

# WeibullNEWS™ *Extra*

From Dr. Bob Abernethy & Wes Fulton

Spring 1996

(WinSMITH Weibull™ is Here!!...See Page 3 for details..)

**Response to the 1995 SAE Weibull Users Conference** There was much discussion about the how to determine if there is a significant difference between data sets. The attendees requested the authors research the various methods to identify best practice. We have spent much of the ensuing year on this research. The following is an abstract of the paper presented by Wes at the 1996 SAE Automotive Congress (with SAE's kind permission) to respond to the attendees request. The full paper with all the references may be obtained from SAE . It will be included in the Second Edition of The New Weibull Handbook.

## A Standard Method for Design Comparison...Is There Difference Between Data Sets?

By Wes Fulton & Dr. Bob Abernethy

**INTRODUCTION** The question is...are the two data sets from the same parent distribution or are they significantly different? The solutions to this question were researched by the authors. The two sets might be from a new design versus an old design or different geographic sources, different fleets or customers, different usage of the product or different production lots, etc.

Some methods already in wide use require the data set comparison at a particular percentile ("B level" or "B life" in Weibull parlance). The authors suggested the use of plotted likelihood ratio (LR) contours. Dr. Wayne Nelson, the Keynote Speaker at the 1995 SAE Weibull User Conference suggested the LR test. This paper is a status report on the research conducted and preliminary conclusions considering (1) accuracy and (2) ease of application.

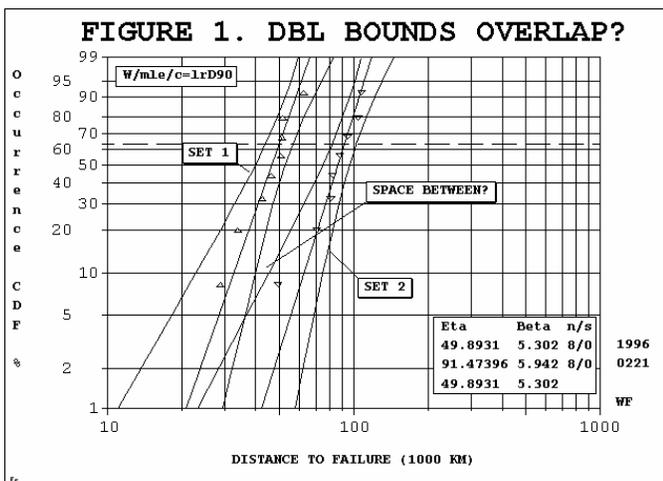
**METHODS** Parametric models and Weibull in particular are studied since they can be broadly applied and when appropriate give much greater accuracy than non-parametric models. The null hypothesis (baseline) is that the two data sets come from the same parent Weibull.

**Weibayes Methods** - It is logical to assume both sets have the same slope (beta =  $\delta$ ). There may be prior supporting experience and/or data. If this assumption is acceptable, Weibayes methods apply as described in the USAF Weibull Handbook, [Abernethy et al 1983] and The New Weibull Handbook . This technique is well established, accepted, and widely employed. Therefore Weibayes methods were not included in the current research. When  $\delta$  is known well , Weibayes is the best practice.

**B Life Methods** - Another simplifying assumption is that comparison can be done at a single B life. The widely accepted B10 level recommended by Wallodi Weibull for comparing bearings is now used at Ford for comparing vendor redesigns [Abernethy 1993]. These methods, designated "B Life Methods", are: (1) Both Median Lines Outside Bounds, (2) Double Bounds Do Not Overlap, (3) Ford Method and (4) Double Bounds on the Merged Set Do Not Include Either Median Line. B life comparisons may be required by contract or by industry practice; otherwise more general methods may be preferred.

**No Assumption - General Methods** - More general approaches avoid these assumptions on beta or B life. These methods "General Methods" are the (5) LR Test and (6) Likelihood Contour Plots. These methods should be used as best practice if the above the  $\delta$  assumption is shaky and the B life comparison is not required.

The study was conducted mainly on Weibull data sets from the same parent data with WeibullSMITH™, Visual\*SMITH™ and MonteCarloSMITH™ [6] software. The term "alike" is used here to mean "not significantly different". Ideally, results for same-parent samples should trend toward 90% alike at 90% confidence.



### B Life Methods -

1. Both Median Lines Outside Bounds. Sets are significantly different if both median Weibull lines lie outside 90% bounds of the other set at the B10 level. Values from 50-60% alike were obtained with significant bias tendency in thousands of trials. This method is not acceptable.

2. Double Bounds Do Not Overlap. Sets are significantly different if there is space between the upper bound of the lower set and the lower bound of the upper set at the B10 level. See Figure 1. Approximately 90% of the pairs are alike, if 80% double confidence is used on the plot. Alike % is biased somewhat with confidence level, sample size, choice of B level and Weibull slope differences. This method warrants additional research.

3. Ford Method Jim Lempke's Ford Method is

included in The New Weibull Handbook. Both Fisher Matrix and likelihood ratio confidence bounds make some of the same assumptions, so this method was not evaluated separately.

4. Double Bounds on the Merged Set Do Not Include Either Median Line. This method overestimates alike % by several percentage points and has some bias with respect to fit method, B life selected and sample size (slight). Bias trends are difficult to predict. This method is not acceptable.

**General Methods -**

5. LR Test This test is described fully in [Nelson 1990] and can be extended to more than two sets. The maximum log likelihood (MLL) is obtained for each set separately and merged. The sum of the two separate MLL's minus the MLL for the merged data is compared to a chi-squared critical value. Various sample sizes, confidence levels and distribution types were investigated. The standard LR test produces low alike % for small samples. It showed no confidence level bias and very little distribution type bias. Wes Fulton discovered a simple adjustment factor and created a modified LR test removing most sample size bias.

The standard LR Test (biased at small samples) uses inequality Equation (1) below.

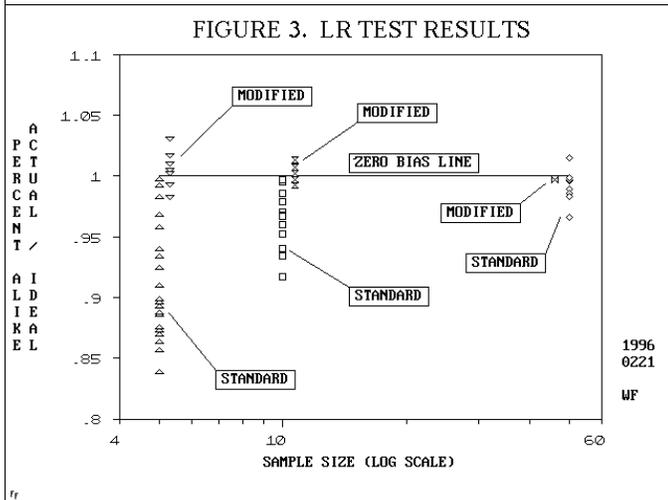
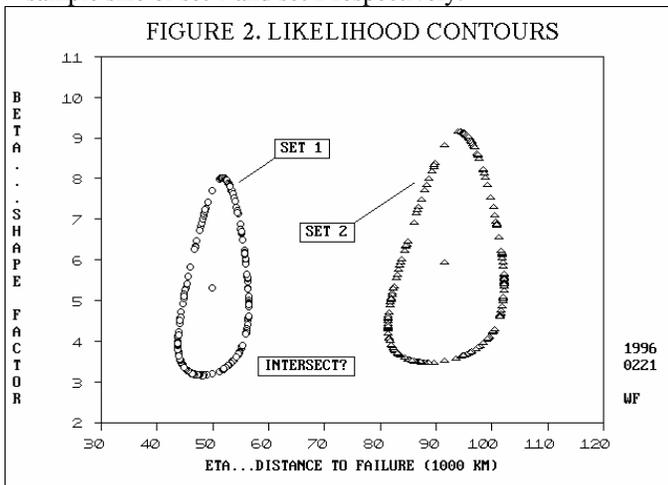
$$\text{Significant difference. When } \overset{\wedge}{L}_1 + \overset{\wedge}{L}_2 - \overset{\wedge}{L}_M > \chi_{cd}^2 / 2 \quad (1)$$

Note:  $\overset{\wedge}{L}$  = log likelihood,  $\overset{\wedge}{L}_1$ ,  $\overset{\wedge}{L}_2$ ,  $\overset{\wedge}{L}_M$  = maximums for sets 1, 2 and merged respectively, explained in [Nelson 1990].

$\chi_{cd}^2$  = Chi<sup>2</sup> at c confidence, d degrees of freedom (here d = 2). The modified LR Test uses Equation (2) below:

$$\text{Significant difference. when } (\overset{\wedge}{L}_1 + \overset{\wedge}{L}_2 - \overset{\wedge}{L}_M) \times (FF) > \chi_{cd}^2 / 2 \quad (2)$$

Note: **FF** = Fulton Factor = (N - 1) / (N + PHI), PHI = golden section = 0.618, N = composite size = 2 / (1 / N1 + 1 / N2), and N1 and N2 = sample size of set 1 and set 2 respectively.



6. Likelihood Contour Plots If the 90% likelihood contours for the parameters eta and beta do not intersect, the authors believe this indicates significant difference. See Figure 2, the contour plot for confidence in Figure 1. Note that this test and method 2 may give conflicting results. 32 data sets were sampled and cross compared with 444 out of 496 intersecting (90% alike). Preliminary investigation of sample size 4 indicate small sample bias, expected since the likelihood contour method is similar to the standard LR test.

**CONCLUSIONS**

The modified LR test using the proposed Fulton Factor is the most accurate general method found in this study. It is easy to use with WinSMITH software. Figure 3 shows averages of thousands of LR test simulations at different sample sizes.

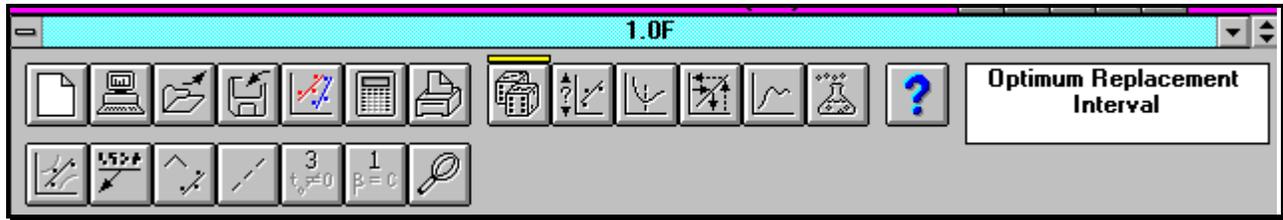
If the B life comparison is required, the fastest and easiest method by far is Double Bounds Do Not Overlap. It is somewhat biased with respect to sample size, confidence level, Weibull slope disparity and the B level chosen. More research is needed. The related Likelihood Contour method provides plotted results and works well at the confidence level and sample size analyzed. Additional investigation into different confidence levels and sample sizes for the contour method is needed.

Weibayes is best practice if the  $\delta$  is well known as it eliminates the uncertainty in one parameter.

**RECOMMENDATIONS FOR FURTHER RESEARCH**

The three most useful methods (Double Bounds Do Not Overlap, Modified LR Test and Likelihood Contour Plots) should be researched further for clarification and validation including suspensions and other distributions. The correction addressed with the Fulton Factor may also apply to LR confidence.

# WinSMITH Weibull<sup>TM</sup> is Here!!...



What is smarter than WeibullSMITH<sup>TM</sup> and also does Windows?? WinSMITH Weibull of course! Now released, WinSMITH is designed for Windows. It contains all features of our popular WeibullSMITH probability plotting software plus many exclusive best practice enhancements not available elsewhere... and it is even **easier to use than WeibullSMITH!**

- ☞ Unlimited occurrences/suspensions in each of 10 sets maximum with frequency table
- ☞ Data entry with built-in spreadsheet, ASCII file and Windows clipboard
- ☞ Weibull 2 and 3 parameter, WeiBayes (1 parameter.), Exponential, Rayleigh, Log Normal, Normal
- ☞ Rank regression X on Y or Y on X, and maximum likelihood fit methods

**These WinSMITH Weibull features are NOT AVAILABLE ELSEWHERE:**

- ☞ Kaplan-Meier analysis with Greenwood's confidence bounds
- ☞ Critical correlation coefficient (CCC) for goodness of fit
- ☞ Special methods for small samples including the new modified likelihood ratio test
- ☞ Distribution analysis based on the critical correlation coefficient (CCC)
- ☞ Reliability with confidence test substantiation for Weibull, Normal and log Normal, with allowed failures
- ☞ Optimum parts/component replacement for minimum cost
- ☞ Failure forecasting with and without renewal (risk analysis)
- ☞ Monte Carlo simulator sampling from parent populations, probit and Kaplan-Meier
- ☞ Fisher matrix and likelihood ratio confidence bounds for B life, reliability,  $\Omega$  &  $\approx$
- ☞ Sole source for the likelihood contour which is the Royal Statistics Society's "most important single statistic"
- ☞ Special methods for inspection data analysis...Sherwin's Method, and Probit analysis.
- ☞ Easy visual access to recent results with 5 plots simultaneously
- ☞ MonteCarloSMITH options to measure the bias and precision of the WinSMITH methodology
- ☞ Friendly customer technical support... call Wes 310-548-6358 or Dr. Bob 407-842-4082

WinSMITH Weibull is the only software that implements all of the Weibull analysis techniques in The New Weibull Handbook by Dr. Robert Abernethy, the standard source on Weibull analysis. It is compatible with all popular SuperSMITH<sup>TM</sup> software for DOS, WeibullSMITH<sup>TM</sup>, Visual\*SMITH<sup>TM</sup>, MonteCarloSMITH<sup>TM</sup> and BiWeibullSMITH<sup>TM</sup>. WinSMITH Weibull's \$480.00 list price (\$380.00 until April '96) is well below competition and allows for purchase of needed capability only..... Our software is developed by engineers for engineers.



**Weibull User Conferences.** (For on-site Weibull User Conferences to update your analysts call Dr. Bob 407-842-4082)

☞ The Third Annual 1996 SAE User Conference with Dr. Lawless was outstanding! Warranty analysis and Kaplan-Meier were hot topics. Everybody loved WinSMITH. The interchange between attendees and the experts was just great. We were asked to investigate some aspects of using the Kaplan-Meier method for interval analysis such as warranty analysis.

☞ ASME will host the **Second Annual ASME Weibull User Conference in Boston** June 10-11, 1996. Think about joining Wes and me as this will be an outstanding conference. Dr. Dimitri Kececioglu will be the keynote speaker. Expert Panelists include the top Weibull analysts from CHIPCOM - Joseph Dzekevich, American Nuclear Insurers - Stan Focht, Volvo of Sweden - Sten-Ake Irell, Reliability Consultant - Paul Barringer, and from the Federal Aviation Administration - Tom Boudreau. Call ASME at 212-705-7398. Book early as attendance will be limited.

**Weibull Seminars & Workshops, 1996 -1997:** (For on-site Weibull Workshops to train your engineers call Dr. Bob 407-842-4082)

☞ SAE Detroit, May 13-15, & August 27-29, 1996 Call 412-776-4841

☞ ASME Seattle, June 26-28, 1996 & Phoenix, November 20-22, 1997 Call 212-705-7398

☞ **Reliability Analysis Center** Cooperstown, NY, October 28-29, 1996, a 5 day seminar with Dr. Bob & Wes, Dr. Larry Crow and Dr. Wayne Nelson (This conference was filled early last year so book early). Call RAC at (800) 526 4803.

☞ **Gulf Publishing Sponsor at the Process Plant Reliability Conference**, Houston, Texas, September 29-October 1, 1996; and Amsterdam, Holland, November 14-16, 1996 Call 713-520-4444.

**Recommended Related Seminars** are available from Dr. Wayne Nelson (518) 346 5138, Paul Barringer (713) 852 6810, and James McLinn (612) 498 8814.

**The "Newest" Weibull Handbook:** The Second Edition of the The New Weibull Handbook will be available midyear, July 1st, at the same price. We are rewriting most of the chapters to include all of our new techniques, research, and the basis for the new options in WinSMITH. Where alternatives methods are available, best practices will be identified and there will be an Index.