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**HANDBOOK  
UNCERTAINTY IN GAS TURBINE MEASUREMENTS**

**Dr. R. B. Abernethy et al.  
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**and**

**J. W. Thompson, Jr.  
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**Approved for public release; distribution unlimited.**

## FOREWORD

The work reported herein was sponsored by the Arnold Engineering Development Center, Air Force Systems Command, United States Air Force, under Program Element 65802F.

The results presented were compiled by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), Arnold Air Force Station, Tennessee, under Contract F40600-73-C-0004. The preparation of the text was accomplished by Dr. R. B. Abernethy, Senior Project Engineer, Billy D. Powell, David L. Colbert, and Daniel G. Sanders, Pratt & Whitney Aircraft, under subcontract to ARO, Inc. The contracted work consisted of a revision to the material in the "Interagency Chemical Rocket Propulsion Group (ICRPG) Handbook for Estimating the Uncertainty in Measurements made with Liquid Propellant Rocket Engine Systems," CPIA Publication No. 180 (same authors as above), substituting treatment of gas turbine measurement errors for rocket engine treatment and writing additional material applicable to gas turbine measurement errors. The report was prepared under ARO Project No. RW5245, and the manuscript was submitted for publication on May 8, 1972.

The authors are indebted to the many engineers and statisticians who have contributed to the work. A few must be noted for their particular contributions, Dr. Joan Rosenblatt, Dr. H. H. Ku, and J. M. Cameron of the National Bureau of Standards for their helpful discussions and comments on both this handbook and CPIA 180, and similarly, R. E. Smith, Jr., Chief of T-Cells Division, as well as T. C. Austin, C. R. Bartlett, W. O. Boals, Jr., and T. J. Gillard of ARO, Inc., at the Arnold Engineering Development Center. Engineers at Pratt & Whitney Aircraft, Florida and Connecticut facilities, provided the authors with constructive and spirited criticism in every section. Various technical committees under the American Society of Mechanical Engineers (ASME), the American Institute of Aeronautics and Astronautics, and the International Standards Organizations expressed interest and comments.

This technical report has been reviewed and is approved.

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Author's Update:

January 1980

"The reception of this handbook far exceeded our expectations when it was published in 1973. Many of the applications are a direct result of the I.S.A. Tutorials on Test Measurement Accuracy given at the International Instrumentation Symposia in 1975, 1977 and 1979. A few examples include the U.S. Naval Air Propulsion Test Center, the Royal Canadian Air Force at Cold Lake, Alberta, and the U.S. National Engineering Laboratory. Our corresponding handbook for rocket engines published by the ICRPG (JANNAF), also continues as a well used standard. Three committees have adopted the AEDC handbook as the foundation of new standards now being drafted:

- o The American Society of Mechanical Engineers Committee on the Measurement of Fluid Flow in Closed Conduits has a draft ASME/ANSI standard, "Fluid Flow Measurement Uncertainty" in process.
- o The Society of Automotive Engineers Committee E33C, Aircraft Propulsion Measurement Uncertainty, adopted the model and is drafting as SAE Aerospace Recommended Practice.
- o The International Organization of Standardization (ISO) has adopted this model for their new international standard on fluid flow measurement and calibration uncertainty in process.

A fourth committee, A.S.M.E. Performance Test Codes 19.1, is reviewing the AEDC handbook as a possible basis for their proposed draft ASME/ANSI standard, "Performance Measurement Uncertainty."

The handbook is referenced in several British uncertainty standards and is under consideration as a text at the Technion Institute in Isreal.

During these years we have continued our research and Monte Carlo comparisons with other models. Our conclusion is that this model and method is the best technology available today.

The authors would like to express their appreciation for the continued help and counsel that they receive from Dr. Joan Rosenblatt, Dr. Harry Ku and Mr. Cameron of the National Bureau of Standards. The original model is described in NBS Handbook 91 and NBS Special Publication 300."

We welcome comments and invite questions and criticisms.

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