

## **Problem Of The Month**

### **April 2001—Weibull Beta Slopes For Ball Bearings**

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What's the correct Weibull beta value to characterize the life distribution for ball bearings? Weibull equations, named after [Waloddi Weibull](#) (1887-1979), are considered the statistical method of describing the variable life of "identical" ball bearings tested under identical conditions as a statistical equation. You need an idea of the Weibull distributions for modeling purposes.

The life of a ball bearing for a given size, load, speed, lubrication, etc can be characterized by the time  $t = \eta * (\ln(1/(1-F(t))))^{1/\beta}$  where  $\eta$  is the characteristic life,  $\beta$  is the slope of the Weibull distribution, and  $F(t)$  is the cumulative distribution function. [The New Weibull Handbook](#), 4<sup>th</sup> edition by Dr. Robert B. Abernethy describes the Weibull equation.

The typical rating for the life of a ball bearing is determined at the  $L_{10}$  or  $B_{10}$ , where L (in English, L=life) or B (in German, B= "Bruchzeit"), the life at which the initial crack appears in a ball bearing or as Fred Geitner says:

Bruch = breakage/fracture,  
Einleit(ung) = initiation/introduction,  
Zeit = time or the fracture/breakage initiation period or incipient failure time/period.

Other common design percentage values for B or L are described in section 2.8 of [The New Weibull Handbook](#), 5<sup>th</sup> edition, by Dr. Robert B. Abernethy, ISBN-13: 978-0-9653062-3-2 or ISBN-10: 0-9653062-3-2 which was published December 2006. In short,  $L_{10} \equiv B_{10}$ .

The  $L_{10}$  or  $B_{10}$ , refers to the life at which 10% will fail (90% will survive) under a given load. For a typical Weibull beta value, the characteristic life  $\eta$  is roughly 5 times greater than the  $L_{10}$  or  $B_{10}$ , life (see details near the bottom of this web page). You can find the  $\eta$  and  $\beta$  with [WinSMITH Weibull](#) software to construct a probability plot to produce the statistical values on a straight-line plot.

Typical Weibull slope values are shown in [databases](#), and many people ask why the database has a smaller beta value than often cited. My clients tend to mix water, chemicals, and other nasty stuff with their lubricating oils, which adds scatter to the distributions of life and thus produce flatter (smaller) beta values than usually expected. The database shows  $\beta = 1.3$  (for reasons noted above) whereas the values below will show  $\beta = 1.43$ .

In the mid 1950's a controversy was underway about the exponents to use in the Lundberg/Palmgren bearing equation. The Lundberg/Palmgren bearing equation specifies the life at which 10% of the population will fail (90% will survive)  $L_{10} =$

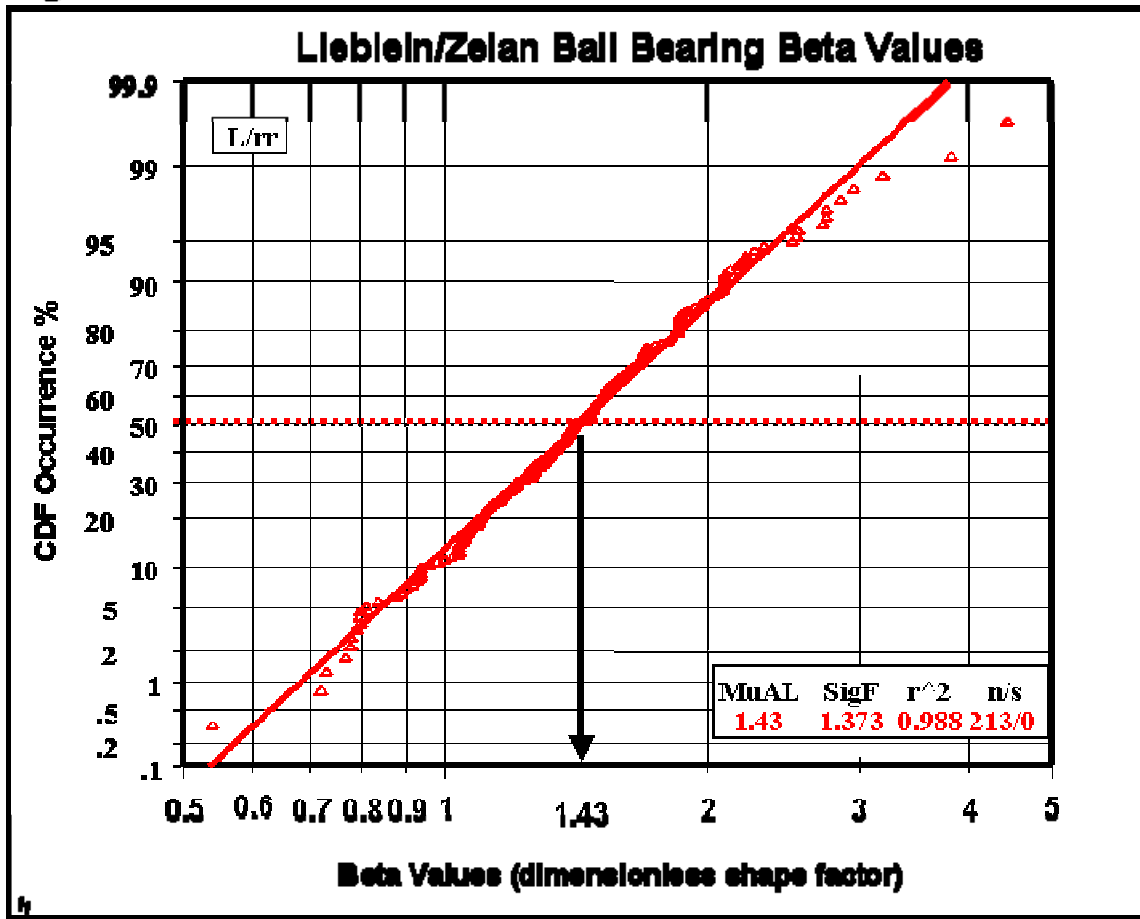
$((C/F)^P) \cdot 10^6$  revolutions where  $C$  = dynamic capacity of the bearing, and  $F$  = equivalent load. The outcome of the analysis showed  $P = 3$  for ball bearing and  $P = 3.333$  for roller bearings. The 44 page report by Lieblein&Zelen concerning the statistical life of deep groove, radial, ball bearings (which is not so easy to retrieve from the [NIST](#)) is available for download as a [PDF file](#) (2.8 Meg file) from this website.

When the bearing load,  $F$ , shows  $F > \sim 10$  to 15% of the  $C$  value in the Lundberg/Palmgren equation, the bearing is **highly loaded** and the bearing life will be very short!! So don't get greedy! Most machine design books do not flag this criterion as a problem.

Bearing load limits are real. More information is available from the [SKF Interactive Engineering Catalogue](#) (please note this website requires your browser to be Java enabled) accessible from the Internet. Also remember when bearings are too lightly loaded, the balls or rollers will skid and this also contributes to very short bearing life—this is the reason bearings on aircraft landing gear are heavily preloaded as they go from full rest to full RPM in a few milliseconds when an airplane lands and this preload prevents the bearings from skidding. The SKF electronic manual refers to two other criteria for extending  $L_{10}$  or  $B_{10}$ , life by consideration of oil contamination factors (super filtered oil for use in oil mist systems have demonstrated far longer lives just as occurs when magnetic oil shields are employed to prevent incursion of contaminants from outside sources into the lubricant cavities), and a  $P_u$  factor which is the limited equivalent load for achieving greatly extended bearing life (probably relates the to three parameter Weibull equation with a  $t_0$  shift in life by which you have a failure free interval as it takes a specified minimum number of revolutions before any bearing failure can occur).

The Lieblein/Zelen report summarized tests to failure carried out over a period of years by four major ball bearing manufacturers on endurance test of bearings with inclusion of the suspended data on test terminated before all bearings in the test group failed. The report contains 213 data values (from four different ball bearing manufacturers) for the Weibull slope values  $\beta$ . The Weibull slope value  $\beta$  is related to dispersion in the life data, and  $\beta$  varied from a minimum value of 0.54 to a maximum value of 4.44! The data are shown in Figure 1 as a log normal distribution. The log normal is the best statistical distribution for beta values from Weibull distributions of life data based on a distributional analysis by [WinSMITH Weibull](#) software.

**Figure 1**

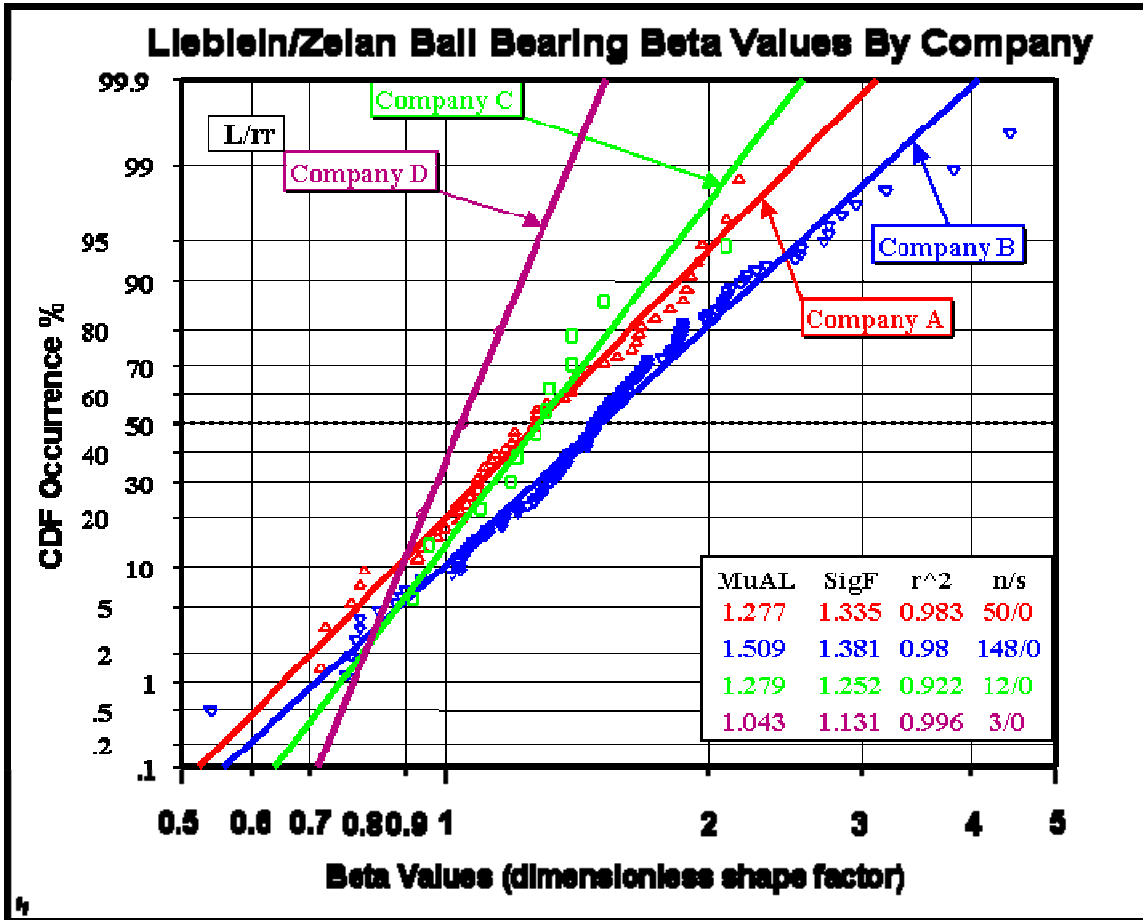


You can download the Excel file used for Figure 1 with the summary data from the Lieblein/Zelan report listed as [Ball Bearing Data by Lieblein and Zelan.XLS](#). Notice the beta values in Figure 1 at ~2.5 and above seem to be indicating a different type of failure mode [because they deviate from the straight line] than the remainder of the data as most of the 90% confidence outliers will also fall outside of the boundaries compared to only one point in the lower left hand corner of the plot. The coefficient of determination for the merged data from four suppliers in Figure 1 is 0.988. The critical value for Figure 1 is 0.9882, which for practical purposes, says we have a good curve fit when the r<sup>2</sup> values are rounded to three decimal places.

If the combined data in Figure 1 is plotted on a Weibull plot the data is obviously not on a straight line with each end of the data set clearly trending downward to form a concave downward plot and suggest a  $t_0$  correction is needed. The curve fit is a miserable 0.968 compared to the critical value of 0.99 for  $t_0$  conditions, which clearly says the three parameter Weibull, for these combined values, is obviously a poor choice compared to the log normal plot in Figure 1 for the combined beta values. This says the “right” value for  $\beta = 1.43$ .

Figure 2 shows the fatigue test data from bearing manufactures for Weibull beta values in Figure 1 by individual manufacturer. Notice the quantity of test results in each data set is substantially different. The data, for values above beta ~2.5, comes from Company B with the largest quantity in the data set.

**Figure 2**



The probability plot in Figure 2 results in another question: Are the differences in the statistical results of eta and beta of meaningful difference or just different due to noise on the signal?

Figure 3 helps answer the question of significant differences by use of the likelihood ratio test. The likelihood ratio test uses confidence intervals in WinSMITH Weibull software to produce contour lines for each set of data. Data sets of significant difference do not overlap contours. For engineers, this is an obvious test of significant differences.

**Figure 3**

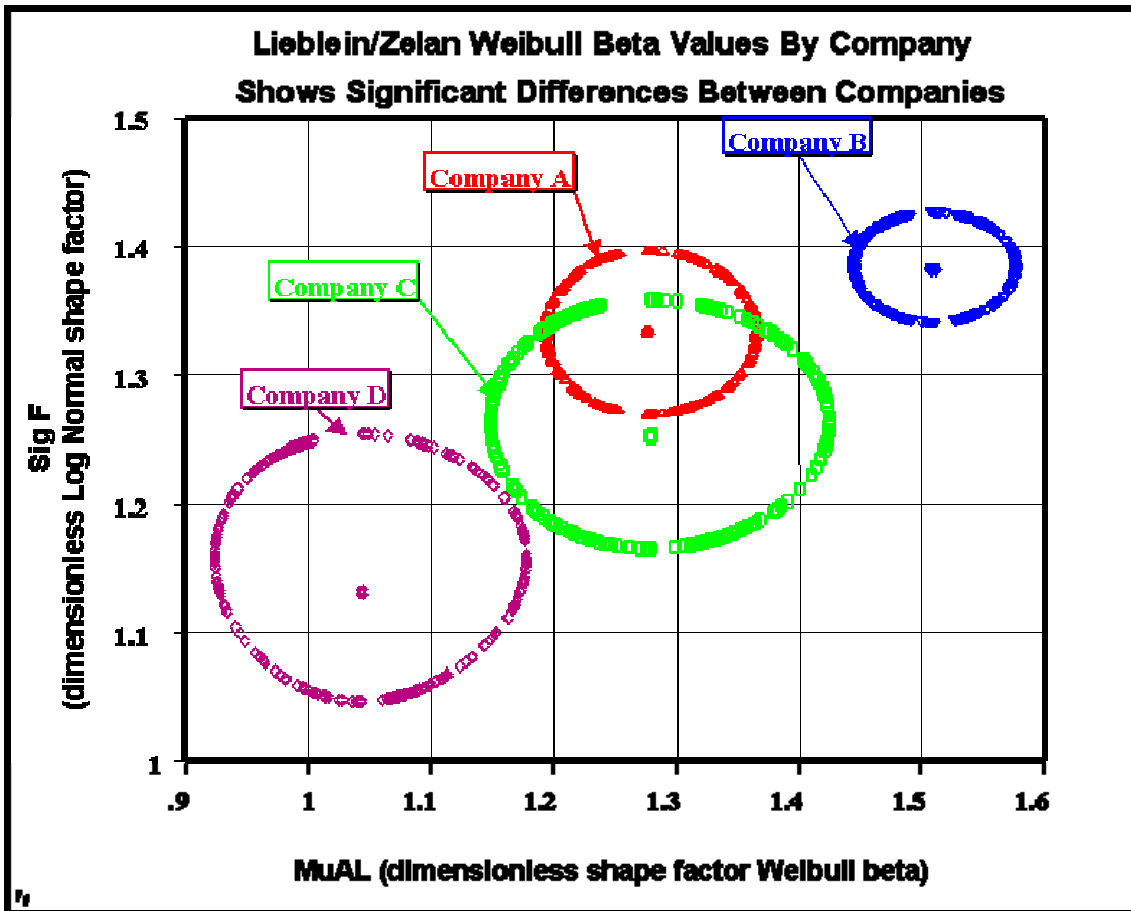


Figure 3 shows Company A and Company C lack significant differences in the data. Company B and Company C have significant differences between Company A/C and themselves. This suggests only the data from Company A and Company C could be merged rather than lumping all four data sets together.

Figure 3 says the performance of ball bearings from different manufacturers can have different results. Figure 3 validates the differences between manufacturers you suspected but couldn't prove in your operating environment. Liebein & Zelan's document, [see PDF \(2.8 Meg file size\)](#), refers to "workmanship" factors,  $a_0$  or  $f_c$ , for each bearing and each company (see page 313 of the document or page 41 of 44 listed for the PDF file). Note that Figure 3 shows a data point in the "middle" of the contours which represents the "top of the mountain" for the maximum likelihood estimate and these are the same statistics reported in Figure 2 which says the Weibull beta can vary between 1.043 and 1.509 depending upon manufactures---thus the real number for your case may line between these values rather than having the "best" single point estimate of 1.43 shown in Figure1.

So you'll probably ask, do I plan to change the value of 1.3 in the database? No, it's still an OK number for representing data from my clients who use bearings from all manufacturers.

Does the Weibull beta value for ball bearing represent the same approximate value for tapered roller bearings, angular contact bearings, etc.? Probably—maybe the manufacturer of these bearing can provide some specific details.

**API Standard 610: Centrifugal Pumps for Petroleum, Heavy Duty Chemical, and Gas Industry Services**, Table 2.7-Bearing Selection, contains a footnote concerning rolling element bearing life and specifically the  $L_{10}$  or  $B_{10}$  must have a basic rating of “...at least 25,000 hours with continuous operation at rated conditions, and at least 16,000 hours at maximum radial and axial loads and rated speed” for centrifugal pumps requires a minimum  $L_{10}$  or  $B_{10}$  life for the antifriction bearings supporting the shaft.


Given the API minimum  $L_{10}$  or  $B_{10}$  requirements, what minimum  $\eta$  Weibull characteristic life should you expect? The amount of life at  $L_{10}$  or  $B_{10}$  gives a point location on the Weibull probability plot. If  $\beta$  the Weibull slope is known, then the characteristic life  $\eta$  can be calculated.

The summary data from the Lieblein/Zelen report listed as [Ball Bearing Data by Lieblein and Zelan.XLS](#), includes a calculated value for a multipliers to find the Weibull characteristic life. This multiplier was not included in the original Lieblein/Zelen’s report. The last column of the spreadsheet shows the multiplier (i.e., multiply the  $L_{10}$  or  $B_{10}$  life by a factor to get the calculated  $\eta$  value). Calculated values for each Lieblein/Zelen test result multiplication factors is shown in Figure 4 as a probability plot—please note the best curve fit is a two-parameter log normal and it is poor fit for a straight line. The infant mortality failure modes where  $\beta < 1$  result in multipliers greater than 9.5 and multipliers smaller than  $\sim 3.5$  are the result when  $\beta > 2$  for a strong wear-out failure signal. The lognormal plot shows a central tendency of 5.22 and the median calculation in Excel is 4.93. Thus a reasonable rule of thumb (without decimal points) for the multiplier is  $\sim 5 * L_{10} = \eta$  (it is **NOT** the often referred value of  $\sim 10 * L_{10} = \eta$  which assumes the bearing failure mode is exponential rather than a wear out failure mode).

Thus the API Standard 610 Weibull characteristic life  $\eta$  is  $5 * 25,000 = 125,000$  hours for continuous conditions or  $5 * 16,000 = 80,000$  hours under maximum load conditions. At the characteristic value,  $\eta$ , 63.2% of the population will have failed and 36.8% will survive.

When the quantity of bearings tested is larger than 20 pieces (some say greater than 100 pieces) you can begin to see the life data fit a three-parameter Weibull distribution. The 3-P Weibull distributions show the failure-free zone as a properly designed and loaded rolling element bearing require a large number of revolutions to form a bearing crack that results in bearing failure. These minimum number of revolutions describe the failure-free zone and the Weibull curve is concave downward when plotted in the 2-parameter Weibull format. Likewise, many bearings are destroyed by sitting on the shelf awaiting use, and the destruction is caused by false brinnelling and by corrosion cells forming as the bearings sit and wait—these “prefailed” issues result in a concave upward Weibull curve when plotted in the 2-parameter Weibull format.

Refer to the caveats on the [Problem Of The Month Page](#) about the limitations of the following solution. Maybe you have a better idea on how to solve the problem. Maybe you find where I've screwed-up the solution and you can point out my errors as you check my calculations. E-mail your comments, criticism, and corrections to: Paul

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