Weibull NEWS™

The Latest in Life Data Analysis Research ℠

Fourteenth Edition | From: Dr. Bob Abernethy & Wes Fulton | Spring 1999

An Irish Odyssey Leads to Improved Maximum Likelihood Estimates **- by Dr. Bob Abernethy.** Last June after doing a Weibull Workshop for Rolls Royce in Bristol England; my wife, Sally, and I toured Ireland. Except for the rain it was wonderful. One of the high points was a visit to the huge Guinness brewery in Dublin, where Gossett did his famous work on Student's t and later the F test. I came home and reread the story of this brewmaster trying to improve the accuracy of small sample statistical analysis a century ago. His aim was to improve the quality and taste of Guinness, an admirable objective. Like most of us he was forced to use small samples. He focused on maximum likelihood (MLE) of standard deviation (sigma) from a normal distribution. His derivation of the Student's t statistic indicated not to use the MLE of sigma, but rather the square root of an unbiased estimate of variance or SQR(Var-U). This square root is still biased, but much less so than the MLE sigma. When Gossett showed his derivation to Sir Ronald Fisher, there was disagreement. Fisher said to use the MLE sigma. Some years later Fisher changed his mind and agreed that Student's t should be used with SQR(Var-U). To convert an MLE variance to the unbiased version, we multiply by N/(N-1). Today engineers always use this N/(N-1) correction because the sigma estimate is more accurate, even though it does not completely unbias the MLE sigma.

With life data analysis, the small sample Weibull, normal and the log normal MLE estimates are all biased by the probability plot line being too steep and therefore, optimistic. This is the same problem Gossett was working on long ago. For small samples most of the uncertainty is in Weibull beta and normal sigma; Weibull eta and normal mu have much less uncertainty. The thought came to me, if the three distributions have the same disease, would Gossett's medicine help the Weibull the way it helped the normal? Turning to the Table 5.1 in The New Weibull Handbook I found that by dividing the MLE estimates by the same factor, N/(N-1), the bias in the median MLE beta was eliminated! Further, if I multiplied the MLE sigma by the same factor it eliminated that bias too. Well, I was very excited! However, statisticians generally estimate the bias as the mean value minus the true value rather than the median minus the true value. As I redid the analysis using the mean value, the N/(N-1) factor reduced but did not eliminate the bias in either beta or sigma.

Recalling my early days teaching statistical quality control, I remembered the C4 factor, which eliminates the bias in SQR(Var-U). It is given in several texts. For example, see ASTM "Manual on Presentation of Data and Control Analysis," Sixth Edition and Acheson J. Duncan's Text, "Quality Control and Industrial Statistics," Sixth Edition, Irwin. Therefore, the *Reduced Bias Adjustment (RBA)* factor for the normal and log normal standard deviation is the square root of N/(N-1) divided by C4. This produces an unbiased estimate of the standard deviation (S) as follows: (Note: ^ is the symbol for MLE)

$$S_u = \hat{S} * RBA_\sigma$$

$$RBA_\sigma = \left(\sqrt{(N/(N-1))}\right) / (C4)$$

Where:

C4 = SQR(2 / (N - 1)) * ((N - 2) / 2)! / ((N - 3) / 2)!

N = sample size (number of failures only)

The equation presented above works for the normal and the lognormal. I believe it is well-established theory, although has not been applied to life data analysis.

The next problem is the Weibull beta. Trying the same solution did not work. More research showed that empirically the RBA factor for the Weibull beta is C4 raised to the sixth power. If we multiply the MLE beta by (C4)^6 it essentially eliminates the bias in the MLE beta!

The average value of RBA beta in the table is 2.9992! I am excited about these results as it means that mathematically oriented analysts, who prefer MLE, now have a much more accurate solution for small samples of life data. (See the table below.)

$$\beta_{\mathbf{u}} = \hat{\beta} * \mathbf{RBA}_{\beta}$$

$$\mathbf{RBA}_{\beta} = (\mathbf{C4})^{6}$$

Wes Fulton suggested I look at other betas. Using 4,000 Monte Carlo trials, and sample size 4:
at a true beta = 0.70, RBA mean beta = 0.70;
at true beta = 1.0. RBA mean beta = 1.006,
And at true beta = 6.0, RBA mean beta = 6.11…so it appears RBA works at other betas.

Mean Values Applying RBA Factors to Maximum Likelihood Estimates-*Italics*=*Minimum Bias*

MRR – Median Rank Regression   MLE – Maximum Likelihood Estimate

True Weibull β= 3.0, η = 1000, 10,000 Trials   **True Normal μ = 100, σ = 3.0**

MRR β	MLE β	RBA β	N	MRR η	MLE η	MRR μ	MLE μ	MRR σ	MLE σ	RBA σ
4.922	6.467	***3.133***	3	***992.13***	970.09	***100.02***	100.04	3.099	2.221	***3.069***
3.716	4.896	***2.994***	4	***1000.58***	981.36	***100.08***	100.12	3.107	2.422	***3.036***
3.477	4.296	***2.964***	5	***993.97***	986.83	100.05	***100.00***	3.089	2.515	***2.991***
3.313	4.030	***2.991***	6	***994.27***	991.00	***100.00***	99.97	3.080	2.634	***3.032***
3.259	3.820	***2.979***	7	***999.81***	991.50	***99.99***	99.97	3.123	2.663	***2.998***
3.213	3.676	***2.969***	8	***995.40***	990.59	***100.00***	100.07	3.074	2.701	***2.992***
3.142	3.654	***3.031***	9	***993.80***	991.36	99.93	***100.01***	3.060	2.739	***2.997***
3.192	3.505	***2.969***	10	***997.75***	994.80	99.97	***100.01***	3.065	2.759	***2.990***
3.034	3.220	***2.975***	20	***998.59***	995.87	100.04	***100.00***	3.062	2.905	***3.020***
2.989	3.149	***2.990***	30	***999.75***	998.42	***99.98***	99.97	3.014	2.936	***3.012***
2.978	3.051	***2.996***	100	***999.77***	1000.48	***100.00***	100.01	3.027	2.977	***3.001***

Median rank regression (MRR) is still the recommended best practice and the engineering standard as it simpler, generally unbiased and provides a good plot of the data. However, we will now recommend comparisons with the new Reduced Bias Adjusted MLE estimates of beta or sigma. They should compare well when we have a good fit. Note that the MLE eta and mu are not adjusted, as they are reasonably accurate.

There has not been time yet to thoroughly check out the method for data sets with suspensions. The preliminary results look very good as long as the N in the Reduced Bias Adjustment is the number of failures, not including suspensions. Volunteers will be welcome to do a larger study with suspensions using MonteCarloSMITH™.

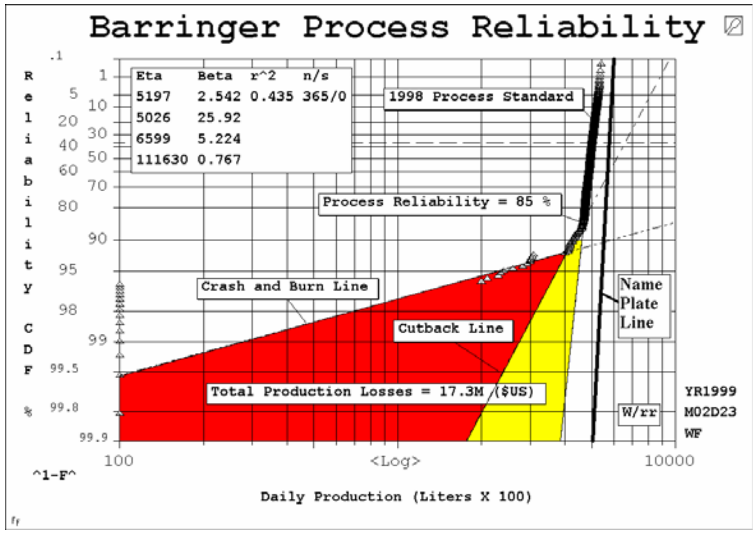
There is one concern. If this is as accurate as it appears to be, why hasn't someone suggested these methods for life data before? (Perhaps they have and I am not aware of it.) The procedure for the normal and log normal has been known for decades! J. M. Juran's text, "Quality Control Handbook," McGraw-Hill, 1951, actually tables the RBA factor for the normal as C2 and references the 1945 edition of the ASTM report listed above. It may be that with the usual normal applications for measurements we always have complete samples and calculate SQR(Var-U) directly and then apply C4. With life data we have suspensions, we cannot directly calculate the square root, and so we have to use the MLE sigma times the RBA factor for an unbiased slope estimate.

In the next printing of The New Weibull Handbook the method will be described in more detail. Wes Fulton has already added the RBA method to the WinSMITH Weibull software.

SuperSMITH™ Software Improvements **- by Wes Fulton.** The software is getting to a stage in development where I can take a breather. Both WinSMITH™ Weibull (WSW) and WinSMITH™ Visual (WSV) have been upgraded significantly to improve ease of use and capability. The executable files for both programs have been compacted more to keep the small memory footprint (10 times smaller than others). This is only possible with high code reuse and efficiency. Downloading the software from the website is a breeze. Many people have downloaded the DEMO software and then purchased the password to get FULL status. All of this has been done in a few minutes as far away as Australia and Europe. We have 100 % electronic distribution capability before Y2K (and no Y2K bug!). Some other changes are…

RBA Factor (WSW): The Method / Input selection gives modified MLE (mmle) for Weibull, normal and lognormal using the best-practice RBA developed by Dr. Bob. The RBA factor is used exclusively for Weibull mmle, while you can choose between RBA factor and (N / (N - 1)) ^ 0.5 for normal and lognormal.

Graphical Delete/Cut/Copy: The Zoom menu has graphical delete/cut/copy capability as well as an option to add points or lines on the plot graphically. This feature facilitates **Production Process Reliability**. Paul Barringer developed this method to improve production processes. Process reliability and capability can be assessed, and losses from production cutbacks and downtime (often millions of $US) are quantified.



Risk Analysis (WSW): The Abernethy risk analysis model for failure forecasting evaluates effects of scheduled replacement, variable usage rates (seasonal equipment), variable production rates, competing warranty periods, etc. for Weibull, normal, and lognormal distribution families. This special tool is not available elsewhere.

ACH (WSV): WinSMITH™ Visual can eat a WinSMITH™ Weibull data file to make an aggregate cumulative hazard (ACH) plot. Such a plot is used at Rolls Royce by Geoff Cole and David Nevelle to identify inconsistent models and subset (batch) problems.

On-graph position indicators: Precise Plot Reading provides gage boxes that move with the cursor along the plot axes for quick value estimation from the graph.

Text box on plot: The Label / Style icon provides a small note mini-icon for notation on the plot. This selection will allow a text box to be edited and then placed anywhere on the plot.

Test Requirements (WSW): The test planning section has the Sica-Luko correction factor from Charles Sica and Stephen Luko to account for late stoppage of sudden death testing.

Accelerated Testing (WSW): Parameter as Function of Engineering Variable, recommended by Wayne Nelson, has multivariate capability for up to nine independent stress variables.

Design Comparison (WSW): The set comparison selection in the calculator is unique to SuperSMITH, providing best practice for competitive design evaluation. Up to ten data sets can be compared at once using the modified likelihood ratio technique. If multiple sets are currently in the data grid, only those sets not hidden (by the Zoom menu) will be compared.

Dr. Bob would like to hear from you at weibull@worldnet.att.net, Wes can be reached at wes@weibullnews.com, and you can get free demo software, updates, seminar schedules, and tutorial info at http://www.weibullnews.com, our website. Links to other organizations and suppliers are provided to help connect you to the World of Weibull.