and t = 2.0 can be used.

$$df_{TSFC} = \frac{\left[\left(\frac{\partial TSFC}{\partial W_f} S_{W_f}\right)^2 + \left(\frac{\partial TSFC}{\partial F_N} S_{F_N}\right)^2\right]^2}{\frac{\left(\frac{\partial TSFC}{\partial W_f} S_{W_f}\right)^4}{df_{W_f}} + \frac{\left(\frac{\partial TSFC}{\partial F_N} S_{F_N}\right)^4}{df_{F_N}}} \quad \text{(I-14)}$$

$$= \frac{\left[\left(\frac{1}{F_N} S_{W_f}\right)^2 + \left(\frac{-W_f}{F_N^2} S_{F_N}\right)^2\right]^2}{\frac{\left(\frac{1}{F_N} S_{W_f}\right)^4}{df_{\dot{W}_f}} + \frac{\left(\frac{-W_f}{F_N^2} S_{F_N}\right)^4}{df_{F_N}}} \quad \text{(I-15)}$$

$$= \frac{\left[\left(\frac{1}{10{,}000} \times 50\right)^2 + \left(\frac{-10{,}000}{10{,}000^2} \times 37.8\right)^2\right]^2}{\frac{\left(\frac{1}{10{,}000} \times 50\right)^4}{60} + \frac{\left(\frac{-10{,}000}{10{,}000^2} \times 37.8\right)^4}{57}}$$

$$= 110$$

The t value is 2, and the uncertainty is

$$U = \pm(B + t_{95}S) = \pm[0.0053 + (2.0)(0.0063)] = \pm 0.0179 \text{ lbm/lbf-hr}$$

The results of the error analysis are presented in Table IV.

The uncertainty limit as a percent of the nominal value may be calculated by dividing the uncertainty limit in engineering units by the corresponding nominal value and then multiplying by 100.

The propagation of error formulas used in this section are derived and discussed in Appendix B.

Table IV Uncertainty Components

Parameter	Nominal Value	Bias Limit	Precision Error	Degrees of Freedom	Uncertainty
Thrust, F_N	10,000 lbf	18.1 lbf	37.8 lbf	57	93.7 lbf
Fuel Flow, W_f	10,000 lbm/hr	50 lbm/hr	50 lbm/hr	60	150 lbm/hr
Thrust Specific Fuel Consumption	1.0 lbm/lbf-hr	0.0053 lbm/lbf-hr	0.0063 lbm/lbr-hr	110	0.018 lbm/lbf-hr

1.6 MEASUREMENT PROCESS

In making uncertainty analyses, definition of the measurement process is of utmost importance. Uncertainty statements are based on a well-defined measurement process. A typical process is the measurement of thrust specific fuel consumption (TSFC) for a given gas turbine engine at a given test facility. The uncertainty of this measurement process will contain precision errors due to variations between installations, test stands, and measurement instruments. This uncertainty will be greater than the uncertainty for comparative tests to measure TSFC on a single test stand for a single engine, a different measurement process. The single stand, single engine, back-to-back test would assume that most installation and calibration errors would be biases rather than precision errors. Biases may be ignored in comparative testing in that the same equipment is used for all testing, and biases do not affect the comparison of one test with another (the test objective being to determine if a design change is beneficial). The single stand, single engine model and other comparative tests are treated in Section 8.3.

Because the definition of the measurement process is a prerequisite to defining the mathematical model, all the elemental bias and precision error sources which affect the measurements must be listed. Then, it must be determined how the bias and precision errors are related to the engine performance parameter. Based on this defined measurement process, the errors may be biases or precision errors.

The bias and precision errors related to the defined measurement process for thrust specific fuel consumption are listed in Section II. Uncertainty analyses should be repeated periodically. Continuous validation is essential.

1.7 REPORTING ERROR

The definition of the components, bias limit, precision index, and the limit (U) suggests a format for reporting measurement error. The format will describe the components of error, which are necessary to estimate further propagation of the errors, and a single value (U) which is the largest error expected from the combined errors. Additional information, degrees of freedom for the estimate of S, is required to use the precision index. These numbers provide all the information necessary to describe and use the measurement error. The reporting format is:

1. S, the estimate of the precision index, calculated from data.
2. df, the degrees of freedom associated with the estimate of the precision index (S).

3. B, the upper limit of the bias error of the measurement process or B^- and B^+ if the bias limit is nonsymmetrical.

4. $U = \pm(B + t_{95} S)$, the uncertainty limit, beyond which measurement errors would not reasonably fall. The t value is the 95th percentile of the two-tailed Student "t" distribution.

5. U, the interval between $U^- = B^- - t_{95} S$ and $U^+ = B^+ + t_{95} S$. These limits should be reported when the bias limit is nonsymmetrical.

The model components, S, df, B, and U, are required to report the error of any measurement process. As recommended in Section 1.4, for simplification, the first three components may be relegated to the detailed sections of uncertainty reports and presentations. The first three components, S, df, and B, are necessary to propagate the errors further, to propagate the uncertainty to more complex parameters, and to substantiate the uncertainty limit.

1.8 TRACEABILITY

In recent years the demanding requirements of military and commercial aircraft have led to the establishment of extensive hierarchies of standards laboratories within the military and the aerospace industry. The NBS is at the apex of these hierarchies, providing the ultimate reference for each standards laboratory. It has become commonplace for Government contracting agencies to require contractors to establish and prove traceability of their measurements to the NBS. This requirement has created even more extensive hierarchies of standards within the individual standards laboratories. At each level of these hierarchies, formal calibration procedures are used. These procedures not only define calibration methods and intervals but also specify just what information must be recorded during a calibration, i.e., meter model, serial number, calibration date, etc., in addition to actual measurement data.

The measurement process takes place over a long period of time. During this period, many calibrations occur at each level. Therefore, the precision errors of each comparison are precision errors affecting the measurement process. The overall effect on the measurement of force is a random (precision error) one. Therefore, the resultant overall precision index is the root-sum-square of the individual precision indices. For each comparison, the resultant calibration value is usually the average of several readings. The associated precision index would be a standard error of the mean (or standard error of estimate) for that number of readings. The precision index is

$$S = \sqrt{s_1^2 + s_2^2 + s_3^2 + s_4^2} \qquad \text{(I-16)}$$

for four steps in the calibration process.

The degrees of freedom for each precision index may be combined using the Welch-Satterthwaite formula to provide an estimate of the degrees of freedom for the combined precision index.

$$df = \frac{\left(s_1^2 + s_2^2 + s_3^2 + s_4^2\right)^2}{\frac{s_1^4}{df_1} + \frac{s_2^4}{df_2} + \frac{s_3^4}{df_3} + \frac{s_4^4}{df_4}} \qquad \text{(I-17)}$$

This technique was simulated for various sample sizes and provides the best known estimate of the equivalent degrees of freedom. The results of the simulation and the further use of the technique are discussed in CPIA No. 180 (AD855130). If non-integral values of df result from the Welch-Satterthwaite estimate, appropriate Student's t values can be found by interpolating from the table in Appendix E.

The unknown bias error limit for the end instrument is usually a function of many elemental bias limits, perhaps ten or twenty. It is unreasonable to assume that all of these biases are cumulative. There must be a cancelling effect because some are positive and some are negative. For this reason, the arbitrary rule that the bias limit B will be the root-sum-square of the elemental bias limit estimates was adopted:

$$B = \sqrt{b_1^2 + b_2^2 + b_3^2 + \cdots + b_L^2} \qquad \text{(I-18)}$$

where L is the number of sources of bias

In combining elemental nonsymmetrical bias limits, the upper limits should be root-sum-squared to determine the combined upper limit. The lower limits should be root-sum-squared to determine the combined lower limit. The resulting will be nonsymmetrical bias limits. An example of an error analysis containing nonsymmetrical bias limits is given in Section VIII.

The uncertainty in the measurement instrument due to calibration is calculated using the uncertainty formula:

$$U = \pm(B + t_{95}S) \qquad \text{(I-19)}$$

where S is the precision index calculated from Eq. (I-16).

List of References on Statistical Quality Control Charts

Basic References

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