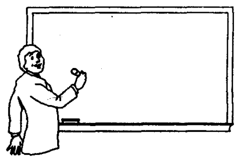

Weibull News

Fifth Edition

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Fall 1991

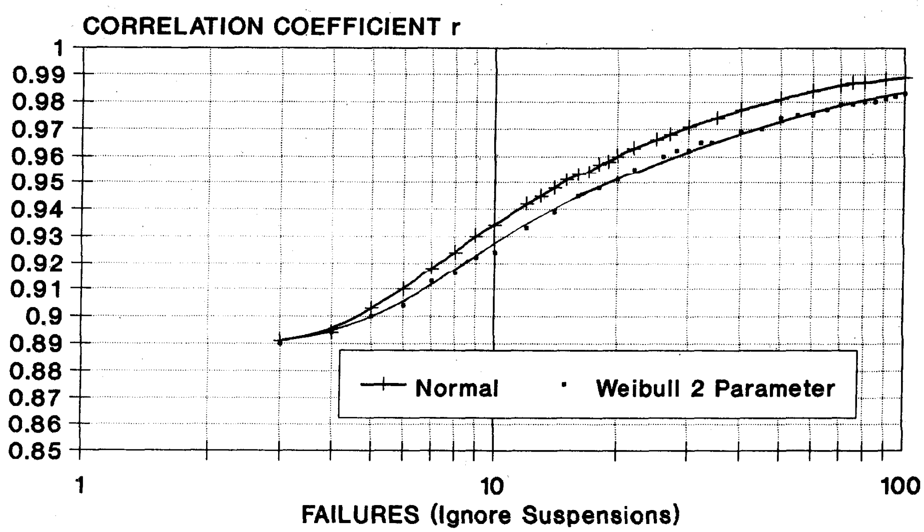
Weibull Research... Two areas that need improvement are (1) a better measure of goodness-of-fit for the Weibull plot and (2) confidence statements for Weibull data sets that have suspensions. I believe you will find these preliminary results useful. This study is based on special software Wes Fulton developed. We have been busy so we hope you will understand why the fifth edition is a bit tardy.

Weibull & Log Normal Goodness-of-Fit... How good is your fit? Chi-squared and Komogorov-Smirnoff tests will work but there is an easier way. The correlation coefficient is a simple measure of the strength of a linear relationship and would be ideal except we build in correlation when we assign median rank plotting positions. Rigorously, it is an illegal application but almost everyone uses it. The correlation coefficients are artificially too high. All probability plots, normal, log normal, Weibull, etc. suffer this deficiency, but we now have a solution.

Wes produced a program called MonteCarloSMITH (just for me)...and I have been having lots of fun with it. My approach was to use MonteCarloSMITH to produce a thousand random Weibull data sets for each sample size. In my PC the sets were plotted and the correlation coefficients calculated and ranked from smallest to largest. From the results, I have plotted the lower 90th percentile as an estimate of the lower 90% confidence bound for the correlation coefficient, given that the data is Weibull distributed. To use this plot, compare your coefficient with the value from the curve. If your correlation coefficient equals or exceeds the critical value on the curve, you have a "good" Weibull fit. I used 1000 trials for each point. Other statisticians applied the same technique for normal probability paper so I plotted their 90th percentile also. I believe the normal critical coefficients also apply to the Log Normal. I will to verify this later. The normal results are based on 120,000 trials and they used Blom's plotting position instead of median ranks but that should be a small difference. Note my old rule-of-thumb of 0.95 was not too bad. Future research will be reported in the New Weibull Handbook on three parameter Weibulls and Log Normals. Wes will expand MonteCarloSMITH to allow this research.

Critical Correlation Coefficient

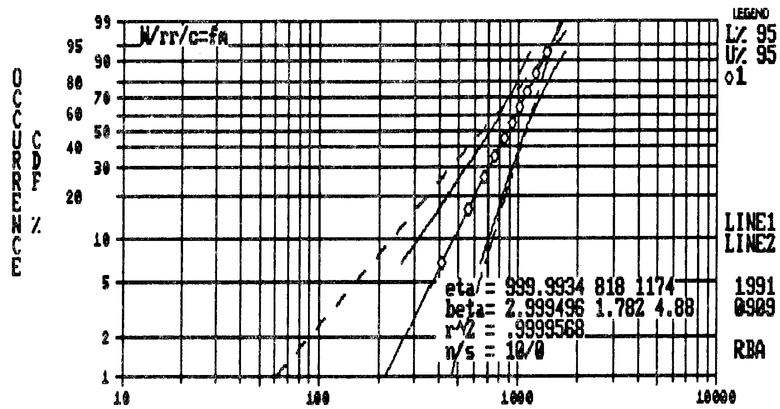
Normal (Blom) & Weibull (Johnson)
Probability Plotting - 90% Confidence



•Am.Stat.Feb.'85 & ASQC Rel.Rev.Jun.'90

Weibull Confidence Intervals... Beta-Binomial confidence bounds are used in WeibullSMITH™. Some suggest Wayne Nelson's bounds based on Fisher's Incomplete Matrix might be better. Others including Wayne, suggest the Likelihood Ratio method described in Gerald Lawless's text may have advantages. Wes has programmed a special WiebullSMITH that has all three as options. Together with MonteCarloSMITH this gives us the tools to make valid comparisons...and I am in the middle of it. Very preliminary results indicate that for rank regression Fisher's Matrix is a little better for complete samples and much better for suspensions. And it is almost instantaneous! I have not looked at Maximum Likelihood or Log Normal as yet. "Better" to me means close to 90% of the Weibull "B" lives should be within the 90% confidence bounds. Should these other confidence methods be added as options to WeibullSMITH™? Wes & I need your opinion. More on this research later in the New Weibull Handbook or next Newsletter.

NELSON'S FISHER MATRIX BOUNDS



MONTECARLOSMITH LINES 50TH 950TH RANKED ETAS AND BETAS

Comparing Two Weibull Datasets... None of the three confidence methods are satisfactory for determining if two datasets come from the same distribution when we have random suspensions. However, the 50th and 950th Betas & Etas (from MonteCarloSMITH) plotted as two lines are excellent for this purpose. See Figure. The procedure is input your beta, eta, suspension histogram & sample size into MonteCarloSMITH. Output ranked 50th & 950th Etas and Betas. Plot these two lines on your Weibull plot using the **Only Line** option. If the second Weibull dataset lies within the two Beta/Eta lines (below the B60 line), the two Weibull sets are not significantly different and may be **Merged**. However, if the second Weibull intersects either Beta/Eta line (below B60), the two sets are significantly different. If there is a much demand for this test, Wes will make MonteCarloSMITH available to you. Again please let us know. MonteCarloSMITH can also provide confidence bounds for Beta, Eta, any "B" life, r and r squared, with and without suspensions, MLE or RR.

New WeibullSMITH™ Capabilities...■ One large title on your plots or two small titles. ■ Hard copy output to Laser or plotters directly from the plot... hit "H". ■ A table of ranked data and parameters like Beta, Eta, B lifes with confidence bounds ■ Plots may have a parameter inset or a Beta-B life inset or no inset. ■ The Probit Inspection Data option will now accept % and fixed sample inspection size. ■ WeibullSMITH™ will run on a **MAC** using the "SoftPC" program.

Weibull Rally For Weibull Workshop **Alumni**, Wes Fulton and I will sponsor a Weibull Rally on an advance Weibull technology, selected case studies, consulting on your problems, and a presentation of new WeibullSMITH software. Plan on two days in the spring in the Palm Beaches. **Call me** if you want to reserve one of the limited spaces.
Public Weibull-LogNormal Workshops... ■ **UTSI** (615) 455 0631, 4 Day Workshop, Palm Beach, October 22-25, 1992 ■ ASME (212) 705 7123, 2 Day Weibull & 3 Day Measurement Uncertainty Workshops, Phoenix, January 16-17, 1992 ■ SAE (412) 776 4841, 4 Day Workshop, Detroit, February 25-28, 1992.

Reliability Assurance Index... As you know I always recommend avoiding confidence intervals whenever possible. Part of the reason is the confusion over how to select the confidence level a priori. Wes Fulton has come up with an interesting solution that I believe has merit. Please read his paper which is attached.

New Weibull Handbook... The third chapter is almost complete. One chapter will be devoted to case studies from many different industries. Please send me an interesting case study, 1-3 pages, simple prose with Weibull plots. I will return the my final edited version for your review. We will use your letterhead with your byline. You will help young novice students around the world. In return, I will provide you with a copy of the **New Handbook**. Call me to discuss this. I would like to use the same examples in our Weibull Rally and our computer tutorial for WeibullSMITH™ unless you object.

Call me at (407) 842 4082... Dr. Robert B. Abernethy (407) 842 4082 (Florida time).
For software questions, call Wes Fulton at: (213) 548 6358 (California time).

ASSURANCE INDEX SIMPLIFICATION OF RELIABILITY WITH CONFIDENCE

by Wes Fulton

23 September 1991

ABSTRACT: Reliability with confidence is explored. The co-specification of equal reliability and confidence, called "assurance", is found to have many benefits.

"Confidence" conjures up a mystic aura with many statistics, probability, reliability and quality control people. It can be explained loosely as the average frequency of being correct when making a statement. By coupling confidence to reliability in a simple fashion, the mystery can be dispelled.

Military and commercial specifications require ample arguments to support a manufacturer's claim of high reliability. They even require substantiation that the claim itself is intrinsically trustworthy by attaching a minimum confidence value to the reliability requirement. This can produce confusion among prospective suppliers who begin the process of reliability demonstration without a grasp of its meaning. Dr. Robert B. Abernethy sums up confidence in his seminars by saying "Avoid the use of confidence intervals when the parameter of interest is a probability. Two probability-like numbers in one sentence is one more than most of us can handle." Usually there is no solid rationale given for the selection of reliability requirement let alone that for confidence.

Most people would agree that 90 percent confidence about a statement is good. Then 95 percent confidence is better, right? Wrong if you are the manufacturer, because higher confidence requires more substantiation which means more testing, more analysis, more cost. Is 80 percent confidence enough? Is 95 percent confidence too much? There is a straightforward answer. Make the confidence requirement the same as reliability and call it "assurance".

A single specification of 95 percent assurance index would mean the equipment shall have at least 95 percent reliability with 95 percent confidence. For special applications a higher assurance index such as 99 percent may be needed. This would require 99 percent reliability with 99 percent confidence.

The assurance index for specifying high quality designs would help in many ways. First, there is no confusion about why a particular confidence value was chosen. Second, requiring high confidence for high reliability is consistent with respect to failure criticality. Third, it would provide a simple, frugal expression without the need for all the "nines" used in standard reliability-with-confidence specification. Fourth, the a priori selection of confidence is guaranteed, avoiding the most common abuse of confidence intervals. Fifth, it avoids confusion about the demonstrated reliability of competing designs as the following example shows:

Suppose a boastful electronics firm claims to make a 98 percent failure-free computer. When pressed on this claim, the president of the company acknowledges that confidence is 80 percent. That is, the best guess for this computer's reliability is somewhat higher than 98 percent. There is an 80 percent chance that the actual reliability is higher than 98 percent and a 20 percent chance that the reliability is lower. How does this compare with a computer from a quiet electronics firm that rates their product at 97 percent reliability with 95 percent confidence? Which computer is better? The answer requires some assumptions.

If computer failures can be adequately represented by a Weibull distribution, then comparison of the test effort required to demonstrate reliability with confidence will provide insight. It turns out that a lot of aerospace specifications require one component to be tested four lifetimes without failure. This is approximately 95 percent assurance if a Weibull with slope 3.0 is used. Eight lifetimes is approximately 99 percent assurance with Weibull slope 3.0. The 3.0 Weibull slope is standard for early wearout failures and is used here for comparison.

The testing required on one computer to show 97 percent reliability with 95 percent confidence is approximately seven percent more than the testing required for 98 percent reliability with 80 percent confidence. The "lower-reliability" machine happens to be actually better, since it must pass more rigorous testing. This confusion can be avoided by using the assurance index.

The assurance index for the boastful company's computer is based on 4.3 lifetimes and is 96 percent. The quiet company's computer with 4.6 lifetimes of testing would earn 96.6 percent assurance index. Comparing the assurance index gives the clear winner.

A 96.6 percent assurance index indicates a significantly higher requirement than a 96 percent assurance index. For each increase in assurance, both reliability and confidence are tightened. Existing specifications can be restated in terms of assurance to provide a more concise and understandable goal. Measurement of attained assurance then follows naturally.

An assurance line can be added to any cumulative probability distribution plot. There is only one assurance line for any set of measured data. This makes it easy for interpretation and validation of specification requirements. Figure 1 shows a plot with an assurance line as well as the straight fit line.

The co-specification of equal probability and confidence can also apply to probability of defect-free products. Therefore, the assurance index concept may be useful in a wide range of quality control applications as well as engineering.

The underlying relationships for assurance are:

let Φ (PHI) = assurance index in percent

and ϕ (phi) = Φ / 100 = assurance

then for a binomial distribution example...

$$1 - \phi = \sum_{c=0}^{f} C^N_c * \phi \wedge (N-c) * (1 - \phi) \wedge c$$

where C^N_c = combinations of N things taken c at a time
N = total number of data
f = number of failures
* = multiply
^ = to the power
c = summation index

and for Weibull...

$$1 - \phi = \text{beta distribution}$$

More confidence for more critical situations is intuitive. The confidence required for a 99.999999 percent reliable driver's side air bag component should be a lot higher than the confidence required for an 80 percent reliable nail driver. It makes sense to have commensurate reliability and confidence. It makes life easier for statistics gurus and statistics consumers alike.

ADDITIONAL SOURCES OF INFORMATION

1. Dr. R. B. Abernethy, J. E. Brenneman, C. H. Medlin, G. L. Reinman, "Weibull Analysis Handbook AFWAL-TR-83-2079", Published by Pratt & Whitney Aircraft for Air Force Systems Command, November 1983.

3. Nancy R. Mann, Ray E. Schafer, Nozer D. Singpurwalla, "Methods for Statistical Analysis of Reliability and Life Data", Published by John Wiley & Sons, 1974.

2. W. Weibull, "Fatigue Testing and Analysis of Results", published by Pergamon Press for NATO, 1961.

ACKNOWLEDGEMENTS

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THE AUTHOR

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FIGURE 1

Weibull Plot with First Ever 'ASSURANCE' Line

ASSURANCE = CONFIDENCE = RELIABILITY at Each Point on Plot